# On the Beaming of Gluonic Fields at Strong Coupling

Bryan Larios
A.Guijosa & C.Agón

Work based on [arXiv:1206.5005[hep-th]]

Physics Department
National University Autonomous of Honduras

String Phenomenology 2012

# Index

# Index

# Index

1. Motivation.
2. Outline
3. The Origin of Beaming.

# Index

1. Motivation.
2. Outline
3. The Origin of Beaming.
4. Pulverizing the String

# Index

1. Motivation.
2. Outline
3. The Origin of Beaming.
4. Pulverizing the String
5. Conclusions.

# Motivations

- After the discovery of the AdS/CFT correspondence we have at hand a powerful tool to explore certain non-Abelian strongly coupled gauge theories.

# Motivations

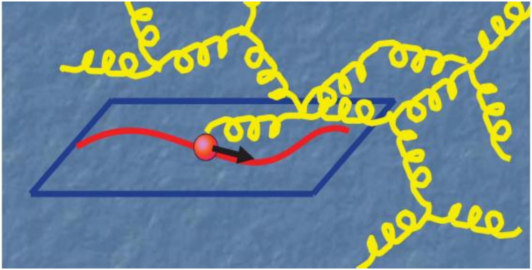
- After the discovery of the AdS/CFT correspondence we have at hand a powerful tool to explore certain non-Abelian strongly coupled gauge theories.
- Particularly, one of the natural aspects to explore in those theories is the way in which the disturbances in the gluonic fields propagate when a quark undergoes an arbitrary motion.

# Motivations

- After the discovery of the AdS/CFT correspondence we have at hand a powerful tool to explore certain non-Abelian strongly coupled gauge theories.
- Particularly, one of the natural aspects to explore in those theories is the way in which the disturbances in the gluonic fields propagate when a quark undergoes an arbitrary motion.
- We can add Heavy quarks to this setup by introducing open strings on the gravity side.

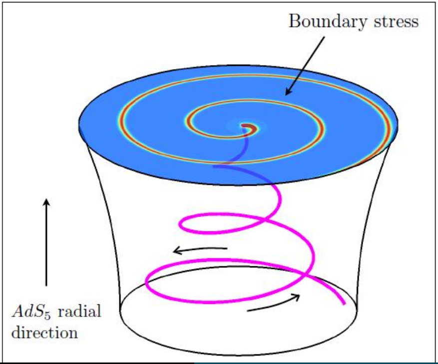
# Expected radiation at strong coupling

- At strong coupling the gauge fields are expected to re-radiate



- We would expect that radiation at strong coupling to promptly isotropize and consequently differ greatly from classical electromagnetism.

- However, in (Athanasiou,Chesler,Liu,Nickel,Rajagopal 2010 [arXiv:1001.3880 [hep-th]]) it was found that a heavy quark undergoing uniform circular motion (U.C.M.) at relativistic speed in the vacuum of the CFT (dual to the pure AdS geometry), give rise to a gluonic field profile that essentially coincides with the familiar synchrotron spiral of classical electromagnetism.



- Soon after the work of [Atanasiou et al], Veronika Hubeny [arXiv:1011.1270 [hep-th]] proposed an interesting explanation for the beaming in strongly coupled gauge theories, which was shown to be successful in the U.C.M.

# VH's proposal

- It was show at least for the particular case of (U.C.M), that strings bits give rise to a localized contribution to the boundary of AdS, because they move ultra-relativistic (at large z), and therefore generate bulk fields that are themselves beamed.

# VH's proposal

- It was show at least for the particular case of (U.C.M), that strings bits give rise to a localized contribution to the boundary of AdS, because they move ultra-relativistic (at large z), and therefore generate bulk fields that are themselves beamed.
- each strings bits should generates a gravitational shock wave.

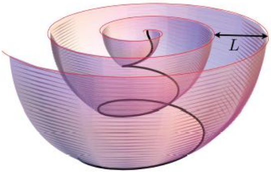
# VH's proposal

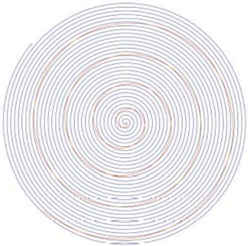
- It was show at least for the particular case of (U.C.M), that strings bits give rise to a localized contribution to the boundary of AdS, because they move ultra-relativistic (at large z), and therefore generate bulk fields that are themselves beamed.
- each strings bits should generates a gravitational shock wave.
- Therefore, the backreaction of the full string should be well approximated by a linear superposition of shock waves perpendicular to the string's transverse velocity.

# VH's proposal

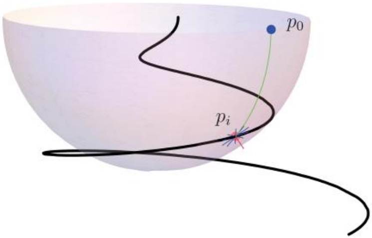
- It was show at least for the particular case of (U.C.M), that strings bits give rise to a localized contribution to the boundary of AdS, because they move ultra-relativistic (at large z), and therefore generate bulk fields that are themselves beamed.
- each strings bits should generates a gravitational shock wave.
- Therefore, the backreaction of the full string should be well approximated by a linear superposition of shock waves perpendicular to the string's transverse velocity.
- This approach allows the gluonic field profile to be determined by means of a construction in terms of spatial geodesics, which is calculationally much more efficient than the tradicional approach.

- With those assumptions VH was able to reproduce the synchrotron radiation pattern.



In $AdS_5$ the shock wave planes turn out to be hemispheres whose equator lies on the boundary at $z = 0$.



# Outline and main results

- For concreteness, in our calculation we used the best understood example of the AdS/CFT correspondence.

| | | |
|---|---|---|
| **Super Yang-Mills** $SU(N_c)$, $\mathcal{N} = 4$ in (3+1) dim.. | $=$ | **Type II B Super String Theory** in $AdS_5 \times S_5$ in (10) dim. |
| $\lambda$ and $N_c$ Large | $\longleftrightarrow$ | $g_s$ and $\frac{l_s}{R}$ small. |
| Complicated Computations | $\longleftrightarrow$ | Easy Computations. |

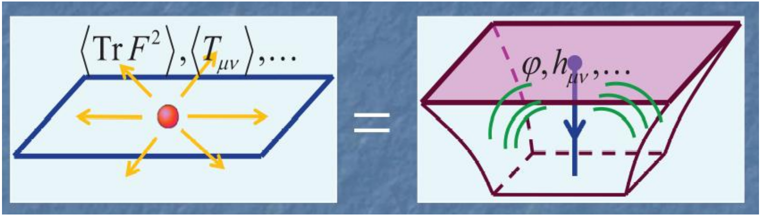
- The aim of this work was to employ the AdS/CFT correspondence to establish how general the phenomenon of beaming is in strongly-coupled gauge theories, and in particular, to identify the conditions under which the shock wave Hubeny's prescription is valid.

- The aim of this work was to employ the AdS/CFT correspondence to establish how general the phenomenon of beaming is in strongly-coupled gauge theories, and in particular, to identify the conditions under which the shock wave Hubeny's prescription is valid.
- To establish whether Hubeny's approach is valid even when the quark is no ultra-relativistic, for a class of trajectories that is much more general than synchrotron case studied.

- The aim of this work was to employ the AdS/CFT correspondence to establish how general the phenomenon of beaming is in strongly-coupled gauge theories, and in particular, to identify the conditions under which the shock wave Hubeny's prescription is valid.
- To establish whether Hubeny's approach is valid even when the quark is no ultra-relativistic, for a class of trajectories that is much more general than synchrotron case studied. (*If so, it would be useful as a calculations tool, and might imply that beaming is a generic property of gluonic radiation at strong coupling, regardless of the motion of the source*)

# Inclusions of quarks

Introduce a quark on the Quantum field theory side of the duality is equivalent to the introduce a classical string on the gravity side.




The string acts as a source of perturbations of the metric field $h_{mn}(x^\mu, z)$ and scalar field $\varphi(x^\mu; z)$ (dilaton) in complete analogy with the quark which is source for the MSYM gluonic observables $\langle Tr(F^2(x^\mu))\rangle$ and $\langle T_{\mu\nu}(x^\mu)\rangle$.

The gluonic field sourced by the quark can be mapped out by computing the expectation value of local MSYM operators. The simplest is the Lagrangian density operator

$$\mathcal{O}_{F^2}(x^\mu) \equiv \frac{1}{2g_{YM}^2} \operatorname{Tr} \left\{ F^2(x^\mu) + [\Phi_I, \Phi_J][\Phi^I, \Phi^J](x^\mu) + \text{fermions} \right\} , \tag{1}$$

which we will abbreviate simply as $\langle \operatorname{Tr} F^2(x^\mu) \rangle$. The explicit connections are:

$$\langle Tr(F^2(x^\mu)) \rangle = -\lim_{z\to 0} \left( \frac{1}{z^3} \partial_z \varphi(x^\mu, z) \right) \tag{2}$$

$$\langle T_{\mu\nu}(x^\rho) \rangle = \frac{R^3}{4\pi G_N^{(5)}} h_{\mu\nu}^{(4)}(x^\rho) \tag{3}$$

# Origin of the Beaming

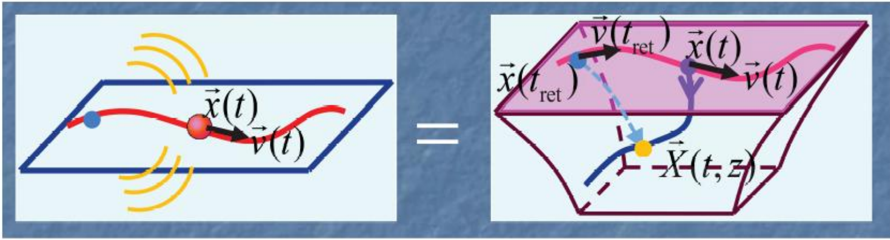
In the large $N_c$, large $\lambda$ limit, the string embedding is determined by extremizing the Nambu-Goto action

$$S_{NG} = -\frac{1}{2\pi l_s^2}\int d^2\sigma\,\sqrt{-\det g_{ab}} = -\frac{1}{2\pi l_s^2}\int d^2\sigma\,\sqrt{\left(\dot{X}\cdot X'\right)^2 - \dot{X}^2 X'^2} \tag{4}$$

where $g_{ab} \equiv \partial_a X^m \partial_b X^n G_{mn}(X)$ $(a, b = 0, 1)$ is the induced metric on the worldsheet, and
$ds^2 = G_{mn}dx^m dx^n = \frac{R^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$ .

**With a general solutions of N-G equations found by Mikhailov (arXiv:hep-th/0305196)**

$$\vec{X}(t_r, z) = \vec{x}(t_r) + \frac{\vec{v}(t_r)z}{\sqrt{1-\vec{v}(t_r)^2}}\,, \tag{5}$$

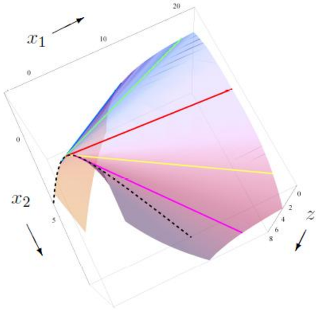


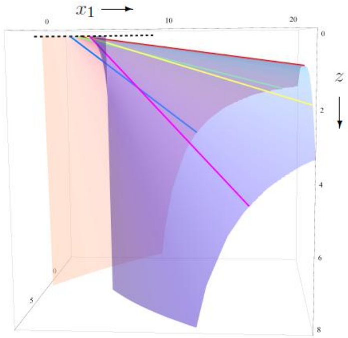

- Endpoint information flows along null lines on the worldsheet which we recall as Mikhailov lines.
- With just this solution we can give a generic explanation for the beaming.

- From the Mikhailov's solution we have

$$z = \sqrt{1 - \vec{v}^2}(t - t_r) = (t - t_r)/\gamma \tag{6}$$

- Given an event $(t_r, \vec{x}(t_r))$ in which the quark is sufficiently relativistic, the null line on the string worldsheet remains near the boundary of AdS even far from the source.
- That implies that the fields generated by this points of the string will be beamed when they reach the boundary.
- Therefore, the fields on the gauge theory side will be correspondingly localized.




- Different null lines (color) on the string worldsheet with the string endpoint following an hyperbolic trajectory $x_2 = \sqrt{c^2 + x_1^2}$ with $v_1(t_r) = 0.999 sech(t_r/T)$ (dotted).

# VHs Proposal (*revised*)

*...the backreaction of the full string is well-approximated by a linear superposition of gravitational shock waves normal to the string's transverse velocity.*

so it is crucial for the beaming calculation the transverse velocity

$$V_\perp^m \equiv \dot{X}^m - \left( \frac{\dot{X} \cdot X'}{X' \cdot X'} \right) X'^m , \tag{7}$$

Additionally, the integrand of the Nambu-Goto Lagrangian can be written as

$$\sqrt{\left(\dot{X} \cdot X'\right)^2 - \dot{X}^2 X'^2} = \sqrt{X'^2}\sqrt{-V_\perp^2} . \tag{8}$$

# Gravitational Field

The Fields sourced by the string are obtained by Green function methods.

$$\varphi(x) = \int d^5\bar{x} J(\bar{x}) D(x;\bar{x})$$

$$h_{mn}(x) = \int d^5\bar{x}\sqrt{-G}\mathcal{G}_{mn;\bar{m}\bar{n}}(x,\bar{x}) T^{\bar{m}\bar{n}}(\bar{x})$$

where the sources are

$$J(\bar{x}) \sim \frac{\delta S_{NG}}{\delta\varphi(\bar{x})}, \qquad T^{\bar{m}\bar{n}} \sim \frac{1}{\sqrt{-G}}\frac{\delta S_{NG}}{\delta G_{mn}(\bar{x})}$$

# Pulverizing the String

## Dilaton Field (String)

$$\varphi(x) = \frac{1}{4\pi l_s^2} \int d\tau\, d\sigma \sqrt{X'^2(\tau,\sigma)}\sqrt{-V_\perp^2(\tau,\sigma)}\, D(x;X(\tau,\sigma)) \ , \quad (9)$$

Our interest is to determine under what conditions (9) can be approximated by using point particles in place of the string, as Hubeny proposed.

## Dilaton Field (Particle)

$$\varphi(x) = \frac{m}{2} \int d\tau \sqrt{-V^2(\tau)}\, D(x;X(\tau)) \ . \quad (10)$$

# Pulverizing the String

For the purpose of computing the dilaton field (and consequently, $\langle \mathrm{Tr}\, F^2(x) \rangle$), the string (or a segment of it) can be approximated as a collection of particles moving with velocities $V^m = V_\perp^m$ only to the extent that

$$V_\perp^m \simeq \dot{X}^m \quad \textit{and} \quad X'^2 \simeq \text{constant} \, .$$

# Pulverizing the String

## Graviton Field (String)

$$\mathrm{T}^{mn}(x) = \frac{1}{2\pi l_s^2} \int \frac{d^2\sigma}{\sqrt{-G}\sqrt{-V_\perp^2}} \mathcal{V}^{mn}\, \delta^{(5)}\left(x - X(\tau,\sigma)\right) \;, \qquad (11)$$

With

$$\mathcal{V}^{mn} = \sqrt{X'^2}\, V_\perp^m V_\perp^n + \frac{V_\perp^2}{\sqrt{X'^2}} X'^m X'^n \;. \qquad (12)$$

## Graviton Field (Particle)

$$\mathrm{T}^{mn}(x) \equiv \frac{2}{\sqrt{-G}} \frac{\delta S_{\text{part}}}{\delta G_{mn}(x)} = m \int \frac{d\tau\, V^m V^n}{\sqrt{-G}\sqrt{-V^2}}\, \delta^{(5)}\left(x - X(\tau)\right) \;. \qquad (13)$$

# Pulverizing the String

we learn that the string energy-momentum tensor can be well approximated by the aggregated effect of a collection of particles moving with velocities $V^m = V^m_\perp$ only if conditions $V^m_\perp \simeq \dot{X}^m$ and $X'^2 \simeq \text{constant}$ are satisfied, together with the requirement that

$$\left|\frac{1}{V^2_\perp} V^\mu_\perp V^\nu_\perp\right| \gg \left|\frac{1}{X'^2} X'^\mu X'^\nu\right| \quad \forall \quad \mu, \nu \; . \tag{14}$$

This demands that the transverse velocity of the string be ultra-relativistic, in the sense that

$$\frac{z^2}{R^2} V^2_\perp \simeq 0 \; . \tag{15}$$

# Required conditions for the validity of VH proposal

- $V_{\perp}^{m} \simeq \dot{X}^{m}$
- $X'^{2} \simeq Constant$
- $\frac{z^2}{R^2} V_{\perp}^{2} \simeq 0 \rightarrow \vec{V}_{\perp}^{2} \simeq (V_{\perp}^{0})^{2}$

# Conclusions

- The AdS/CFT duality teaches a new feature of the radiation pattern in some non-Abelian strongly coupled CFT's: the presence of beaming. Surprisingly that effect is in complete analogy with the weak coupling expectations.

# Conclusions

- The AdS/CFT duality teaches a new feature of the radiation pattern in some non-Abelian strongly coupled CFT's: the presence of beaming. Surprisingly that effect is in complete analogy with the weak coupling expectations.
- We provide a simple and intuitive explanation to the beaming in such a theories using just the shape of the string structure of the Mikhailov solution.

# Conclusions

- The AdS/CFT duality teaches a new feature of the radiation pattern in some non-Abelian strongly coupled CFT's: the presence of beaming. Surprisingly that effect is in complete analogy with the weak coupling expectations.
- We provide a simple and intuitive explanation to the beaming in such a theories using just the shape of the string structure of the Mikhailov solution.
- We show that the interesting VH's proposal seems to be valid for U.C.M., but unfortunately it is not for much more general trajectories.

THANK YOU.....