Local wavefunctions in F-theory

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Local models motivation 1:

String theory currently lacks a top-down vacuum selection principle

We do not know the full landscape

An important aspect of string phenomenology is the sensitivity of low energy observables to the choice of vacuum (=assumptions)

Typically the more localised an interaction is the less sensitive and the more general the analysis

Local models motivation 2:

Calculating observables in string compactifications is hard

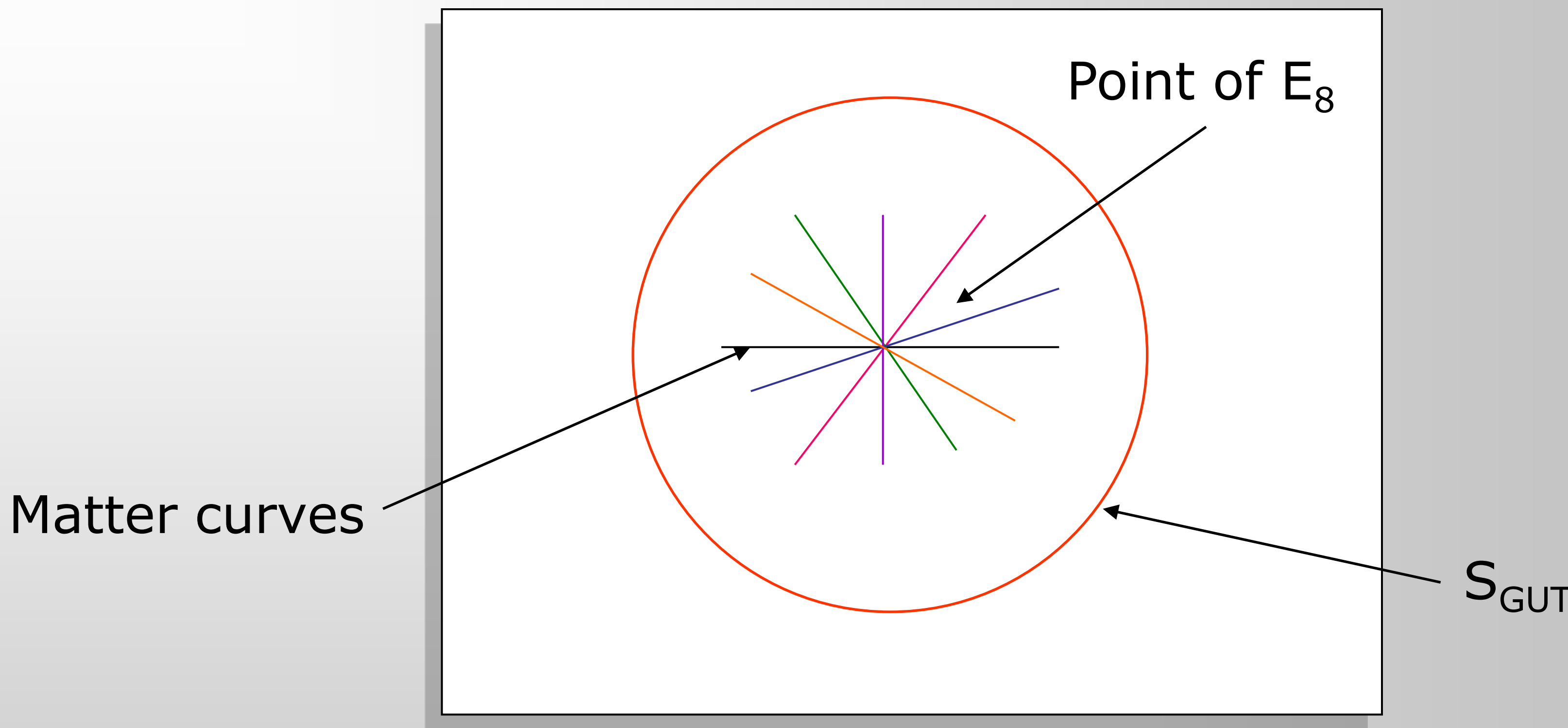
Typically there are many uncalculated "O(1)" factors in constructions – and typically the phenomenology can change significantly if these turns out to be O(100)…

Many calculations simplify considerably in a local approximation – and such an approximation is particularly good for particle physics operator coefficients

(Typically used to study Yukawa couplings in F-theory)

Heckman, Vafa, Cecotti, Ibanez, Font, Conlon, Dudas, EP, Aparicio, Marchesano, Martucci, Leontaris, King, Ross, Callaghan, …

In F-theory(/Heterotic) the ultimate manifestation of locality is the point of E_8



If such a point of E_8 (or small deformation of) exists then there is a very rich local theory around it which allows us to study many phenomenological features explicitly and generally

Talk will be report on initial exploration of the idea of such a complete local theory and the calculation of operators within it

Current description of the theory on exceptional F-theory branes is through the IR limit as canonical 8-dimensional twisted super Yang-Mills (Gauge field + complex Higgs)

$\langle \varphi_H \rangle$ Spatially varying Higgs localises matter onto curves

$\langle F \rangle$ Background flux generates chirality

Matter fields arise as fluctuations of the 8D fields

$$\begin{aligned} \mathbf{A}_{\bar{m}} &= \{A_{\bar{m}}, \psi_{\bar{m}}, \mathcal{G}_{\bar{m}}\} , \\ \mathbf{\Phi}_{mn} &= \{(\varphi_H)_{mn}, \chi_{mn}, \mathcal{H}_{mn}\} , \\ \mathbf{V} &= \{\eta, A_\mu, \mathcal{D}\} , \end{aligned}$$

$$\Psi_{8D} = \phi_{4\mathrm{D}} \times \psi_{\mathrm{int}}$$

Operator coefficients arise as overlaps of wavefunctions

$$\int_{4D\times S} \Psi^1\Psi^2\Psi^3 = \int_{4D} \phi^1\phi^2\phi^3 \left(\int_S \psi^1\psi^2\psi^3 \right)$$

Schematically the wavefunctions take the form

$$\psi \sim e^{-|\langle \phi_H \rangle|^2} e^{-\langle F \rangle}$$

Exponential localisation onto matter curves implies that it is possible to consider studying them on a local patch

Locally the most general (Abelian) Higgs and Flux backgrounds are

$$\langle \varphi_H \rangle = M_K R \, m_i^a z_i \, Q_a dz_1 \wedge dz_2 + \dots \ ,$$
$$\langle A \rangle = -M_K \, \mathrm{Im}(M_{ij}^a z_i d\bar{z}_j) Q_a + \dots \ ,$$

$$M_K = \frac{M_*}{R_{\|}} \ .$$
$$R \equiv R_{\|} R_{\perp} \ .$$

Wavefunctions are solutions to the Dirac equation $\mathbb{D}^- \Psi = 0 \ ,$

$$\mathbb{D}^{\pm} = \begin{pmatrix} 0 & D_1^{\pm} & D_2^{\pm} & D_3^{\pm} \\ -D_1^{\pm} & 0 & -D_3^{\mp} & D_2^{\mp} \\ -D_2^{\pm} & D_3^{\mp} & 0 & -D_1^{\mp} \\ -D_3^{\pm} & -D_2^{\mp} & D_1^{\mp} & 0 \end{pmatrix} , \qquad \Psi = \begin{pmatrix} \eta \\ \psi_{\bar{1}} \\ \psi_{\bar{2}} \\ \chi \end{pmatrix} ,$$

Can solve for the most general local wavefunction

$$\varphi = f\left(-\hat{\xi}_{1,2} z_1 + \hat{\xi}_{1,1} z_2\right) e^{-p_1|z_1|^2 - p_2|z_2|^2 + p_3 \bar{z}_1 z_2 + p_4 \bar{z}_2 z_1} ,$$

$$\begin{aligned}
p_1 &= \frac{1}{2} M_{11} - R m_1 \left(\frac{\hat{\xi}^*_{3,2}\hat{\xi}^*_{2,3} - \hat{\xi}^*_{2,2}\hat{\xi}^*_{3,3}}{\hat{\xi}^*_{3,1}\hat{\xi}^*_{2,2} - \hat{\xi}^*_{2,1}\hat{\xi}^*_{3,2}}\right) , \\
p_2 &= \frac{1}{2} M_{22} + R m_2 \left(\frac{\hat{\xi}^*_{3,1}\hat{\xi}^*_{2,3} - \hat{\xi}^*_{2,1}\hat{\xi}^*_{3,3}}{\hat{\xi}^*_{3,1}\hat{\xi}^*_{2,2} - \hat{\xi}^*_{2,1}\hat{\xi}^*_{3,2}}\right) , \\
p_3 &= -\frac{1}{2} M_{21} + R m_2 \left(\frac{\hat{\xi}^*_{3,2}\hat{\xi}^*_{2,3} - \hat{\xi}^*_{2,2}\hat{\xi}^*_{3,3}}{\hat{\xi}^*_{3,1}\hat{\xi}^*_{2,2} - \hat{\xi}^*_{2,1}\hat{\xi}^*_{3,2}}\right) , \\
p_4 &= -\frac{1}{2} M_{12} - R m_1 \left(\frac{\hat{\xi}^*_{3,1}\hat{\xi}^*_{2,3} - \hat{\xi}^*_{2,1}\hat{\xi}^*_{3,3}}{\hat{\xi}^*_{3,1}\hat{\xi}^*_{2,2} - \hat{\xi}^*_{2,1}\hat{\xi}^*_{3,2}}\right) .
\end{aligned}$$

$$\xi_i = \begin{pmatrix} \frac{1}{2}\left(\bar{M}_{12} + M_{21}\right)\lambda_i + R^2 m_2 \bar{m}_1 \\ \lambda_i^2 - \frac{1}{2}\left(\bar{M}_{11} + M_{11}\right)\lambda_i - R^2|m_1|^2 \\ R m_2 \lambda_i + \frac{1}{2} R m_1 \left(\bar{M}_{12} + M_{21}\right) - \frac{1}{2} R m_2 \left(\bar{M}_{11} + M_{11}\right) \end{pmatrix} ,$$

$$\begin{aligned}
-\lambda_i^3 \quad + \quad & \lambda_i \left(R^2|m_1|^2 + R^2|m_2|^2 + \frac{1}{4}\left|M_{12} + \bar{M}_{21}\right|^2 + \frac{1}{4}\left|M_{11} + \bar{M}_{11}\right|^2\right) \\
& + R^2 \,\mathrm{Re}\left[\left(M_{12} + \bar{M}_{21}\right)\bar{m}_1 m_2 + M_{11}\left(|m_1|^2 - |m_2|^2\right)\right] = 0 .
\end{aligned}$$

Wavefunction profiles along curve and a local notion of chirality

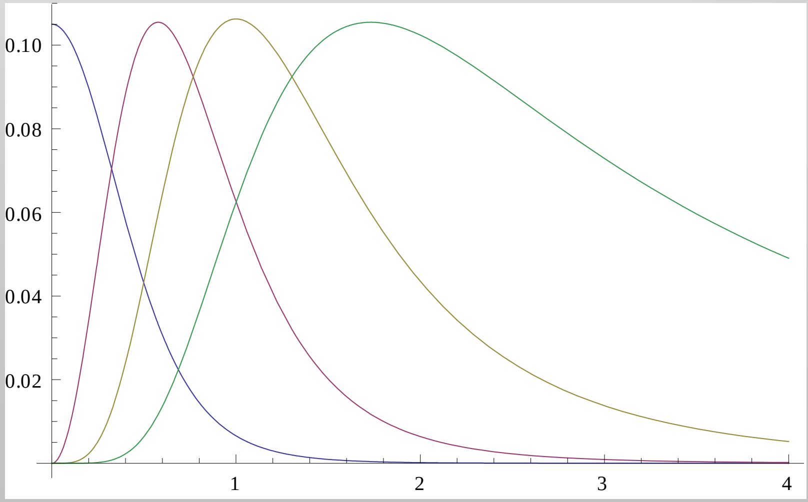
Example matter curve P^1, wavefunctions basis takes the form

$$\begin{aligned} \psi_- &= f_-(\bar{z})\,(1+z\bar{z})^{\frac{1-M}{2}} \;, \\ \psi_+ &= f_+(z)\,(1+z\bar{z})^{\frac{1+M}{2}} \;. \end{aligned}$$

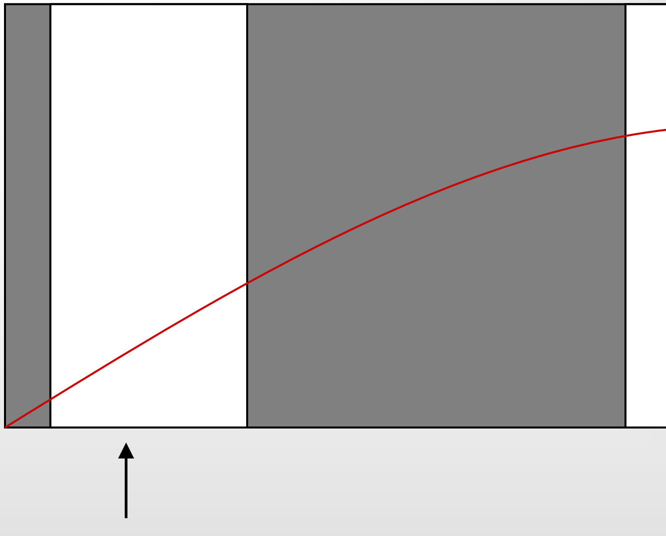
$$\chi_{\text{global}} = -\frac{1}{2\pi}\int_C F = M \;.$$

Conlon, Maharana, Quevedo

Chirality manifests in the unitary frame as the number of normalisable holomorphic polynomial prefactors



Interested in picture around a patch inside the curve where the enhancement point is



Case I: No turning point – mode is `global`

Case II: Turning point – mode is `local`

Type of mode is fixed by boundary condition and orthonormality – can not be fixed in the local theory due to arbitrary holomorphic prefactor

$$z_1 \rightarrow z_1 + m_2 a \,, \quad z_2 \rightarrow z_2 - m_1 a \,,$$

$$\varphi \rightarrow \varphi \, C_{wl}(a) \, \varphi_{wl}(a) \,, \qquad \varphi_{wl} = e^{z_1 c_1 + z_2 c_2} \,,$$

Whether a mode should be local or global is constrained by phenomenology, since operator coefficient suppressed for global modes

Expect 3rd generation + Higgs for example to be local modes to give O(1) Yukawa coupling

(Additional motivation for the point of E_8)

For `local` modes chirality is related to the local form of the flux

$$\chi_{\mathrm{local}}\left(q^a\right) = -\mathrm{Re}\left[\left(M_{12} + \bar{M}_{21}\right)\bar{m}_1 m_2 + M_{11}\left(\left|m_1\right|^2 - \left|m_2\right|^2\right)\right] \ .$$

Gives strong constraints on the fluxes for a point of E_8 - many states and few fluxes – local spectrum model building!

For example: not find models where all matter is local and generations distributed on different curves

Massive modes

Massive mode wavefunctions can also be studied locally – they are created from massless wavefunctions by raising operators

$$\tilde{D}_p^- = \sum_j \hat{\xi}_{p,j}\, D_j^- \ , \quad \tilde{D}_p^+ = \sum_j \hat{\xi}^*_{p,j}\, D_j^+ \ ,$$

$$\left[\tilde{D}_p^+ , \tilde{D}_q^-\right] = -\delta_{pq}\lambda_p \ .$$

$$\Psi_{P,(n,m,l)} = \frac{\left(i\tilde{D}_1^+\right)^n \left(i\tilde{D}_2^-\right)^m \left(i\tilde{D}_3^-\right)^l}{\sqrt{m!n!l!}\,(-\lambda_1)^{n/2}\,\lambda_2^{m/2}\lambda_3^{l/2}}\Psi_P \ ,$$

Like massless modes Massive modes come in N=4 groupings

Landau-Levels arise from flux and are N=1 like

$$\begin{aligned} M^2_{\mathbf{R}} &= M^2_K \left\{-\lambda_1, 0, \lambda_2 - \lambda_1, \lambda_3 - \lambda_1\right\} \ , \\ M^2_{\bar{\mathbf{R}}} &= M^2_K \left\{-\lambda_1, -2\lambda_1, \lambda_3, \lambda_2\right\} \ . \end{aligned}$$

KK modes are N=2 like and arise in the absence of flux $\lambda_3 \to 0$

Coupling of massive modes to massless modes is cubic and is calculated through wavefunction overlaps

$$\int_S \psi_0^1 \psi_0^2 \psi_{\rm M}^3$$

Massive modes couple to each other in separate subsectors

$$W_{ab}^M = i \int_S \mathrm{Tr}\left[\left(\Psi_a^{\bar{\mathbf{R}}}\right)^T \mathbb{D}^- \Psi_b^{\mathbf{R}}\right]$$

$$W \supset i \left(\Phi_0^{\mathbf{R}}\right)^T \cdot \mathbf{M} \cdot \Phi_0^{\mathbf{R}} \, ,$$

$$\Phi_0^{\mathbf{R}} = \begin{pmatrix} \phi_{0,(0,0,0)}^{\mathbf{R}} \\ \phi_{2,(0,1,0)}^{\bar{\mathbf{R}}} \\ \phi_{3,(0,0,1)}^{\bar{\mathbf{R}}} \\ \phi_{1,(0,1,1)}^{\mathbf{R}} \end{pmatrix} , \quad \mathbf{M} = M_K \begin{pmatrix} 0 & -(\lambda_2)^{\frac{1}{2}} & -(\lambda_3)^{\frac{1}{2}} & 0 \\ -(\lambda_2)^{\frac{1}{2}} & 0 & 0 & (\lambda_3)^{\frac{1}{2}} \\ -(\lambda_3)^{\frac{1}{2}} & 0 & 0 & -(\lambda_2)^{\frac{1}{2}} \\ 0 & (\lambda_3)^{\frac{1}{2}} & -(\lambda_2)^{\frac{1}{2}} & 0 \end{pmatrix} .$$

An example model and calculation

$$E_8 \supset SU(5)_{GUT} \times SU(5)_{\perp} \rightarrow SU(5)_{GUT} \times U(1)^4$$

$$\mathbf{248} \rightarrow (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus (\mathbf{10},\mathbf{5}) \oplus (\bar{\mathbf{5}},\mathbf{10}) \oplus (\bar{\mathbf{10}},\bar{\mathbf{5}}) \oplus (\mathbf{5},\bar{\mathbf{10}}) \ ,$$

First need to find fluxes and Higgs values compatible with chirality of the MSSM

Higgs	Value	Flux	Value
m_1^1	1	m_2^4	$-\frac{4}{5}$
m_1^2	$e^{2\pi i/5}$	m_2^5	-1
m_1^3	$e^{4\pi i/5}$	M_{11}^1	$-\frac{11}{5}$
m_1^4	$e^{6\pi i/5}$	M_{11}^2	$\frac{14}{5}$
m_1^5	$e^{8\pi i/5}$	M_{11}^3	$-\frac{11}{5}$
m_2^1	$-\frac{3}{5}$	M_{11}^5	$\frac{8}{5}$
m_2^2	$\frac{7}{5}$	M_{11}^Y	$-\frac{9}{5}$
m_2^3	1		

This implies that the following modes can be taken as local

Representation	χ_{local}
$\left((3,2)_{1/6} \oplus (\bar{3},1)_{-2/3} \oplus (1,1)_1\right) \otimes \mathbf{q}_1$	+1
$\left((3,2)_{1/6} \oplus (\bar{3},1)_{-2/3} \oplus (1,1)_1\right) \otimes \mathbf{q}_2$	+1
$\left((3,2)_{1/6} \oplus (\bar{3},1)_{-2/3} \oplus (1,1)_1\right) \otimes \mathbf{q}_3$	0
$\left((\bar{3},1)_{1/3} \oplus (1,2)_{-1/2}\right) \otimes (-\mathbf{q}_1 - \mathbf{q}_3)$	-1
$\left((\bar{3},1)_{1/3} \oplus (1,2)_{-1/2}\right) \otimes (-\mathbf{q}_4 - \mathbf{q}_5)$	-1
$(3,1)_{-1/3} \otimes (-\mathbf{q}_1 - \mathbf{q}_2)$	0
$(1,2)_{+1/2} \otimes (-\mathbf{q}_1 - \mathbf{q}_2)$	+1
$(3,1)_{-1/3} \otimes (-\mathbf{q}_2 - \mathbf{q}_3)$	0
$(1,2)_{+1/2} \otimes (-\mathbf{q}_2 - \mathbf{q}_3)$	-1

Example calculation of dimension 5 proton decay in presence of Pecci-Quinn U(1)

$$W \supset \hat{Y}_u T_u^{KK} Q_1 Q_2 + \hat{Y}_d T_d^{KK} Q_1 L_1 + \hat{Y}_X X_1 \bar{T}_u^{KK} \bar{T}_d^{KK} ,$$

$$W \supset \frac{\hat{Y}_u \hat{Y}_d \hat{Y}_X}{M_u M_d} X_1 Q_1 Q_2 Q_1 L_1 .$$

Interested in relation to exotics mass

$$W \supset Y_{X_1 E_1 E_2} X_1 E_1 E_2 .$$

The wavefunctions are determined by the flux and Higgs, after…

$$\begin{aligned} \hat{Y}_u &\simeq 5 \times 10^{-4} , \; \hat{Y}_d \simeq 2 \times 10^{-4} , \\ \hat{Y}_X &\simeq 3 \times 10^{-4} , \; Y_{X_1 E_1 E_2} \simeq 5 \times 10^{-3} , \end{aligned}$$

$$\frac{\hat{Y}_u \hat{Y}_d \hat{Y}_X}{Y_{X_1 E_1 E_2}} \simeq 10^{-8} ,$$

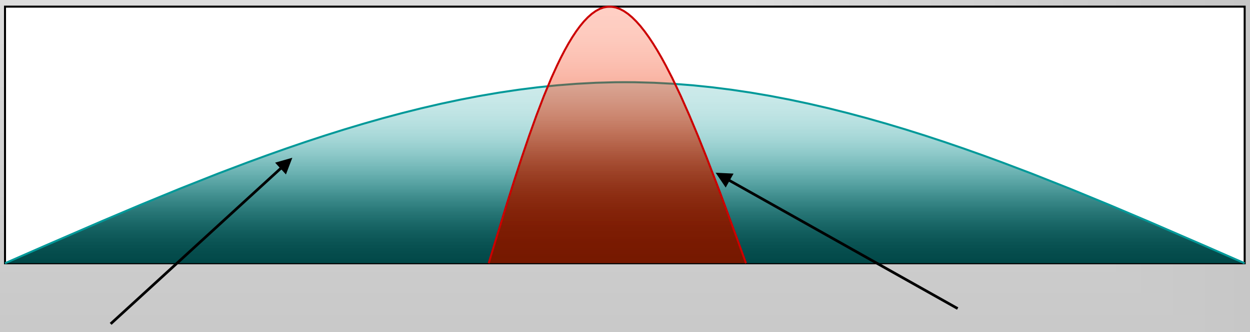
But in the model also find small Yukawa couplings

$$Y_{H_D Q_1 D_1} Y_{H_U Q_1 U_2} \simeq 10^{-5} .$$

Still more supression then would expect from 4D field theory…

General issue:

It is difficult to generate O(1) couplings because wavefunctions are not localised strongly enough by the flux



Localised by flux

Localised by Higgs

Normalisation integral dominates overlap integral – effective suppression by modular weight

General issue:

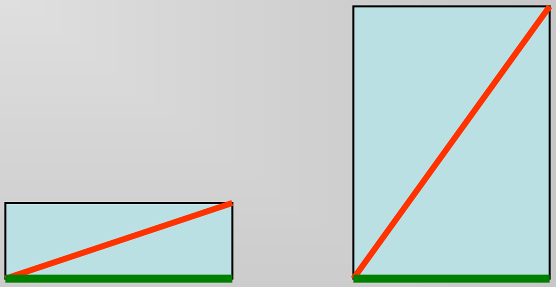
The effective theory is generally not compatible with the decoupling limit!

To keep higher derivative operators under control require

$$\frac{M_{ij}}{R_{\|}^2} \ll 1 \,, \quad \frac{m_i R_\perp}{R_{\|}} \ll 1 \,.$$

$$R_\perp \sim \left(\frac{M_{\text{Planck}}}{M_*} \right)^p$$

Torodial intuition: large intersection angles



Although the holomorphic sector is unaffected the Kahler potential receives large corrections

Must be difference between local normal scale and global one

Summary

Local models around enhanced symmetries (most complete being based on point of E_8) are an interesting set of constructions in F-theory(/Heterotic)

They offer, potentially, highly calculable, general, and predictive theories with few input parameters

Toy models show that many coefficients that by symmetries alone would be of order 1 can be much smaller/larger – may shed new light on some longstanding phenomenological issues

Still some way to go for realistic models, future difficulties likely to be related to differential rather than algebraic geometry

Predictivity

In principle every coupling in the theory can be calculated by a wavefunction overlap integral which receives the bulk of its contribution from a small local patch

Only a few local parameters that are projections of a large set of global parameters

Generality

Only assumes a point of E_8 enhancement (or close to it): potentially applies to a large number of vacua

(some pheno motivation from CKM relating up and down sector)

Talk to outline some of the important concepts and constraints that arise in thinking about such a local theory

The road to a realistic local model is still long:

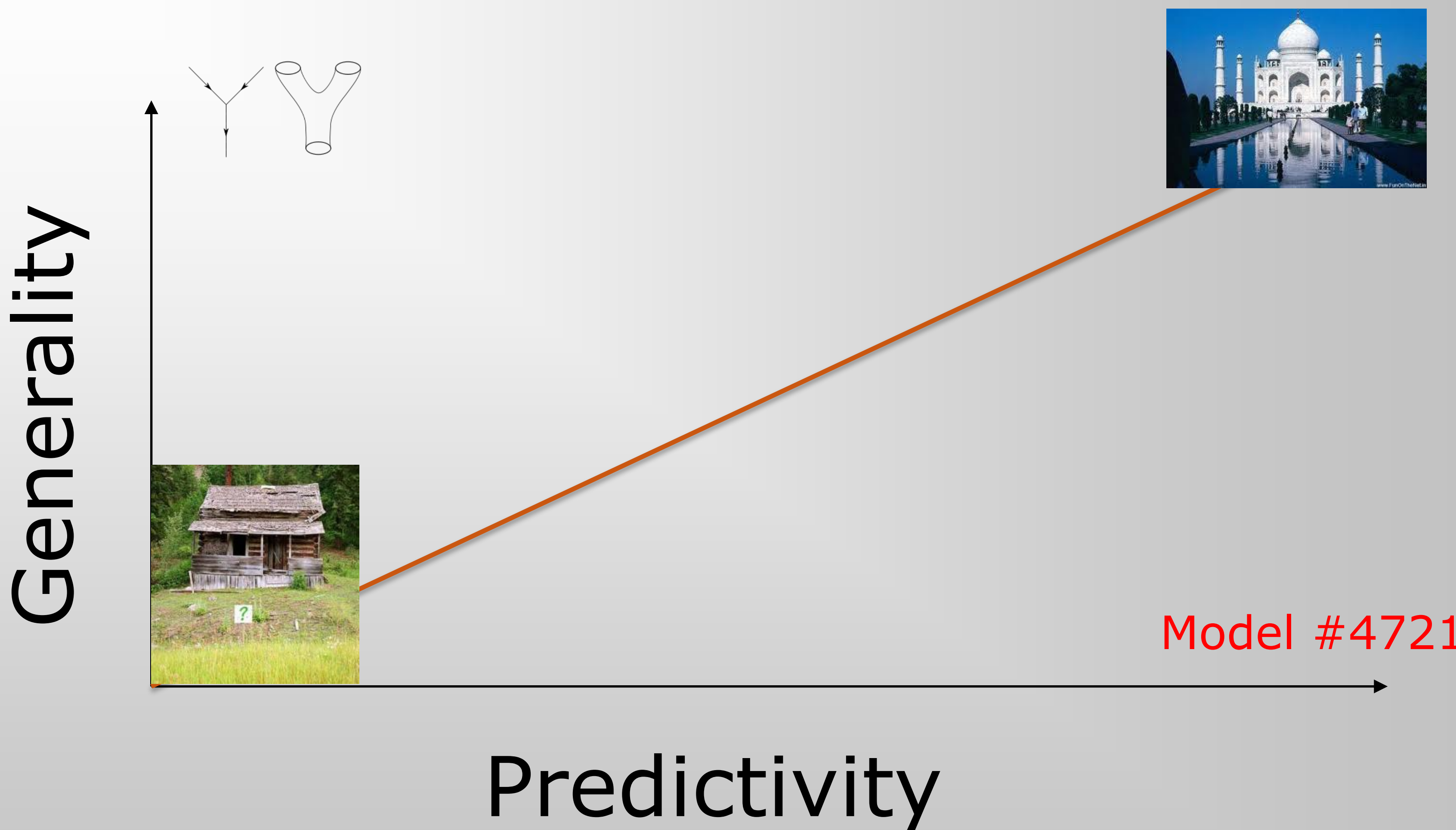
- Find parameter space within valid effective theory that recreates order 1 Yukawa coupling

- Incorporate monodromies: numerical methods?

- Incorporate non-commutative deformations?

- Understand contributions of leading corrections to metric, flux and Higgs from compact embedding (related to accuracy of normalisation integrals)

Motivation: Two great challenges for string model building



Motivation: much of the difficulty relates to the geometry of the compact space - locality as a tool to make headway?

Local models motivation 1:

String theory currently lacks a top-down vacuum selection principle

We do not even know the full landscape

An important aspect of string phenomenology is the sensitivity of low energy observables to the choice of vacuum (=assumptions)

Typically the more localised an interaction is the less sensitive

Cosmology   Susy breaking   Spectrum   Operators

→

Everything   Local open string sector

The amount of experimental constraints also increases with arrow