Gauge fluxes and M5-instantons in F-theory and their Type IIB duals

- 1109.3454 (NPB) with **S. Krause, C. Mayrhofer**
- 1202.3138 with **S. Krause, C. Mayrhofer**
- 1205.4720 with **M. Kerstan**

Timo Weigand

Institute for Theoretical Physics, University of Heidelberg

Introduction

Investigating local and global aspects of F-theory goes hand in hand

✓ Many phenomena governed purely by local data

cf. talks by Vafa, Palti,...

✓ Other important ingredients in **model building** depend on **global aspects of 4-fold geometry**, e.g.

- abelian gauge symmetries and their selection rules
- gauge fluxes
- instanton effects for moduli stabilisation or generation of couplings

see also talks by Halversson, Cvetič, Marchesano

Outline of this talk:

1) Resolution of singularities for $SU(5)[\times U(1)_X]$ models

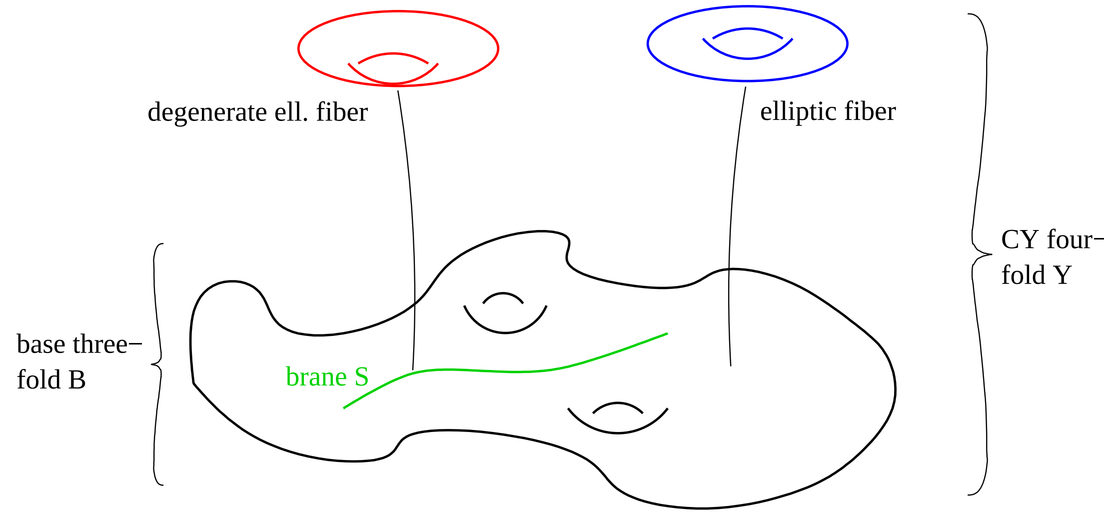
2) Construction of gauge fluxes and use for model building

3) Comparison with Type IIB limit

4) Selection rules for M5-instanton superpotentials

The framework

Elliptic fibration $Y_4 : T^2 \to B$



$$y^2 = x^3 + f(u_i)xz^4 + g(u_i)z^6$$

- fibre coordinates $(x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z)$
- B_6 coordinates: u_i
- singular over $\Delta \equiv 4f^3 + 27g^2 = 0$

- Example: **SU(5) singularity along $w = 0$ in Tate form**:

$$P_T : y^2 + a_1 xyz + a_3 yz^3 = x^3 + a_2 x^2 z^2 + a_4 xz^4 + a_6 z^6$$

$$a_1 = a_1, \qquad a_2 = a_{2,1}\, w, \qquad a_3 = a_{3,2}\, w^2, \qquad a_4 = a_{4,3}\, w^3, \qquad a_6 = a_{6,5}\, w^5$$

$$\Delta = w^5 \cdot \Delta'$$ [Bershadsky et al.'96]

- **Extra $U(1)$s** often desirable for phenomenology

  Simplest example: $a_6 \equiv 0 \leftrightarrow SU(5) \times U(1)_X$ [Grimm,TW '10]

  $\Longrightarrow$ extra curve of SU(2) singularities at $a_3 = 0 = a_4$

G_4-Fluxes - Overview

Gauge fluxes described by $\mathbf{G_4 \in H^{2,2}(Y_4)}$ with **'1 leg along fiber'**

a) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge D_b = 0$ b) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge Z = 0$ $\forall D_i \in H^2(B)$, Z: fibre

Construction requires detailed knowledge of geometry of 4-fold Y_4

$$\mathbf{H^{2,2}(Y_4) = H^{2,2}_{\text{vert}}(Y_4) \oplus H^{2,2}_{\text{hor}}(Y_4)}$$

- $\mathbf{H^{2,2}_{\text{vert}}(Y_4)}$ generated by elements of $H^{1,1} \wedge H^{1,1}$: factorisable fluxes

  $\Longleftrightarrow$ extra 2-forms obtained by **resolution of singularities**

  - fluxes associated with massless U(1)s [Grimm,TW '10], [Braun,Collinucci,Valandro '11], [Krause,Mayrhofer,TW'11],[Grimm,Hayashi'11]

  - extra special fluxes, e.g. 'spectral cover' fluxes [Marsano,Schäfer-Nameki'11]

- $\mathbf{H^{2,2}_{\text{hor}}(Y_4)}$: non-factorisable fluxes [Braun,Collinucci,Valandro '11], [Krause,Mayrhofer,TW'11]

Singularity resolutions - Overview

- make possible definition of effective action see talk by Grimm
- simplify construction of gauge fluxes

1) **Complete resolution via toric methods** for 4-folds over specific torically embedded base B

[Blumenhagen,Grimm,Jurke,TW'09] [Grimm,Krause,TW'09] [Collinucci,Savelli'10], [Chen,Knapp,Kreuzer,Mayrhofer'10] [Knapp,Kreuzer,Mayrhofer,Walliser'11]

2) **Explicit handle on details of matter curves and Yukawa points** via independent approaches

- Algebraic resolution via blow-ups and small resolutions for arbitrary base [Esole,Yau'11] [Marsano,Schäfer-Nameki'11] [Braun,Collinucci,Valandro '11] [Collinucci, Savelli'12] [Küntzler,Schäfer-Nameki'12] [Tatar,Walters'12]
- Algebraic resolution by sequence of blow-ups for arbitrary base B [Krause,Mayrhofer,TW'11] [Krause,Mayrhofer,TW'12]
- Refinement of toric resolution [Grimm,Hayashi'11]

Resolution of SU(5) singularities

Analytic resolution of singularities [Krause,Mayrhofer,TW'11]

- inspired by Tate algorithm [Bershadsky et al.'96]
- proceeds by sequence of blowups
- valid for arbitrary base B

1) Toy model SU(2):

$$P_T : y^2 + a_1 xyz + a_{3,1} yz^3 w = x^3 + a_{2,1} x^2 z^2 w + a_{4,1} xz^4 w + a_{6,2} z^6 w^2$$

- Y_4 singular at point $x = 0 = y$ over W : $w = 0$: $\quad P = 0 = dP$
- Introduce 1 new coordinate e_1 and 1 scaling relation

  $x = \tilde{x} e_1, \quad y = \tilde{y} e_1, \quad w = \tilde{w} e_1 \qquad (\tilde{x}, \tilde{y}, \tilde{w}, e_1) \simeq (\lambda \tilde{x}, \lambda \tilde{y}, \lambda \tilde{w}, \lambda^{-1} e_1)$
- $(\tilde{x}, \tilde{y}, \tilde{w})$ cannot vanish due to extra scaling relations

  singularity at $(x, y, w) = (0, 0, 0)$ replaced by divisor $e_1 = 0$
- **Resolved 4-fold** $\hat{Y}_4$ given by Proper transform (drop tilde):

  $P_T : y^2 + a_1 xyz + a_{3,1} yz^3 w = x^3 e_1 + a_{2,1} x^2 z^2 w\, e_1 + a_{4,1} xz^4 w + a_{6,2} z^6 w^2$

Resolution of SU(5) singularities

2) The real thing: SU(5) [Krause,Mayrhofer,TW'11]

$P_T : y^2 + a_1 xyz + a_{3,2} yz^3 w^2 = x^3 + a_{2,1} x^2 z^2 w + a_{4,3} xz^4 w^3 + a_{6,5} z^6 w^5$

Sequence of 4 blow-ups: 4 new coordinates e_i and scaling relations

- $(x, y, w) \to (x e_1 e_4 e_2^2 e_3^2, y e_1 e_4^2 e_2^2 e_3^3, w e_1 e_2 e_3 e_4)$
- Scaling relations yield sets of not simultaneously vanishing coordinates

3) SU(5) × U(1)$_X$

If $a_6 \equiv 0$: $SU(2)$ singularity remains over curve

$$C_{34} = \{a_{3,2} = 0\} \cap \{a_{4,3} = 0\}$$

1 more blow-up [Grimm,TW '10],[Krause,Mayrhofer,TW '11]

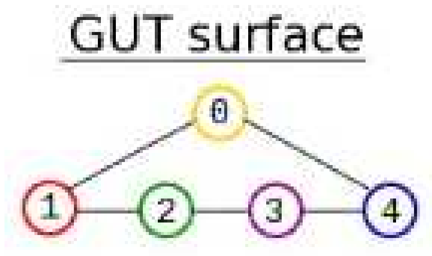
- $y = \tilde{y} s, \qquad x = \tilde{x} s \qquad (\tilde{x}, \tilde{y}, s) \simeq (\lambda^{-1}\tilde{x}, \lambda^{-1}\tilde{y}, \lambda s)$
- $y^2\, s\, e_3\, e_4 + a_1\, x\, y\, z\, s + a_{3,2}\, y\, z^3\, e_0^2\, e_1\, e_4 =$
  $x^3\, e_1\, e_2^2\, e_3\, s^2 + a_{2,1}\, x^2\, z^2\, e_0\, e_1\, e_2\, s + a_{4,3}\, x\, z^4\, e_0^3\, e_1^2\, e_2\, e_4 \qquad e_0 \equiv w$
- **new section** $S : s = 0$ with $\mathbb{P}^1$ pasted in above C_{34}

Structure of resolved fiber

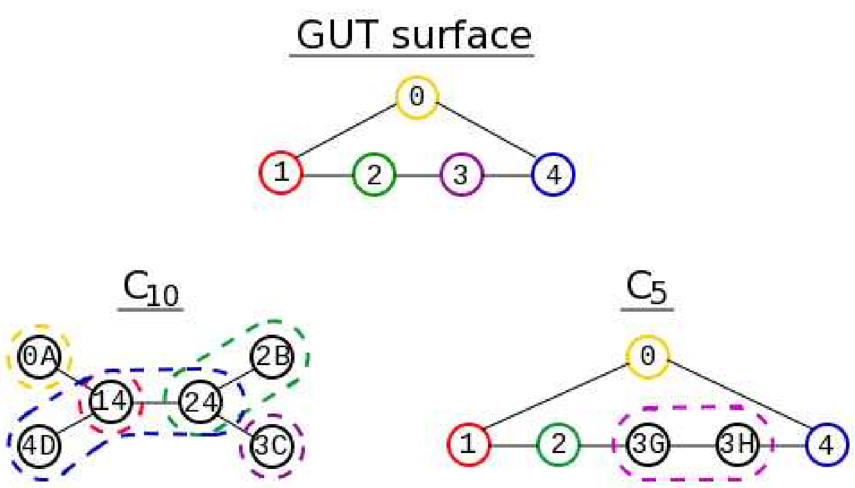
Over divisor: intersecting $\mathbb{P}^1_i$ - 1 for each resolution divisor $E_i : e_i = 0$

$$\mathbb{P}^1_i = \{e_i\} \cap \{P_T|_{e_i=0}\} \quad \cap \quad \{y_a\} \cap \{y_b\}, \qquad i = 0, \ldots, 4$$

⇒ affine Dynkin diagram of SU(5)
(together with $\mathbb{P}^1_0$)



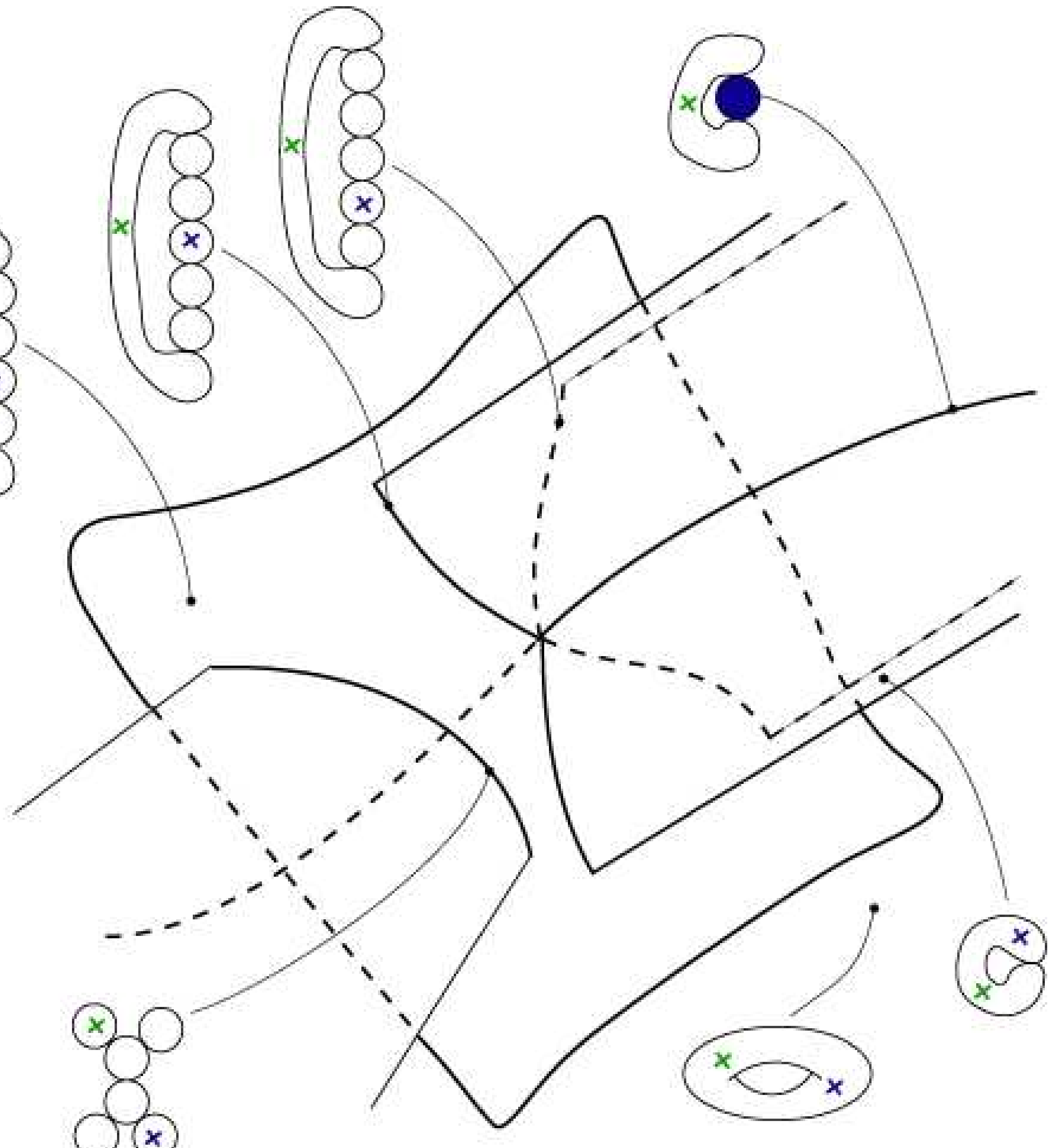
Over certain curves some $\mathbb{P}^1_i$ split → Dynkins of higher groups



Each weight β^k of repr. R ↔ surface C^k_R fibered over curve $\mathcal{C}_R \subset B$

Structure of resolved fiber

[Krause,Mayrhofer,TW'11]



Gauge flux from $U(1)_X$

Extra resolution S gives rise to **extra $U(1)_X$ not contained in SU(5)**

$$\mathsf{w}_X = S - Z - \bar{\mathcal{K}} + \sum t_i E_i \in H^{1,1}(\hat{Y}_4) \rightarrow C_3 = A_X \wedge \mathsf{w}_X$$

$$\text{repr. } R: \quad \mathbf{10}_1, \quad \mathbf{5}_3, \quad \mathbf{5}_{-2}, \quad \mathbf{1}_5 \qquad \mathbf{q_R} = \int_{\mathbb{P}_\mathbf{R}} \mathsf{w}_\mathbf{X}$$

Associated with $U(1)_X$: **Gauge flux G_4** [Grimm,TW' 10]

[Braun,Collinucci,Valandro '11] [Krause,Mayrhofer,TW '11] [Grimm,Hayashi'11]

$$C_3 = A_X \wedge \mathsf{w}_X \Longrightarrow \mathbf{G_4 = F_X} \wedge \mathsf{w}_\mathbf{X}, \qquad F_X \in H^{1,1}(\hat{Y}_4) \cap H^2(B_3)$$

Importance of G_4 flux: induces chirality for charged matter

$$\chi(R) = \int_{C_R} G_4 = \int_{C_R} F_X \wedge \mathsf{w}_X = q_R \int_{\mathcal{C}_R} F_X \qquad \mathbf{q_R} = \int_{\mathbb{P}_\mathbf{R}} \mathsf{w}_\mathbf{X}$$

✓ computation of charged singlets away from SU(5) GUT brane

✓ result accounts for corrections to semi-local split spectral cover due to global features

Flux consistency conditions

- Quantisation: $G_4 + \frac{1}{2} c_2(\hat{Y}_4) \in H^4(\hat{Y}_4, \mathbb{Z})$ [Collinucci,Savelli'10&'12]
- D3/M2 tadpole: $N_{M_2} + \frac{1}{2} \int_{\hat{Y}_4} G_4 \wedge G_4 = \frac{1}{24} \chi(\hat{Y}_4)$

$$\frac{1}{2} \int_{\hat{Y}_4} G_4 \wedge G_4 = \int_{B_3} F_X \wedge F_X \wedge (15\,[W] - 25\,c_1(B_3))$$

[Krause,Mayrhofer,TW'11]

- F-term condition: $G_4 \in H^{2,2}(\hat{Y}_4)$ ✓
- D-term condition: from detailed analysis of F/M- theory effective action [Grimm '10] [Grimm,Kerstan,Palti,TW '11]

$$D_X = -\frac{2}{\mathcal{V}_B} \int_{\hat{Y}_4} J \wedge G_4 \wedge \mathrm{w}_X = \int_{B_3} J \wedge F_X \wedge (15\,[W] - 25\,c_1(B_3))$$

✓ explicit example of 3-generation model on concrete 4-fold

[Krause,Mayrhofer,TW '11]

✓ classification of fluxes and explicit match with Type IIB fluxes

[Krause,Mayrhofer,TW '12]

Fluxes in F-theory vs. Type IIB

Why bother about Type IIB limit?

♡ We never stopped loving Intersecting Branes!

♡ It is useful to check if brane intuition correct also in F-theory.

Sen limit of U(1) restricted Tate model: [Krause,Mayrhofer,TW '12]

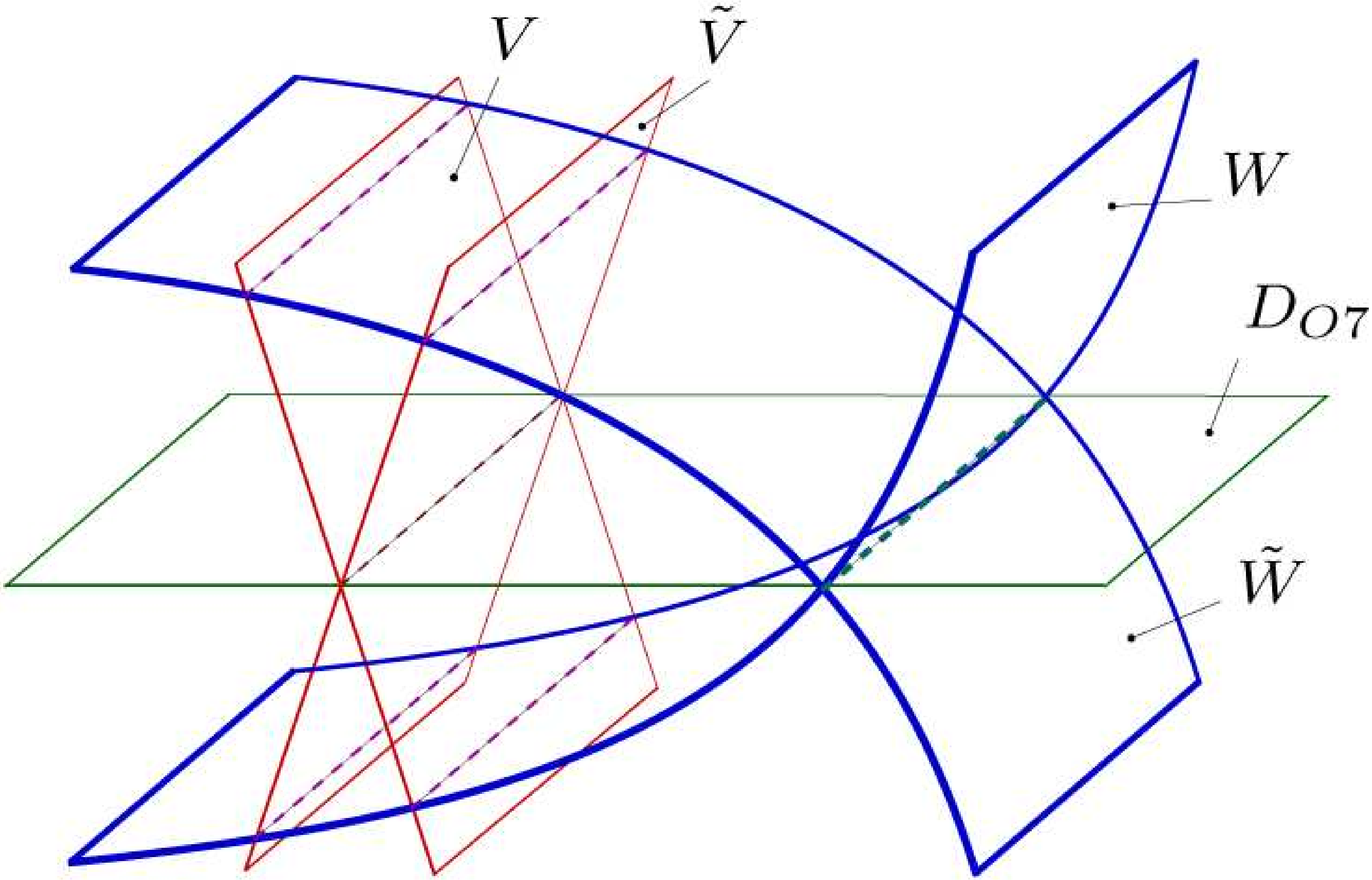
- Type IIB on double cover X_3 such that $B_3 = X_3/\sigma$ $\quad \pi : X_3 \to B$
- Existence of smooth limit typically obstructed by conifold singularity due to E_6 point [Donagi,Wijnholt, '09]
- Match with Type IIB requires topological constraints such that no such singularity is present
- $U(5)_a \times U(1)_b$ model with 2 brane/image-brane stacks:

$$5\mathcal{W} + 5\tilde{\mathcal{W}} \qquad\qquad V + \tilde{V}$$

$$V = 4\,D_{O7} - \left[2\mathcal{W} + 3\tilde{\mathcal{W}}\right] \qquad \mathcal{W}_+ \equiv \mathcal{W} + \mathcal{W} = \pi^* \mathcal{W} \qquad D_{O7} = \pi^*(\bar{K})$$

$\mathcal{W}_- = \mathcal{W} - \mathcal{W}$ constrained by absence of symmetric reps $\leftrightarrow$ smooth W in F-theory $\mathcal{W}_-^2 = \mathcal{W}_+^2 - 2D_{O7}\mathcal{W}$

Fluxes in F-theory vs. Type IIB



Fluxes in F-theory vs. Type IIB

Comparison of abelian gauge factors $U(5)_a \times U(1)_b$

- $U(1)_a$ and $U(1)_b$ massive by Stückelberg in absence of gauge flux
- only $U(1)_X = \frac{1}{5}(U(1)_a - 5U(1)_b)$ is massless
- comparison of $U(1)_X$ charges confirms: $U(1)_X$ is the U(1) in F-theory

Comparison of fluxes

$$G_4^X = \mathcal{F} \wedge \mathrm{w}_X \quad \longleftrightarrow \quad F_a = \mathcal{F}, \quad F_b = -5\mathcal{F} \qquad \mathcal{F} = \pi^* F$$

What about further fluxes in IIB?

- D5-tadpole generically allows only 1 more flux:
  F_λ: $F_a = \frac{\lambda}{5} D_{O7}, \quad F_b = 0 \qquad \leftrightarrow \qquad$ massive $U(1)_a$
- In F-theory: global version of universal spectral cover fluxes
  $G_4^\lambda = \quad \lambda \left(E_2 \wedge E_4 + \frac{1}{5}(2,-1,1,-2)_i E_i \wedge \mathcal{K}\right)$ [Marsano,S-Nameki '11]
- G_4^λ and G_4^X are the only possible factorisable gauge fluxes.
  These exhaust the generically possible $U(1)$ fluxes in Type IIB

Fluxes in F-theory vs. Type IIB

Massive U(1)s $\leftrightarrow$ non-harmonic 2-forms w_a

[Grimm,TW '10],[Grimm,Kerstan,Palti,TW '11]

✓ $U(1)_a$ flux still described by harmonic 4-form

✓ in agreement with proposal of [Grimm,TW '10],[Grimm,Kerstan,Palti,TW '11]

D-term:

- massless $U(1)_X = \frac{1}{5}(U(1)_a - 5U(1)_b)$ realised in F-theory
  D-terms match with Type IIB
- massive $U(1)_{X'} = \frac{1}{2}\,(5\,U(1)_a + U(1)_b)$:
  no D-term in F-theory, but in Type IIB

$$\xi_{X'} \simeq \frac{1}{2\mathcal{V}_X}\Big(5\int_{X_3} D_a \wedge J \wedge (F_a + B^-) + \int_{X_3} D_b \wedge J \wedge (F_b + B^-)\Big).$$

- Interpretation:
  B^- massive in F-theory and adjusts itself such that D-term is satisfied

M5-instantons

D3/M5-instantons in context of F-theory model building include

[Blumenhagen,Collinucci,Jurke'10] [Cvetič,Etxebarria,Halversson'10&'11]

[Donagi,Wijnholt'10] [Marsano,Saulina,Nameki'11] [Grimm,Savelli'11]

$\rightsquigarrow$ see talks by Halversson, Cvetič, Marchesano

[Kerstan,TW'12] analyses

1) M5-partition function in M/F-theory and D3-instanton limit

2) M5-instantons in presence of gauge flux

ad 1) M5-partition function see Kerstan's talk

- M5-instanton wraps vertical divisor $D_M = \pi^{-1} D_M^{\mathrm{b}} \subset \hat{Y}_4$
- hosts 2-form $\mathcal{B}$ with self-dual 3-form field strength $*\mathcal{H} = i\mathcal{H}$

  [Pasti,Sorokin,Tonin '96&'97]
- Auxiliary action [Witten'96]

  $S_{M5} = 2\pi(\mathrm{Vol}_{D_M} + i \int C_6) + S_{\mathcal{B}}, \quad S_{\mathcal{B}} = -2\pi \int \left[\mathcal{H} \wedge * \mathcal{H} - 2i\, \mathcal{H} \wedge C^- + \frac{1}{4} C_3 \wedge * C_3\right]$

M5-instantons

- Of physical interest for superpotential is the partition function

$$W = e^{-2\pi(\mathrm{Vol}_{D_M} + i\int C_6)} \int \mathcal{DB}\, e^{-S_{\mathcal{B}}}$$

classical + quantum piece: $\int \mathcal{DB} \to \sum_{\mathcal{H}} \int d\delta B$

- Evaluation of $\sum_{\mathcal{H}}$ possible [Witten'96],[Henningson,Nilsson,Salomonson'99]

- Evaluation in context of F-theory in [Kerstan,TW'12] $\rightsquigarrow$ see Kerstan's talk

✓quantitative match with fluxed $O(1)$ E3-instantons in Type IIB
as introduced in [Grimm,Kerstan,Palti,TW'11]

✓sum over $\mathcal{H}$-flux $\leftrightarrow$ sum over E_3-flux $\mathcal{F}_E$

✓allows us to fix spin structure of M5-instanton

$$W^{cl.}_{M5} = e^{-2\pi(\mathrm{Vol}_{M5} + i\int C_6)} \mathcal{Z}[{}^{\alpha}_{\beta}]$$

$$\mathcal{Z}[{}^{\alpha}_{\beta}] = e^{\frac{\pi}{2} b^2 C^M_- Z_{MN}(C^N_- - C^N_+)} \times$$

$$\sum e^{i\pi\left((k+\alpha)_M \overline{Z}^{MN}(k+\alpha)_N + 2(k+\alpha)_M(\beta^M - ibC^M_-)\right)} \qquad \alpha = 0 = \beta$$

M5-instantons and gauge flux

ad 2) Flux dependence via $S_{\mathcal{B},G_4} = S_{\mathcal{B}} - 2\pi i \int \mathcal{B} \wedge \iota^* G_4$

Superpotential without 'charged matter operators' $\Longleftrightarrow \iota^* G_4 \stackrel{!}{=} 0$:

- $\int d\mathcal{B}\, e^{2\pi i \int \mathcal{B} \wedge \iota^* G_4} \simeq \delta(G_4)$ [Donagi,Wijnholt'10]
- $\iota^* G_4 \neq 0$ implies Freed-Witten anomaly
  e.g. [Marsano,Saulina,Schäfer-Nameki'11; Grimm,Savelli'11]

Quantitative criterion: **compute $\int_{C_{(4)}} G_4$ with $C_{(4)} \in H_4(D_M)$**

Two types of relevant surfaces $C_{(4)} \in H_4(D_M)$ [Kerstan,TW'12]

1) $\forall\, U(1)_A\ \exists$ surface with dual class $[C^A_{(4)}] \in H^{2,2}_{\text{vert.}}(\hat{Y}_4)$

- $[C^A_{(4)}] = -D_M \wedge \mathsf{w}_A \in H^{2,2}_{\text{vert.}}(\hat{Y}_4)$ $\qquad C_3 = A_A \wedge \mathsf{w}_A$
- $q_A = \int_{C^A_{(4)}} \iota^* G_4 = -\int_{\hat{Y}_4} D_M \wedge \mathsf{w}_A \wedge G_4 \leftrightarrow U(1)_A$ shift of C_6
- This is **sensitive to linear combination of zero modes** with both brane stacks!

M5-instantons and gauge flux

We expect as many constraints as individual brane stacks

2) Proposal

- If $D_M \cap \mathcal{W} \neq 0$ **extra surfaces** exist which are **not in** $H^{2,2}_{\text{vert}}(\hat{Y}_4)$
- These are algebraic only for special complex structure.

  `in different contexts: [Braun,Collinucci,Valandro'11;Collinucci,Savelli'12]`

Evidence:

- Suppose $D_M \cap \mathcal{W} \subset C_{10}$ (10-matter surface) $\Longrightarrow$ take surface $C_{10}|_{D_M}$

  ✓ $\int_{C_{10}|_{D_M}} \iota^* G_4 \neq 0$ yields correct $U(1)_a$ charge in IIB limit!

  ✓ measures effect of intersection with $SU(5)$ stack individually

- If $D_M \cap \mathcal{W}$ not in C_{10}, one can deform intersection locus, possibly on auxiliary space defined in `[Kerstan,TW'12]`

Conclusion:

Consistent with intuition about zero modes even in absence of IIB dual

Conclusions

✓ **Explicit resolution of singularities gives a handle on gauge fluxes**

✓ **Relation to Type IIB demystified**

✓ **Structure of partition function and of selection rules for M5-instantons agrees with Type IIB intuition**

Next steps:

- Better understanding of non-generic fluxes
- Vertex operators/microscopic picture for M5-instantons
- M5-instantons with chiral $\mathcal{H}$ flux?