Scaling Laws in Aeolian Sand Transport

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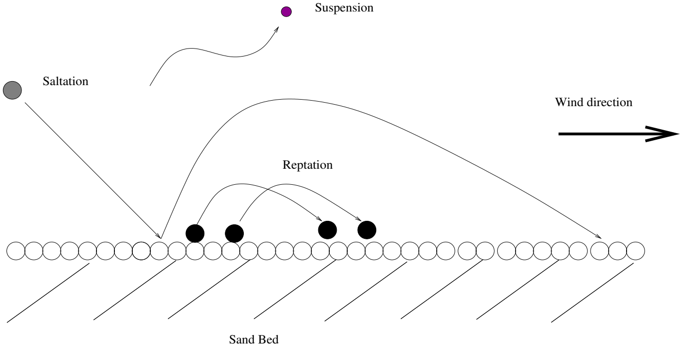
Saltation transport (Bagnold, 1941)

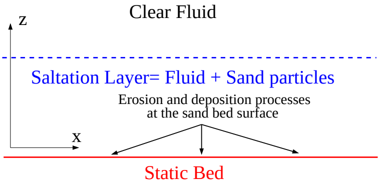


- Two modes of transport : (i) Saltation and (ii) Reptation

Motivation

- Understand the saltation transport and the peculiar role of the sand bed :

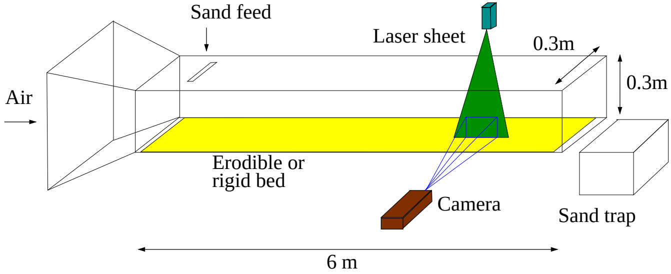
- What sets the height of the saltation layer ?
- What sets the particle velocity ?
- What sets the particle concentration ?



- Strategy : Investigate the salation transport in wind-tunnel using different boundary conditions at the bed : Erodible Bed versus Rigid Bed

Wind tunnel Experiments (LTN, Nantes)

- Length : 10 m
- Section : 0.3 $m \times 0.3$ m
- Two types of basal boundary :
  i) Erodible sand bed and ii) Rigid bed



Instruments and Methods

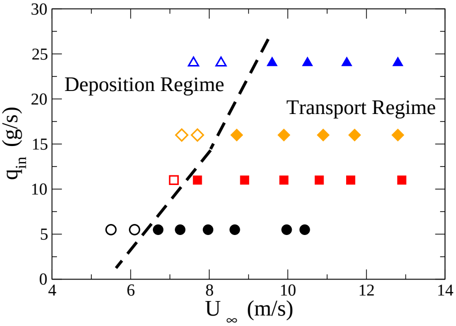
- Median Grain diameter : 230 μm
- Control parameters :
  -Erodible bed : Air shear velocity u^*
  -Rigid bed : Shear velocity u^* and incoming sand flux Q_{in}

Basal shear stress : $S^* = \rho_{air} u^{*2}$

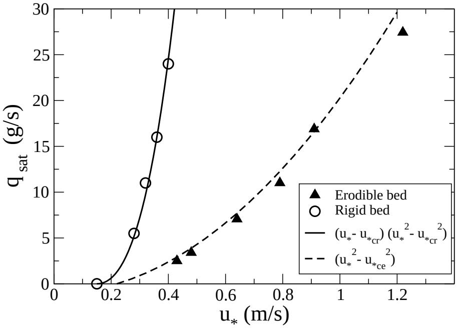
Measured quantities	Experimental Techniques
Air Velocity $U(y)$	Pitot tube
Particle Velocity $u(y)$	Particle Tracking Velocimetry
Particle Volume Fraction $\nu(y)$	Particle counting
Mass Sand Flux Q	Sand trap

Phase diagram and flow transport capacity

- Phase diagram



- Mass flow rate



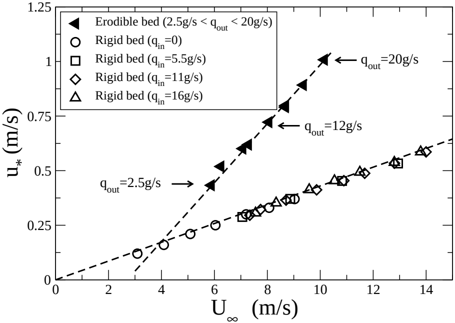
- Erodible bed : $Q_{sat} \propto (u^{*2} - u_c^{*2})$
- Rigid bed : $Q_{sat} \propto (u^* - u_c^*)(u^{*2} - u_c^{*2})$

(Ho et al., PRL 2011)

Air velocity

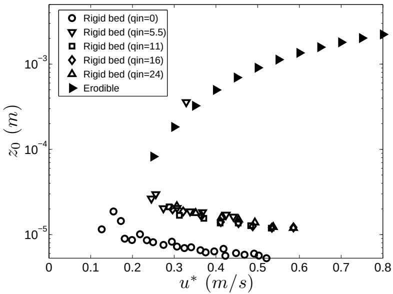
- For both erodible and rigid bed : $U(z) = (u^*/\kappa)\ln(z/z_0)$

- Friction velocity



Basal friction more important on the erodible bed

- Roughness length z_0

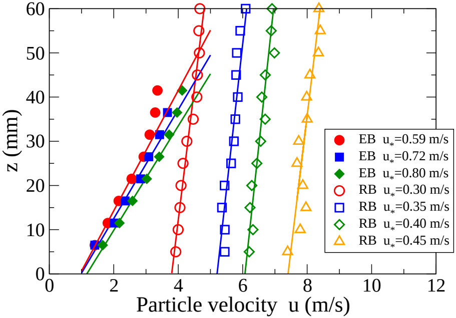


Roughness length larger on the erodible bed

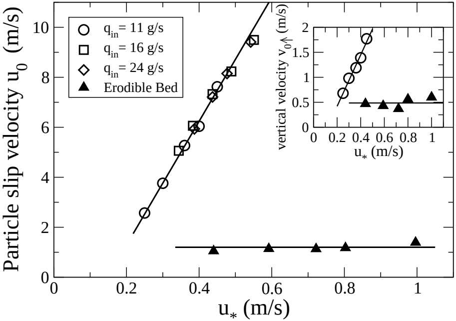
(Ho et al., PRL 2011)

Particle velocity

- Velocity profile

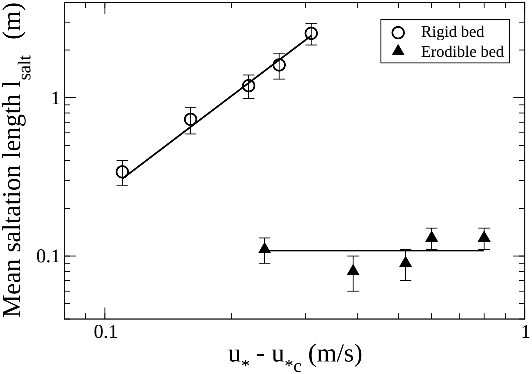


- Slip velocity



- Erodible bed : $u_0 \approx cst$ (independent of u^*)
- Rigid bed : $u_0 \propto (u^* - u^*_c)$

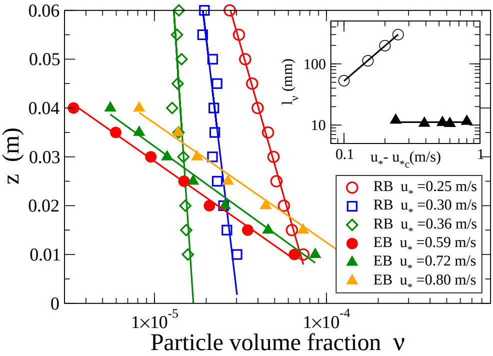
Mean saltation length



- Erodible bed : l_{salt} independent of u^*
- Rigid bed : $l_{salt} \propto (u^{*2} - u_c^{*2})/g$

Particle concentration profile

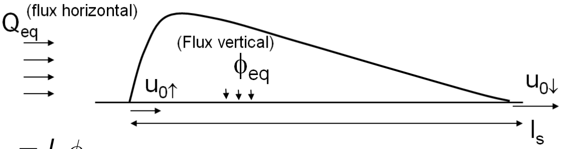
- Exponential profile : $\nu(z) \approx \nu_0 \exp(-z/l_\nu)$



- Erodible bed :
  $l_\nu \approx cst$
  $\nu_0 \propto (u^{*2} - u_c^{*2})$
- Rigid bed :
  $l_\nu \propto (u^{*2} - u_c^{*2})/g$
  $\nu_0 \propto Q_{in}/(u^* - u_c^*)(u^{*2} - u_c^{*2})$

Bagnold law (1)

- Hypothesis :
  We replace the distribution of trajectories by an 'averaged' trajectory of length l_s, height h_s, …



  - $Q_{eq} = l_s \, \phi_{eq}$
  - Particle shear stress : $s_{grain}(z = 0) = \phi_{eq}(u_{0\downarrow} - u_{0\uparrow})$

- Momentum conservation :
  $s_{grain}(z) + S_{air}(z) = cst = \rho_{air} u^{*2} = S^*$

$$\Rightarrow Q_{eq} = l_s \, \phi_{eq} = l_s \frac{(S^* - S_{air}(0))}{(u_{0\downarrow} - u_{0\uparrow})}$$

Bagnold law(2)

- Owen Hypothesis : $S_{air}(z=0) \approx S_{threshold}$
- Bagnold Hypotheses :
  $l_s \propto (u^{*2}/g)$ and $(u_{0\downarrow} - u_{0\uparrow}) \approx u_{0\downarrow} \propto u^*$

$$\Rightarrow Q_{eq} \propto \frac{\rho_{air}}{g} u^* \left(u^{*2} - u^{*2}_{threshold} \right)$$

(Lettau & Lettau 1978)

- Asymptotic Behavior : $Q_{eq} \propto \frac{\rho_{air}}{g} u^{*3}$ *(Bagnold 1941)*

Experimental Scaling laws

- Equilibrium Flux :

$$Q_{eq} = \rho_{air}\, l_s \frac{(u^{*2} - u_c^{*2})}{(u_{0\downarrow} - u_{0\uparrow})}$$

- Experimental Outcomes :
  - Erodible bed

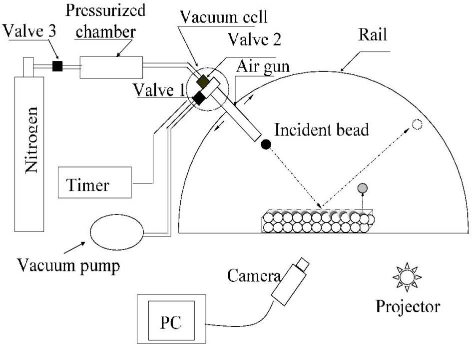
$$\begin{cases} l_s = const \\ u_{0\downarrow}, u_{0\uparrow} = const \end{cases} \Rightarrow Q_{eq} \propto (u^{*2} - u_c^{*2})$$

  - Rigid bed

$$\begin{cases} l_s \sim u^{*2} \\ u_{0\downarrow}, u_{0\uparrow} \sim (u^{*} - u_c^{*}) \end{cases} \Rightarrow Q_{eq} \propto (u^{*} - u_c^{*})(u^{*2} - u_c^{*2})$$

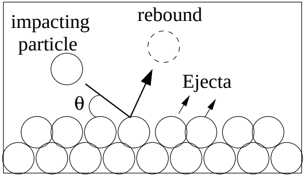
Splash Process

- Model Collision Experiment (Beladjine et al, PRE 2006)

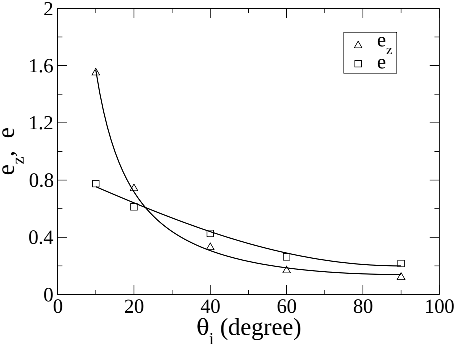


Rebound

- Experimental outcomes *(Beladjine et al, PRE 2007)*

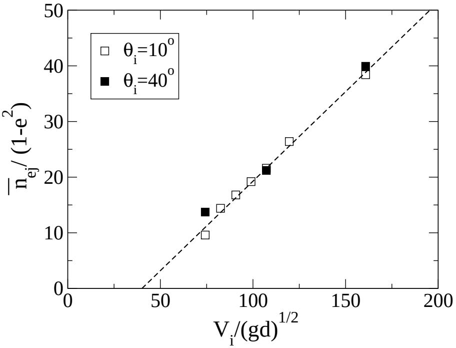


$$\xi' = e\xi = (A - B\sin\theta)\xi$$
$$\xi'_y = -e_y\xi_y = -(A_y/\sin\theta - B_y)\xi_y$$



Splash process : Ejected particles

- Number of ejected particles *(Beladjine et al, PRE 2007)*



- $N_{ej} = N_0\,(1-e^2)\,(\xi/\xi_0 - 1) \quad \text{for } \xi > \xi_0$
- ξ_0 : Threshold velocity for ejection ($\xi_0 \approx 40\,\sqrt{gd}$)

Importance of the splash process

- Balance equations at the bed *(Creyssels et al., JFM 2009)*
  - Mass : $n_{ej} \approx 0 \Rightarrow u_{0\downarrow} \approx \xi_0$
  - Particle Vertical momentum :
    $v_{0\uparrow}/v_{0\downarrow} \gtrsim 1 \Rightarrow e_z(\theta_i) \gtrsim 1 \Rightarrow \theta_i \approx 10°$
  - Particle horizontal momentum :
    $s_0/p_0 = \mu(e_z, e) \approx 0.6$
    with $s_0 \propto \nu_0 v_{0\downarrow}(u_{0\downarrow} - u_{0\uparrow})$ and $p_0 \propto \nu_0 v_{0\downarrow}$
    $\Rightarrow (u_{0\downarrow} - u_{0\uparrow}) \approx \mu v_{0\downarrow}$
    $\Rightarrow \nu_0 \approx (S^* - S_0)/\rho_{air} v_{0\downarrow}^2$

- Consequences
  - The particle velocity is completely controlled by the splash process
  - The particle concentration is driven by the wind strength

Conclusion

- Erodible bed :
  - Saltation layer height, saltation hopping length and particle velocity in the saltation layer are controlled by the Splash process
  - These quantities are invariant with the wind strength
  - The transport rate scales quadratically with the shear velocity
- Rigid bed :
  - Saltation layer height, saltation hopping length and particle velocity in the saltation layer are driven by the wind strength
  - The maximun transport rate scales as the cubic power of the shear velocity