Coupling Selection Rules in Heterotic Orbifold Compactifications

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see also 12XX.XXXX

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Coupling Selection Rules in Heterotic Orbifolds

- Heterotic orbifolds represent simple, globally consistent constructions with clear geometrical interpretation
- Can obtain the MSSM spectrum with no chiral exotics

  Buchmüller et al '06
  Lebedev et al '07
  ...
- To understand dynamics (decoupling of vector-like exotics, moduli stabilization, susy breaking, quark and lepton masses...) we require couplings in LEEFT
- String couplings can be computed via free CFT
- Which couplings are vanishing is determined by string selection rules

Plan

- Orbifold CFT basics
- Coupling selection rules from L-point correlation functions
- Conclusions

Heterotic Orbifold Compactifications

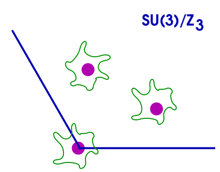
- Heterotic string degrees of freedom:

right-movers: $X_R^M(\sigma-\tau),\ \psi_R^M(\sigma-\tau) \qquad M=1,\ldots,10$

left-movers: $X_L^M(\sigma+\tau),\ X_L^I(\sigma+\tau) \qquad I=1,\ldots,16$

- Torus $\mathbb{R}^6/\Lambda$ *(factorizable)*; orbifold $\mathbb{T}^6/\mathbb{Z}_\mathrm{N}$
- Orbifold boundary conditions $\Rightarrow$

$$X^j(\sigma+\pi,\tau) = (\theta^k X)^j(\sigma,\tau) + \lambda^j, \quad \lambda^j \in \Lambda^j, \qquad j=1,2,3$$

for state in kth twisted sector 

- States in string Hilbert space $\Leftrightarrow$ fields in CFT
- Vertex operator for emission of twisted bosonic field:

$$V_B = e^{\phi} \prod_{j=1}^{3} (\partial X^j)^{N_L^j} (\partial \bar{X}^j)^{\bar{N}_L^j} e^{iq^m H^m} e^{ip^I X^I} \sigma^j_{(k,f)}$$

- Infer $W \sim \Phi^\mathrm{L}$ via $\psi\psi\phi^{\mathrm{L}-2}$ - tree-level couplings $\langle V_F V_F V_B \ldots V_B \rangle$

L-point Correlation Functions

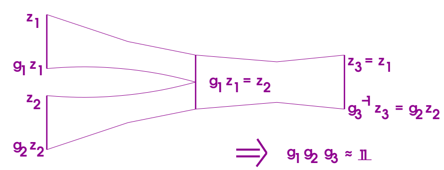
Dixon et al '87
Hamidi & Vafa '87
...

Correlation function factors into several parts:

$$\mathcal{F}_{3-pt} = \langle e^{i\sum_{l=1}^{3} p^I_{sh\,l} X^I} \rangle \times \langle e^{i\sum_{l=1}^{3} q^m_{sh\,l} H^m} \rangle$$
$$\times \prod_{j=1}^{3} \langle (\partial X^j)^{\sum_{l=1}^{3} N^j_{L\,l}} (\partial \bar{X}^j)^{\sum_{l=1}^{3} \bar{N}^j_{L\,l}} \sigma^j_{(k_1,f_1)} \sigma^j_{(k_2,f_2)} \sigma^j_{(k_3,f_3)} \rangle$$

Each part has its own selection rule:

- gauge invariance: $\sum_{l=1}^{3} p^I_{sh\,l} = 0$
- H-momentum conservation: $\sum_{l=1}^{3} q^m_{sh\,l} = 0$
- space group selection rule: *boundary conditions allow twisted strings to join*



Includes point group selection rule: coupling between $\theta^{k_1}\theta^{k_2}\theta^{k_3}$ twisted sectors allowed if $k_1 + k_2 + k_3 = 0 \ mod \ \mathrm{N}$ for $\mathbb{Z}_\mathrm{N}$ orbifold.

Classical and Quantum Splitting

- The non-trivial part of the correlation function is:

$$\mathcal{F} = \prod_{j=1}^{3} \langle (\partial X^j)^{N_L^j} (\partial \bar{X}^j)^{\bar{N}_L^j} \sigma^j_{(k_1,f_1)} \sigma^j_{(k_2,f_2)} \sigma^j_{(k_3,f_3)} \rangle$$

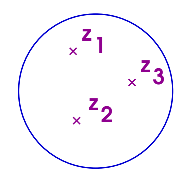
- Fields X^j split into

$$X^j(z,\bar{z}) = X^j_{cl}(z,\bar{z}) + X^j_{qu}(z,\bar{z})$$

with classical instanton solution $\partial\bar{\partial}X^j_{cl} = 0$ 

- Classical solutions determined by local and global monodromy:

$$\partial X^j_{cl}(z) = a^j \nu^j (z-z_1)^{k_1^j-1}(z-z_2)^{k_2^j-1}(z-z_3)^{k_3^j-1}$$
$$\bar{\partial} X^j_{cl}(\bar{z}) = b^j \bar{\nu}^j (\bar{z}-\bar{z}_1)^{-k_1^j}(\bar{z}-\bar{z}_2)^{-k_2^j}(\bar{z}-\bar{z}_3)^{-k_3^j}$$



and vanish if $k^j_{1,2,3}$ are such that classical action does not converge.

New String Coupling Selection Rule

- The correlation function splits as *(in the jth plane)*:

$$\mathcal{F}^j_{3-pt} = \langle (\partial X^j_{cl} + \partial X^j_{qu})^{N^j_L} (\partial \bar{X}^j_{cl} + \partial \bar{X}^j_{qu})^{\bar{N}^j_L} \sigma^j_{(k_1,f_1)} \sigma^j_{(k_2,f_2)} \sigma^j_{(k_3,f_3)} \rangle$$

- OPEs $\Rightarrow \langle (\partial X^j_{qu})^r (\partial \bar{X}^j_{qu})^s \sigma^j_{(k_1,f_1)} \sigma^j_{(k_2,f_2)} \sigma^j_{(k_3,f_3)} \rangle = 0$ unless $r = s$
- Local and global constraints on classical solutions
  $\Rightarrow$ depending on twisted sectors:
  - either anti-holomorphic instantons vanish $\partial \bar{X}^j_{cl} = 0$
  - or holomorphic instantons vanish $\partial X^j_{cl} = 0$
  - or both vanish $\partial \bar{X}^j_{cl} = 0 = \partial X^j_{cl}$
- **Rule 5:**
  - **only holomorphic instantons** $\Rightarrow N^j_L \geq \bar{N}^j_L$
  - **only anti-holomorphic instantons** $\Rightarrow N^j_L \leq \bar{N}^j_L$
  - **no instantons** $\Rightarrow N^j_L = \bar{N}^j_L$

Forgotten String Coupling Selection Rule

Hamidi & Vafa '87
Font, Ibañez, Nilles & Quevedo '88

- Assume e.g. only holomorphic instantons are allowed:

$$\mathcal{F}^{j}_{3-pt} = \sum_{X^j_{cl}} e^{-S_{cl}} (\partial X^j_{cl})^{N^j_L - \bar{N}^j_L} \mathcal{Z}^j_{qu}$$

- Classical solutions are proportional to (shifted) lattice vectors:

$$\partial X^j_{cl} \sim f^j_2 - f^j_1 + \lambda^j, \qquad \lambda^j \in \Lambda^j$$

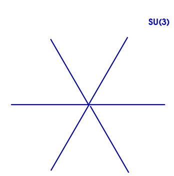
- $\Rightarrow$ Twist invariance: $N^j_L - \bar{N}^j_L = 0 \mod P^j$ for $\mathbb{Z}_{P^j}$ twist

  Together with H-momentum conservation this leads to R-charge conservation

Font et al '88

- **Rule 4: for fields at same fixed point we have** $\partial X^j_{cl} \sim \lambda^j \Rightarrow$

$$N^j_L - \bar{N}^j_L = 0 \mod K^j$$

for $\mathbb{Z}_{K^j}$ lattice symmetry 

Conclusions

To build realistic models we must understand couplings in LEEFT

- Selection rules for superpotential couplings can be identified via L-point string tree-level correlation functions
- Space group selection rule, gauge invariance and R-charge invariance are sufficient for non-oscillator couplings
- Couplings between excited massless states and higher order couplings involve oscillators
- Then the structure of worldsheet instanton solutions leads to additional stringy rules
  - Rule 4: when torus lattice has extra symmetries beyond the orbifold twist
  - Rule 5: when local and global constraints imply instanton solutions are vanishing

  These rules must be applied when computing allowed W couplings
- The ultimate objective is the full LEEFT, K, W, f_a, ξ_{FI}...

Couplings in Explicit MSSM Model

order	no rules 4 & 5	with rules 4 & 5
3	160	152
4	300	282
5	4710	4435(+152)
6	55638	49898(+282)
7	862893	833641(+4587)