Searching for Phenomenologically Attractive Calabi-Yaus with Kähler Classes Variant under Orientifolding

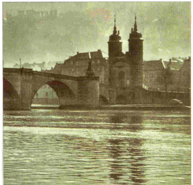
joint work with: M. Cicoli, S. Krippendorf, F. Quevedo and R. Valandro; arXiv:1206.5237;

see also Michele's and Sven's talks

Christoph Mayrhofer

Institute for Theoretical Physics, Universität Heidelberg

Cambridge, June 28th, 2012



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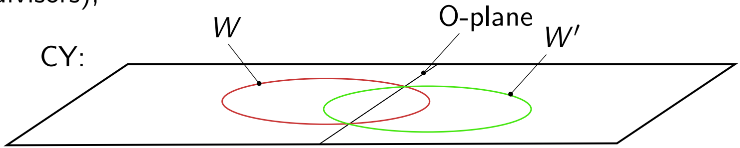

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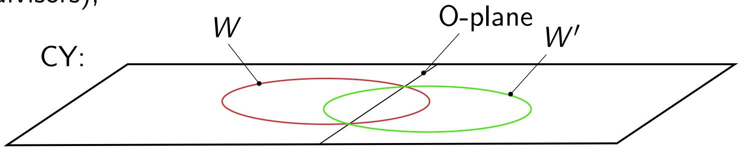
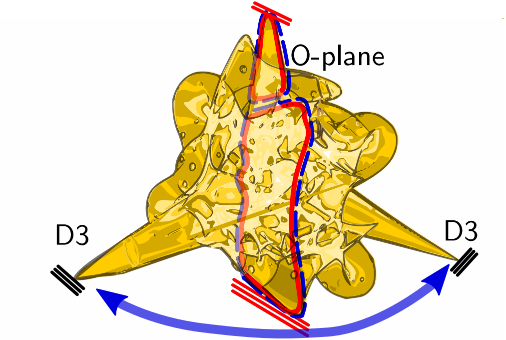


- ... with D3-branes at singularities;



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- S and $S' = \sigma(S)$ not on top of each other $\Rightarrow [S] \neq [S'] \Rightarrow h^{1,1}_- \geq 1$;

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- ...two dP_n divisors and a holomorphic involution exchanging them;
- ...dP_n surfaces do not intersect each other (not necessary for D7's with $SU(n)$-stacks);
- ...further rigid divisor invariant under orientifold involution;

del Pezzo (checks)

Check necessary conditions divisor S has to fulfil:

- Analyse the topological numbers of the toric divisors:

$$\int_S c_1^2(S) \quad \text{and} \quad \int_S c_2(S) = \chi(S) \overset{!}{<} 12 \quad \Rightarrow \quad \chi_h(S) \overset{!}{=} 1\,;$$

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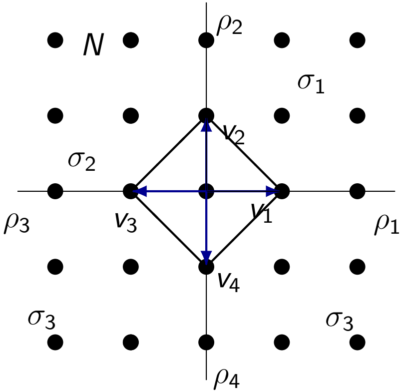
$\mathbb{Z}_2$-symmetry of the N-lattice polytope; E.g. $\mathbb{P}^1 \times \mathbb{P}^1$



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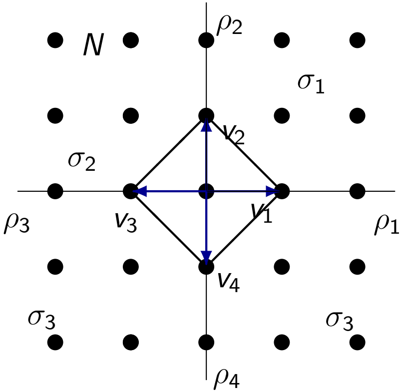

To see action on $H^{1,1}$, look at weight matrix:

z_1	z_2	z_3	z_4
1	0	1	0
0	1	0	1

$$\mathbb{Z}_2\text{-symm.}: \begin{cases} 1: & z_1 \leftrightarrow z_3 \quad \Rightarrow \quad h^{1,1}_- = 0 \\ 2: & z_2 \leftrightarrow z_4 \quad \Rightarrow \quad h^{1,1}_- = 0 \\ 3: & (z_1, z_2) \leftrightarrow (z_4, z_3) \quad \Rightarrow \quad h^{1,1}_- = 1 \\ 4: & (z_2, z_3) \leftrightarrow (z_1, z_4) \quad \Rightarrow \quad h^{1,1}_- = 1 \end{cases}$$

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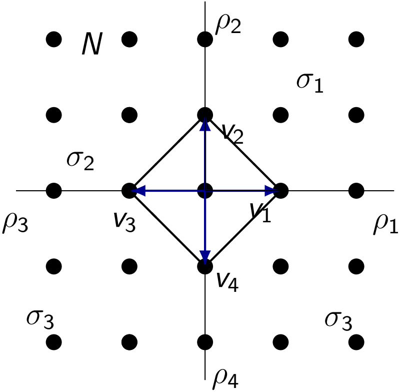

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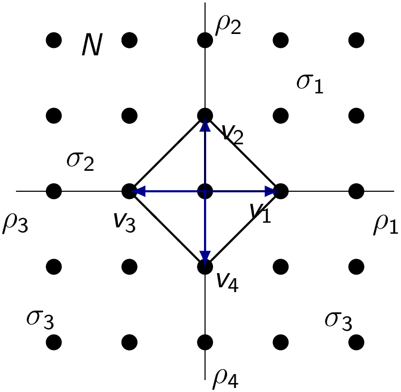

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Search Results

1 197 polytopes with $h^{1,1} = 4$:

	Σ	dP_0	dP_1	dP_2	dP_3	dP_4	dP_5	dP_6	dP_7	dP_8
2 dP_n + O-invo.	82	9	5	-	-	-	2	10	31	25
2 dP_n non-inter.	68	9	2	-	-	-	2	10	27	18
+ rigid divisor	21	3	-	-	-	-	-	4	9	5

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4 990 polytopes with $h^{1,1} = 5$:

	Σ	dP_0	dP_1	dP_2	dP_3	dP_4	dP_5	dP_6	dP_7	dP_8
2 dP_n & O-invo.	386	27	60	21	7	3	13	40	121	94
2 dP_n non-inter.	327	27	55	7	3	1	11	39	112	72
+ rigid divisor	168	14	16	-	-	-	5	28	68	37

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A dP_0-example

The toric (ambient) variety and the multi-degree of the CY hypersurface X:

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0	0	0	1	0	1	0	0	2
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$$\mathrm{SR} = \{z_4\, z_6,\; z_4\, z_7,\; z_5\, z_7,\; z_5\, z_8,\; z_6\, z_8,\; z_1\, z_2\, z_3\}\,;$$

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A diagonal basis:

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- Long term goal: make all computed data easily accessible;

Thank you for your attention!