U(n) Spectral Covers from Decomposition

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(arXiv:1203.3812[hep-th])

Motivation (1/2)

A candidate for beyond the Standard Model
→ Minimal Supersymmetric Standard Model (MSSM)
→ Grand Unified Theory (GUT) around 2×10^{16} GeV

☐ Necessary components for a supersymmetric SU(5) GUT model

· Matter Contents

Matter

$$10_M \to (3,2)_{\frac{1}{6}} + (\bar{3},1)_{-\frac{2}{3}} + (1,1)_1$$
$$\bar{5}_M \to (\bar{3},1)_{\frac{1}{3}} + (1,2)_{-\frac{1}{2}}$$

Higgs

$$5_H \to (3,1)_{-\frac{1}{3}} + (1,2)_{\frac{1}{2}}$$
$$\bar{5}_H \to (\bar{3},1)_{\frac{1}{3}} + (1,2)_{-\frac{1}{2}}$$

A generic superpotential includes

$$W_{GUT} \supset \underbrace{y_u 10_M 10_M 5_H + y_d \bar{5}_M 10_M \bar{5}_H}_{\text{Necessary Yukawa interactions}} + \lambda \bar{5}_M 10_M \bar{5}_M$$

Dimension-4 Proton Decay Operator!

↑

U(1) symmetry, discrete symmetry, et al

How can we realize the U(1) symmetry from String theory, or F-theory?

Motivation (2/2)

- The realization of a U(1) symmetry in F-theory compactifications requires a global analysis.

H.H., T. Kawano, Y. Tsuchiya, T. Watari, 1004

$$\boxed{\text{\# of U(1)'s} = h^{1,1}(CY_4) - h^{1,1}(B_3) - 1}$$

Morrison, Vafa 96
Dasgupta, Rajesh, Sethi 99

Objective: $h^{1,1}(CY_4) \rightarrow h^{1,1}(CY_4') = h^{1,1}(CY_4) + 1$

$\uparrow$

some tuning?

It is not easy to find the tuning purely from the viewpoint of F-theory compactifications.

We have found a parameterization enlightened by heterotic – F-theory duality. We have also checked the evidence that there is an unbroken U(1) symmetry purely from F-theory compactifications.

Other solutions: the split of Tate divisor

Marsano, Saulina, Schäfer-Nameki 1006, 1007

U(1)-restricted Tate model

Grimm, Weigand 1006
Grimm, Kerstan, Palti, Weigand 1107
Krause, Mayrhofer, Weigand 1109

Outline

☑ 1. Motivation

2. Decomposed Spectral Covers in Heterotic String Theory

3. F-theory Interpretation

4. Summary

Decomposed Spectral Covers (1/4)

Heterotic string theory on a background of CY_3 with a vector bundle V whose structure group H realizes a ***N***=1 four-dimensional low energy effective field theory with a gauge group G. ($E_8 \supset G \times H$)

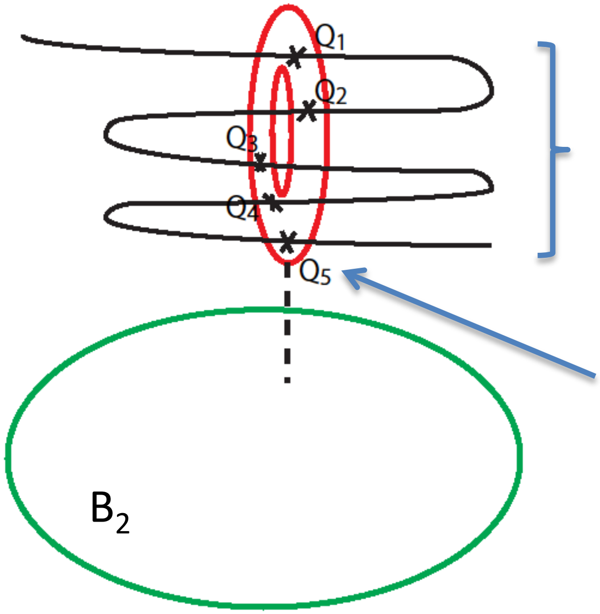
Candelas, Horowitz, Strominger, Witten 85

- When CY_3 has an elliptic fibration with a section (o),

  Stable SU(n) vector bundle V ← “Spectral cover ”+ line bundle on it

Friedman, Morgan, Witten 97
Donagi 97

- Spectral Cover



5-fold cover → SU(5) vector bundle

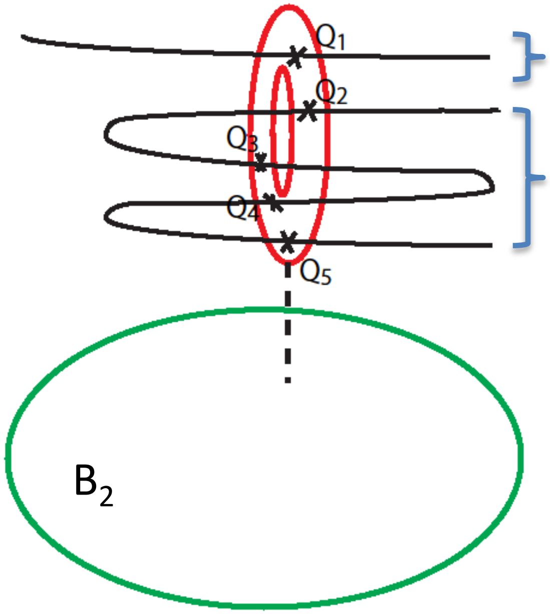
Spectral cover intersects with the elliptic fiber at 5 points

$Q_1 \boxplus Q_2 \boxplus Q_3 \boxplus Q_4 \boxplus Q_5$ = o for $\det V = \mathcal{O}$

Decomposed Spectral Covers (2/4)

- If the structure group reduces to S(U(1) x U(4)), a U(1) symmetry can be present in a low energy effective field theory.

Blumenhagen, Honecker, Weigand 05
Tatar, Watari 06



1-fold cover $Q_1 \neq 0$

4-fold cover $Q_2 \boxplus Q_3 \boxplus Q_4 \boxplus Q_5 \neq 0$

And $Q_1 \boxplus Q_2 \boxplus Q_3 \boxplus Q_4 \boxplus Q_5 = 0$

However, a naïve factorization of the spectral cover equation does not achieve this configuration.
This is essentially because there is no elliptic function which has only one zero point.

- We have found that one can realize the configuration by tuning the complex structure moduli of CY_3 so that the elliptic fibration admits another section different from the zero section.

1-fold cover ← another section

Decomposed Spectral Covers (3/4)

Elliptically fibered CY$_3$: $y^2 = x^3 + fx + g$

$$f \in \Gamma(B_2; \mathcal{O}(-4K_{B_2})), \quad g \in \Gamma(B_2; \mathcal{O}(-6K_{B_2}))$$

Spectral cover: $a_0 + a_2 x + a_3 y + a_4 x^2 + a_5 xy = 0$

$$a_r \in \Gamma(B_2; \mathcal{O}(rK_{B_2} + \eta))$$

- The Higgs bundle ansatz:

$$a_0 s^5 + a_2 s^3 + a_3 s^2 + a_4 s + a_5 = 0$$

$$\rightarrow \ (b_0 s + b_1)(d_0 s^4 + d_1 s^3 + d_2 s^2 + d_3 s + d_4) = 0$$

$$a_0 = b_0 d_0, \ \ 0 = b_0 d_1 + b_1 d_0, \ \ a_2 = b_0 d_2 + b_1 d_1$$
$$a_3 = b_0 d_3 + b_1 d_2, \ \ a_4 = b_0 d_4 + b_1 d_3, \ \ a_5 = b_1 d_4$$

Tatar , Tsuchiya, Watari 0905
Marsano, Saulina, Schäfer-Namaki 0906 0912
Blumenhagen, Grimm, Jurke, Weigand 0908

Then, one of the root of the spectral surface is $Q_1 : (x, y) = \left(\frac{b_0^2}{b_1^2}, -\frac{b_0^3}{b_1^3} \right)$

We can require this point to be a global section.

Decomposed Spectral Covers (4/4)

In order to make Q_1 a global section of the elliptic fibration, Q_1 should be a point on the elliptic fibration regardless of f and g.

Choi, HH 12

$$b_0^2 f + b_1^2 g = 0$$

This equation can be solved by

$$f = b_1^2 F, \; g = -b_0^2 F, \; (d_0 = b_0 d, \; d_1 = -b_1 d)$$

To ensure that this parameterization indeed has a global section at Q_1, one can rewrite the elliptic fibration equation as

$$(b_1^3 y - b_0^3)(b_1^3 y + b_0^3) = (b_1^2 x - b_0^2)(b_1^4 x^2 + b_1^2 b_0^2 x + b_0^4 + b_1^6 F)$$

By using the similar logic, we have also constructed spectral covers for the vector bundle whose structure group is S(U(2) x U(3)) and S(U(1) x U(1) x U(3)). We also constructed another S(U(1) x U(4)) spectral cover which is not equivalent to the one presented here.

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F-theory Interpretation (1/3)

Heterotic String on elliptic CY_3 with a vector bundle

Morrison, Vafa 96
Friedman, Morgan, Witten 97

↕ dual

F-theory on elliptic K3 fibered CY_4 in the stable degeneration limit
(Base S)

$K3 \rightarrow RES \cup_E RES$

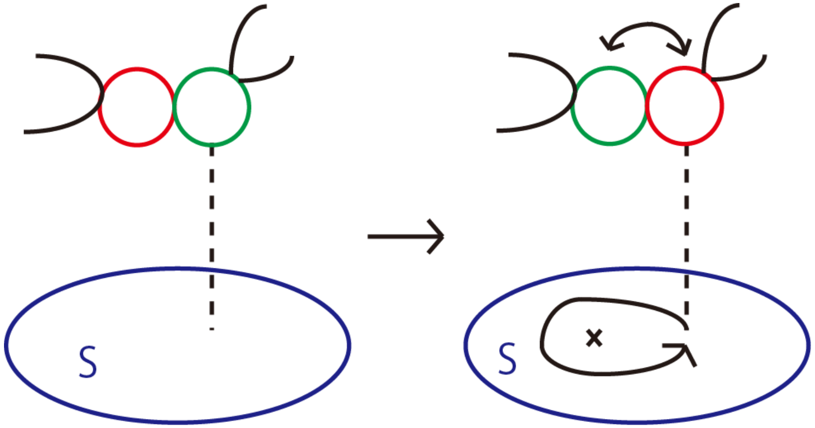
$\boxed{\# \text{ of } U(1)\text{'s} = h^{1,1}(CY_4) - h^{1,1}(B_3) - 1}$ → 2-form has one leg on elliptic fiber and the other leg on 3-fold base

↕ Poincaré dual

monodromy invariant 2-cycle (1-cycle is elliptic fiber) over the base S

Curio, Donagi 98
Donagi, Wijnholt 0802
HH, Tatar, Toda, Watari, Yamazaki 0805

RES → 10 2-cycles
= 8 (simple roots of E_8)
+ 1 (elliptic fiber)
+ 1 (base $\mathbf{P}^1$)



F-theory Interpretation (2/3)

Monodromy of 2-cycles ↔ Monodromy of the locations of 7-branes

·Elliptic K3 fibered CY$_4$ with an A$_4$ singularity in the stable degeneration limit

Bershadsky et al 96

$$\begin{aligned} y^2 = x^3 &+ a_0 z^5 + g z^6 \\ &+ a_2 z^3 x + \underbrace{f z^4 x}_{\text{Complex structure moduli in heterotic string}} \\ &+ a_3 z^2 y \\ &+ a_4 z x^2 \\ &+ \underbrace{a_5 x y}_{\text{Bundle moduli in heterotic string}} \end{aligned}$$

Katz, Mayr, Vafa 97
Berglund, Mayr 98

The discriminant

$$\Delta = \underset{\uparrow}{z^5} \text{ X [degree 7 } \underset{\uparrow}{\text{polynomial}} \text{ in z]}$$

5 7-branes for SU(5)

The locations of the other 7 7-branes
The coefficients are functions of sections a$_r$, f, g

Furthermore, we take the discriminant of the degree 7 polynomial

F-theory Interpretation (3/3)

- The discriminant of the discriminant

$$\tilde{\Delta} = 16777216 a_5^5 \underbrace{A^3 B}_{}$$

Functions of the sections in a_r, f, g
The monodromy here is supposed to give S_5

- Apply the parameterization of the S(U(1)xU(4)) spectral cover equation

$$B \to \underset{\uparrow}{B_1^2} B_2$$

Choi, HH 12

A quadratic component appears!
This may imply the reduction of the monodromy.

We also checked explicitly that the monodromy indeed reduces to Z_2 from S_3 in a subspace of a moduli space in the case of E_6 gauge theories with S(U(1) x U(2)) ($\subset$ SU(3)) structure group.

From the computation, the monodromy coming from the quadratic component is trivial, and this indicates that the appearance of the quadratic factor would be essential for the reduction of the monodromy.

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Summary

· We have explicitly constructed S(U(1) x U(4)) (and S(U(2) x U(3)), S(U(1) x U(1) x U(3))) spectral covers

· In the construction, we tune the complex structure moduli of Calabi-Yau threefolds, and associate the 1-fold cover to a global section.

· In F-theory compactifications, the parameterization makes the discriminant factorize with a quadratic component, which could imply that the monodromy may get reduced.

· The reduction of the monodromy was checked at least in a subspace of the moduli space in the case of the breaking $E_8 \rightarrow E_6$ x SU(3) with the (1+2) decomposition.