# New $\mathcal{N}=1$ dualities from contracting orientifolds

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in collaboration with B. Heidenreich and T. Wrase

to appear soon

# Dualities in field theory and string theory

Equivalence of two different descriptions of a physical system, either as exactly equivalent and complementary descriptions:

- IIB/$\mathcal{N} = 4$ S-duality
- T-duality (and mirror symmetry)
- AdS/CFT
- ...

or, slightly more generally, various different UV descriptions of the same IR physics (Seiberg duality).

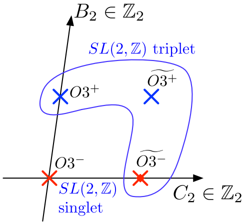
# Stringy realization of $\mathcal{N} = 4$ duality

Can be realized on a $O3^{\pm}/D3$ system in flat space: The four versions of the orientifold are distinguished by discrete $RR$ and $NSNS$ 2-form fluxes $B_2, C_2$ in the transverse space:

$$H^3(S^5/\mathbb{Z}_2, \widetilde{\mathbb{Z}}) = \mathbb{Z}_2 \,.$$

IIB $SL(2,\mathbb{Z})$ exchanges the configurations. [Witten:hep-th/9805112]

- $O3^- + N\, D3s \longrightarrow SO(2N)$
- $\widetilde{O3^-} + N\, D3s \longrightarrow SO(2N+1)$
- $O3^+ + N\, D3s \longrightarrow USp(2N)$
- $\widetilde{O3^+} + N\, D3s \longrightarrow USp(2N)$




**Under S-duality**

$$\widetilde{O3^-} \longleftrightarrow O3^+ \quad : \quad SO(2N+1) \longleftrightarrow USp(2N)$$

# A proposed $\mathcal{N}=1$ duality

(See B. Heidenreich's talk)

| | $USp(\widetilde{N}+4)$ | $SU(\widetilde{N})$ | $SU(3)$ | $U(1)_R$ | $\mathbb{Z}_3$ |
|---|---|---|---|---|---|
| $A^i$ | $\overline{\square}$ | $\square$ | $\square$ | $\frac{2}{3}-\frac{2}{\widetilde{N}}$ | $1$ |
| $B^i$ | $1$ | $\overline{\square\square}$ | $\square$ | $\frac{2}{3}+\frac{4}{\widetilde{N}}$ | $-2$ |

(here $\widetilde{N}\in 2\mathbb{Z}$) is dual to

| | $SO(N-4)$ | $SU(N)$ | $SU(3)$ | $U(1)_R$ | $\mathbb{Z}_3$ |
|---|---|---|---|---|---|
| $A^i$ | $\overline{\square}$ | $\square$ | $\square$ | $\frac{2}{3}+\frac{2}{N}$ | $1$ |
| $B^i$ | $1$ | $\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ | $\square$ | $\frac{2}{3}-\frac{4}{N}$ | $-2$ |

in both cases with $W=\frac{1}{2}\epsilon_{ijk}\mathrm{Tr}\,A^iA^jB^k$.

Global anomalies, the moduli spaces and the spectra of operators match if $\boxed{\widetilde{N}=N-3}$. (As far as we have been able to check so far.)

# Superconformal index matching

A very powerful and refined indicator of duality comes from putting the theory on $S^3 \times \mathbb{R}$, and computing the index [Romelsberger:hep-th/0510060,0707.3702], [Kinney, Maldacena, Minwalla, Raju:hep-th/0510251]:

$$\mathcal{I}(t,x,f) = \int dg\, \mathrm{Tr}\,(-1)^F e^{-\beta\mathcal{H}} t^{\mathcal{R}} x^{2\overline{J}_3} f g\,, \qquad (1)$$

with $2\mathcal{H} = \{Q, Q^\dagger\}$. For $SO(3)\times SU(7) \leftrightarrow USp(8)\times SU(4)$ we get:

$$\begin{aligned}
\mathcal{I}_{SO/USp}(t,x,f) = 1 &+ t^{\frac{2}{3}}\left[\chi_{0,2}(f) + \chi_{4,0}(f)\right] \\
&+ t^{\frac{4}{3}}\left[2\chi_{0,4}(g) + 2\chi_{2,0}(f) + \chi_{3,1}(f) + 2\chi_{4,2}(f) + \chi_{8,0}(f)\right] \\
&+ t^{\frac{5}{3}}(x+x^{-1})\left[\chi_{0,2}(f) + \chi_{4,0}(f)\right] \\
&+ t^2\big[3\chi_{0,6}(f) + \chi_{12,0}(f) + \chi_{1,4}(f) + 5\chi_{2,2}(f) + 3\chi_{3,3}(f) \\
&\quad + 2\chi_{4,1}(f) + 3\chi_{4,4}(f) + \chi_{5,2}(f) + 4\chi_{6,0}(f) + \chi_{6,3}(f) \\
&\quad + \chi_{7,1}(f) + 2\chi_{8,2}(f) + 4\big] + \ldots
\end{aligned}$$

# A proposed $\mathcal{N} = 1$ duality

Forgetting the change in rank, this seems to be essentially a $SO \leftrightarrow USp$ duality, as in $\mathcal{N} = 4$ under S-duality.

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Why the change in rank? $(\widetilde{N} = N - 3)$
Can we derive the duality from the known properties of IIB under S-duality?

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**Plan of attack**

1. Reformulate the problem in terms of large volume objects: $D7$ branes and $O7$ planes wrapping a vanishing 4-cycle.
2. Read the action on the branes from known (or at least more understandable) 10d results in flat space.

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2. Read the action on the branes from known (or at least more understandable) 10d results in flat space.

String picture already provides classification of expected theories, by Witten's argument [Witten:hep-th/9805112]:

$$H^3(X_5/\mathbb{Z}_2, \widetilde{\mathbb{Z}}) = \mathbb{Z}_2$$

# Outline

# Branes at singularities as large volume objects

We can think of the fractional branes at the singularity as large volume D-branes continued to small volume, receiving strong $\alpha'$ corrections.

These $\alpha'$ corrections affect the conditions for supersymmetry, and the masses of states, but we can still think of the object in large volume terms, and compute the chiral spectrum in that picture.

The right way of thinking about this, taking into account all the subtleties, is in terms of the derived category of coherent sheaves. [Douglas:hep-th/0011017]

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The subtleties *are* important for us, but for this talk, this just means (anti-)$D7$s with vector bundles on top: $\mathcal{E}[n]$, with $\mathcal{E}$ a bundle on the collapsing $\mathbb{P}^2$ surface, and $n \in \mathbb{Z}$ distinguishes branes from anti-branes.

# Orientifolds in the derived category

Given a $D7$ brane wrapping $\mathcal{S} = \mathbb{P}^2$ with bundle $\mathcal{E}[n]$, an orientifold wrapping $\mathcal{S}$ acts as:
[Diaconescu,Garcia-Raboso,Karp,Sinha:hep-th/0606180]

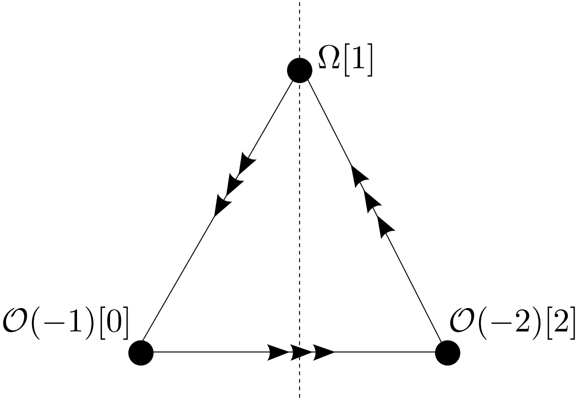
$$i_*\mathcal{E}[n] \longrightarrow i_*(\mathcal{E}^\vee \otimes K_\mathcal{S})[2-n]$$

with $K_\mathcal{S}$ the canonical bundle of $\mathcal{S}$, and $i$ the inclusion $i : \mathcal{S} \hookrightarrow X$ of $\mathcal{S}$ in the ambient Calabi-Yau $X$. In the presence of a $B$-field, this generalizes to:

$$i_*\mathcal{E}[n] \longrightarrow i_*(\mathcal{E}^\vee \otimes K_\mathcal{S} \otimes \mathcal{L}_{2B})[2-n]$$

with $c_1(\mathcal{L}_{2B}) = 2B$.

# $\mathbb{C}^3/\mathbb{Z}_3$ without orientifold




The three branes become mutually supersymmetric at $B = J = 0$.
[Aspinwall:hep-th/0403166]

# Orientifold action

- $\mathcal{O}(-1)[0] \longleftrightarrow \big(\mathcal{O}(-1)^\vee \otimes \mathcal{O}(-3)\big)[2-0] = \mathcal{O}(-2)[2]$
  $\rightsquigarrow$ $SU$ factor.
- $\Omega[1] \longleftrightarrow \big(\Omega^\vee \otimes \bigwedge^2 \Omega\big)[2-1] = \Omega[1]$
  $\rightsquigarrow$ $SO/USp$ factor.

| Number of chiral multiplets | Representation |
|---|---|
| $\langle \mathcal{E}_i, \mathcal{E}_j \rangle$ | $(\square_i, \bar{\square}_j)$ |
| $\langle \mathcal{E}_i, \mathcal{E}'_j \rangle$ | $(\square_i, \square_j)$ |
| $\frac{1}{2}\langle \mathcal{E}_i, \mathcal{E}'_i \rangle \pm \frac{1}{8}\langle e^{-B}\mathcal{E}_i, O7^\pm \rangle$ | $(\square\square, 1)$ |
| $\frac{1}{2}\langle \mathcal{E}_i, \mathcal{E}'_i \rangle \mp \frac{1}{8}\langle e^{-B}\mathcal{E}_i, O7^\pm \rangle$ | $(\begin{smallmatrix}\square\\\square\end{smallmatrix}, 1)$ |

Here $\langle \mathcal{E}_i, \mathcal{E}_j \rangle$ counts the chiral multiplets between the branes:

$$\langle \mathcal{E}_i, \mathcal{E}_j \rangle = \sum_k (-1)^k \int_X \Gamma^{(2k)}(\mathcal{E}_i) \ \wedge \ \Gamma^{(6-2k)}(\mathcal{E}_j)\,,$$

with $\Gamma(\mathcal{E}_i)$ the $RR$ charge vector of $\mathcal{E}_i$.

# RR charge vectors

The RR charges for $D7$ branes described by a bundle $\mathcal{E}[n]$ on $\mathcal{S}$ are:

$$\Gamma(\mathcal{E}[n]) = (-1)^n \int_{\mathcal{S}} \mathrm{ch}(\mathcal{E}) \sqrt{\frac{\mathrm{Td}(T_{\mathcal{S}})}{\mathrm{Td}(N_{\mathcal{S}|X})}}$$

while for a $O7^{\pm}$ plane wrapping $\mathcal{S}$ it is:

$$\Gamma(O7^{\pm}) = \pm 8 \int_{\mathcal{S}} \sqrt{\frac{\hat{L}(T_{\mathcal{S}}/4)}{\hat{L}(N_{\mathcal{S}|X}/4)}} = \pm\left(8 - \frac{\chi(\mathcal{S})}{6}\right)$$

[Minasian,Moore:hep-th/9710230],
[Morales,Scrucca,Serone:hep-th/9812071], [Stefanski:hep-th/9812088],
[Scrucca,Serone:hep-th/9903145]

# RR charge vectors

$$\Gamma(\mathcal{O}(-1)[0]) = [\mathcal{S}] \wedge \left(1 + \frac{1}{2}\ell + \frac{1}{4}\ell^2\right)$$

$$\Gamma(\Omega[1]) = [\mathcal{S}] \wedge \left(-2 + \frac{1}{2}\ell^2\right)$$

$$\Gamma(\mathcal{O}(-2)[2]) = [\mathcal{S}] \wedge \left(1 - \frac{1}{2}\ell + \frac{1}{4}\ell^2\right)$$

$$\Gamma(O7^\pm) = \pm[\mathcal{S}] \wedge \left(8 - \frac{1}{2}\ell^2\right) .$$

Notice:

- $\sum \Gamma(\mathcal{E}_i) = \ell^2$.
- $O7^+ + 4\Omega[1]$ cancels compact tadpoles.
- $O7^- + 4(\mathcal{O}(-1)[0] + \mathcal{O}(-2)[2])$ cancels compact tadpoles.

# Microscopic description of the quiver

The quiver is thus given by, for $O7^+$

| | $USp(\widetilde{N}+4)$ | $U(\widetilde{N})$ | $SU(3)$ |
|---|---|---|---|
| $A^i$ | $\overline{\square}$ | $\square$ | $\square$ |
| $B^i$ | $1$ | $\overline{\square\square}$ | $\square$ |

and for $O7^-$:

| | $SO(N-4)$ | $U(N)$ | $SU(3)$ |
|---|---|---|---|
| $A^i$ | $\overline{\square}$ | $\square$ | $\square$ |
| $B^i$ | $1$ | $\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ | $\square$ |

As we expected from CFT.

# $O7^+$ at strong coupling

We need to understand the strongly coupled limit of the $O7^+$ in flat space. In F-theory, the $O7^+$ is given by a (frozen) singularity with $D_8$ monodromy. [Witten:hep-th/9712028]

Such a monodromy can be achieved by considering a $BCA^8$ system, where $A$ is a $(1,0)$ 7-brane (a $D7$), $B$ a $(1,1)$ 7-brane, and $C$ a $(1,-1)$ 7-brane.

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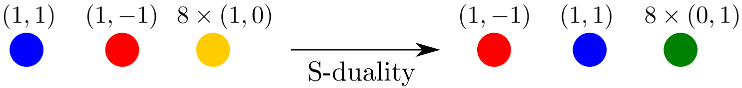
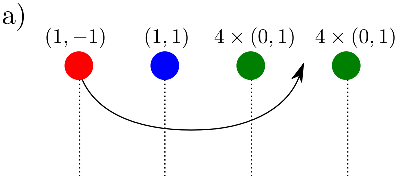
Under S-duality, this configuration becomes $CBX^8_{(0,1)}$. We want to describe this as a $O7^-$ plane plus other 7-branes.

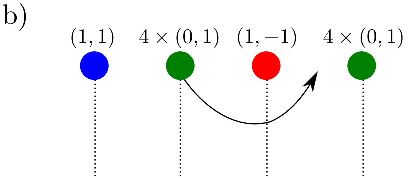


# $O7^+$ at strong coupling

## S-duality

$$O7^+ \longleftrightarrow O7^- + 4A + 4X_{(0,1)}$$



# Application to the $\mathbb{C}^3/\mathbb{Z}_3$ example

In our case:

$$O7^+ \to O7^- + 4\big(\mathcal{O}(-1)[0] + \mathcal{O}(-2)[2]\big) + 4\big(\widehat{\mathcal{O}(-1)[0]} + \widehat{\mathcal{O}(-2)[2]}\big) \\ + n\, D3s\,.$$

Here $n$ measures the mismatch in $D3$ charge, due to choosing the fractional branes as the $A$ branes. We have $n = -5$.
Starting with $O7^+ + 4\Omega[1] + \widetilde{N}\, D3s$, the dual is:

$$O7^- + 4\big(\mathcal{O}(-1)[0] + \mathcal{O}(-2)[2]\big) + 4\big(\widehat{\mathcal{O}(-1)[0]} + \widehat{\mathcal{O}(-2)[2]} + \widehat{\Omega[1]}\big) \\ +(\widetilde{N} - 5)\, D3s \\ = O7^- + 4\big(\mathcal{O}(-1)[0] + \mathcal{O}(-2)[2]\big) + (\widetilde{N} - 1)\, D3s\,.$$

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$$USp(\widetilde{N}+4) \times SU(\widetilde{N}) \longleftrightarrow SO(\widetilde{N}-1) \times SU(\widetilde{N}+3)$$

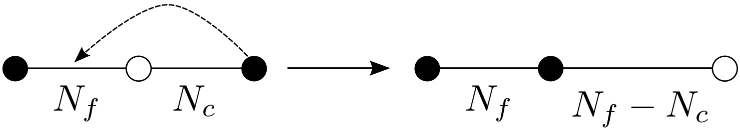
# Conclusions

- New class of possible $\mathcal{N} = 1$ duals.
- The duality seems to follow from S-duality.
- Other examples: phase II of $\mathbb{C}^3/\mathbb{Z}_3$, phase II of $\mathcal{C}(\mathbb{F}_0)$, $\mathcal{C}(dP_1)$, $\mathbb{C}^3/\mathbb{Z}_k$, ...
- Developed tools for analyzing orientifolds at small volume [Diaconescu, Garcia-Raboso, Karp, Sinha], [Brunner, Herbst], [Gao, Hori], [Franco, Hanany, Krefl, Park, Uranga, Vegh], ...
- Got some information about the strongly coupled behavior of the $O7^+$ (although still somewhat mysterious).

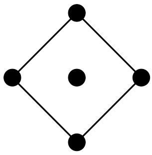
# Stringy realization of Seiberg duality

Statement of the duality: $\mathcal{N}=1$ $SU(N_c)$ SQCD with $N_f$ flavors flows to the same IR as $SU(N_f-N_c)$ with $N_f$ flavors $Q,\widetilde{Q}$, a meson $M$, and a cubic superpotential $W=QM\widetilde{Q}$.

It can typically be realized as motion in moduli space.

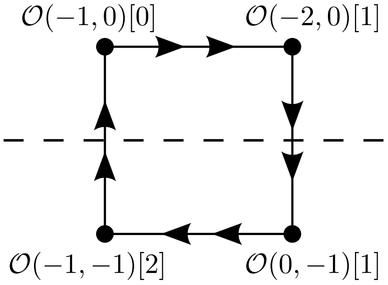


# Complex cone over $\mathbb{F}_0 = \mathbb{P}^1 \times \mathbb{P}^1$



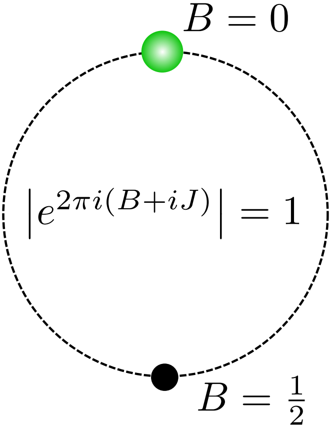
Can be seen as a $\mathbb{Z}_2$ orbifold of the conifold (so not an orbifold of flat space).

Also, from field theory it seems that there is a single possible orientifold one could take:



# Complex cone over $\mathbb{F}_0 = \mathbb{P}^1 \times \mathbb{P}^1$

Interestingly, the orientifold seems to require $B = \left(0, \frac{1}{2}\right)$. This agrees with what one expects from looking to the quiver locus in Kähler moduli space: [Aspinwall,Melnikov:hep-th/0405134]



# Complex cone over $\mathbb{F}_0 = \mathbb{P}^1 \times \mathbb{P}^1$

Taking $\mathcal{L}_{2B} = \mathcal{O}(0,1)$ orientifold now acts as expected, giving:

$$
\begin{aligned}
\mathcal{O}(-1,0)[0] \longrightarrow & (\mathcal{E}^\vee \otimes K_\mathcal{S} \otimes \mathcal{L}_{2B})[2-n] \\
&= \mathcal{O}(1,0) \otimes \mathcal{O}(-2,-2) \otimes \mathcal{O}(0,1)[2-0] \\
&= \mathcal{O}(-1,-1)[2] \\
\mathcal{O}(-2,0)[1] \longrightarrow & \mathcal{O}(2,0) \otimes \mathcal{O}(-2,-2) \otimes \mathcal{O}(0,1)[2-1] \\
&= \mathcal{O}(0,-1)[1]
\end{aligned}
$$

# Complex cone over $\mathbb{F}_0 = \mathbb{P}^1 \times \mathbb{P}^1$

We can cancel the $O7^+$ tadpole by adding $4(\mathcal{O}(-2,0)[1] + \mathcal{O}(0,-1)[1])$. The resulting theory is given by:

| $U(N)$ | $U(N+4)$ | $SU(2)_1$ | $SU(2)_2$ |
|---|---|---|---|
| $\square$ | $\overline{\square}$ | $\square$ | $1$ |
| $\overline{\square\!\square}$ | $1$ | $1$ | $\square$ |
| $1$ | $\begin{smallmatrix}\square\\ \square\end{smallmatrix}$ | $1$ | $\square$ |

Alternatively, we can introduce a $O7^-$, and cancel the tadpole by $4(\mathcal{O}(-1,0)[0] + \mathcal{O}(-1,-1)[2])$. The resulting theory is given by:

| $U(N+4)$ | $U(N)$ | $SU(2)_1$ | $SU(2)_2$ |
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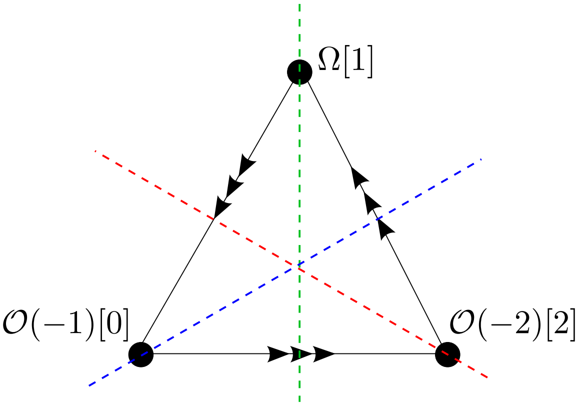
| $\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ | $1$ | $1$ | $\square$ |
| $1$ | $\square\square$ | $1$ | $\square$ |

Isomorphic theories

6 Another example: the complex cone over $\mathbb{F}_0$

7 An aside: ærientifolds (quantum orientifolds)

# Other orientifolds?




Clearly not $\mathcal{E}[n] \longrightarrow (\mathcal{E}^\vee \otimes K_\mathcal{S} \otimes \mathcal{L}_{2B})[2-n]$ ...

# Quantum symmetries

Three special points in the ($\mathbb{P}^1$) Kähler moduli space of $\mathbb{C}^3/\mathbb{Z}_3$:

- **Large volume:** Monodromy around this point sends $B \to B+1$, or equivalently:

$$\mathcal{M}_{LV} : \mathcal{E} \to \mathcal{E} \otimes \mathcal{O}(-1)$$

- **The conifold point:** A brane ($\mathcal{O}(-1)$) becomes massless. Monodromy around this point gives:

$$\mathcal{M}_C : \Gamma(\mathcal{E}) \to \Gamma(\mathcal{E}) - \langle \mathcal{E}, \mathcal{O}(-1) \rangle \Gamma(\mathcal{O}(-1))$$

- **The quiver point:** Monodromy around this point can be obtained as the inverse of the products of the monodromies above: [Aspinwall:hep-th/0403166]

$$\mathcal{M}_Q = (\mathcal{M}_{LV}\mathcal{M}_C)^{-1} = \begin{pmatrix} -\frac{1}{2} & 3 & 0 \\ \frac{1}{4} & -\frac{1}{2} & 0 \\ \frac{1}{8} & \frac{1}{4} & 1 \end{pmatrix} .$$

# Quantum orientifolds

We can thus dress the ordinary orientifold action by the quantum symmetry: [Brunner,Hori,Hosomichi,Walcher:hep-th/0401137] [Diaconescu,Garcia-Raboso,Karp,Sinha:hep-th/0606180]

$$\mathcal{P}' = \mathcal{M}_Q^{-1}\mathcal{P}\mathcal{M}_Q = \begin{pmatrix} -\frac{1}{2} & 3 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{8} & -\frac{1}{4} & 1 \end{pmatrix}.$$

with

$$\begin{aligned} \mathcal{P}'\mathcal{O}(-1)[0] &= \mathcal{O}(-1)[0] \\ \mathcal{P}'\mathcal{O}(-2)[2] &= \Omega[1] \\ \mathcal{P}'\Omega[1] &= \mathcal{O}(-2)[2] \end{aligned}$$

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$$\mathcal{P}' = \mathcal{M}_Q^{-1}\mathcal{P}\mathcal{M}_Q = \begin{pmatrix} -\frac{1}{2} & 3 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{8} & -\frac{1}{4} & 1 \end{pmatrix}.$$

with

$$\begin{aligned} \mathcal{P}'\mathcal{O}(-1)[0] &= \mathcal{O}(-1)[0] \\ \mathcal{P}'\mathcal{O}(-2)[2] &= \Omega[1] \\ \mathcal{P}'\Omega[1] &= \mathcal{O}(-2)[2] \end{aligned}$$

Essential in other cases, for example $\mathcal{C}(dP_1)$.