Scaling Laws in Aeolian Sand Transport

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**Saltation transport (Bagnold, 1941)**

- Two modes of transport: (i) Saltation and (ii) Reptation
Motivation

- Understand the saltation transport and the peculiar role of the sand bed:
  - What sets the height of the saltation layer?
  - What sets the particle velocity?
  - What sets the particle concentration?

- Strategy: Investigate the salation transport in wind-tunnel using different boundary conditions at the bed: Erodible Bed versus Rigid Bed
Wind tunnel Experiments (LTN, Nantes)

- Length : 10 m
- Section : 0.3 m × 0.3 m
- Two types of basal boundary :
  1) Erodible sand bed and 2) Rigid bed
Instruments and Methods

- Median Grain diameter: 230 $\mu m$
- Control parameters:
  - Erodible bed: Air shear velocity $u^*$
  - Rigid bed: Shear velocity $u^*$ and incoming sand flux $Q_{in}$
- Basal shear stress: $S^* = \rho_{air} u^*^2$

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Introduction

Experimental Set-up

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Discussion and Interpretation

Conclusion
Phase diagram and flow transport capacity

Phase diagram

- Deposition Regime
- Transport Regime

Mass flow rate

- Erodible bed: \( Q_{sat} \propto (u^* - u_{cr})(u^* - u_{cr}^2) \)
- Rigid bed: \( Q_{sat} \propto (u^* - u_{cr})^2(u^* - u_{cr}^2) \)

(Ho et al., PRL 2011)
**Air velocity**

- For both erodible and rigid bed: \( U(z) = \left( \frac{u^*}{\kappa} \right) \ln\left( \frac{z}{z_0} \right) \)

**Friction velocity**

**Roughness length \( z_0 \)**

Basal friction more important on the erodible bed

Roughness length larger on the erodible bed

\((Ho \text{ et al., PRL 2011})\)
Particle velocity

- **Velocity profile**

- **Slip velocity**

- Erodible bed: \( u_0 \approx \text{cst} \) (independent of \( u^* \))

- Rigid bed: \( u_0 \propto (u^* - u^*_c) \)
Mean saltation length

- Erodible bed: $l_{salt}$ independent of $u^*$
- Rigid bed: $l_{salt} \propto (u^2 - u_c^2)/g$
Particle concentration profile

- **Exponential profile**: \( \nu(z) \approx \nu_0 \exp\left(-\frac{z}{l_\nu}\right) \)

- **Erodible bed**: 
  \[ l_\nu \approx \text{cst} \]
  \[ \nu_0 \propto (u^* - u_c^*) \]

- **Rigid bed**: 
  \[ l_\nu \propto \frac{(u^* - u_c^*)}{g} \]
  \[ \nu_0 \propto \frac{Q_{in}}{(u^* - u_c^*)}\left(u^2 - u_c^2\right) \]
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**Hypothesis:**
We replace the distribution of trajectories by an 'averaged' trajectory of length \( l_s \), height \( h_s \), . . .

\[ Q_{eq} = l_s \phi_{eq} \]

**Particle shear stress:**
\[ s_{grain}(z = 0) = \phi_{eq}(u_0\downarrow - u_0\uparrow) \]

**Momentum conservation:**
\[ s_{grain}(z) + S_{air}(z) = cst = \rho_{air}u^*2 = S^* \]

\[ \Rightarrow Q_{eq} = l_s \phi_{eq} = l_s \frac{(S^* - S_{air}(0))}{(u_0\downarrow - u_0\uparrow)} \]
Bagnold law (2)

- Owen Hypothesis: \( S_{air}(z = 0) \approx S_{threshold} \)

- Bagnold Hypotheses:
  \[ l_s \propto \left( \frac{u^*}{g} \right)^2 \quad \text{and} \quad (u_0↓ - u_0↑) \approx u_0↓ \propto u^* \]
  \[ \Rightarrow Q_{eq} \propto \frac{\rho_{air}}{g} u^* \left( u^* - u_{threshold}^* \right) \]

(Lettau & Lettau 1978)

- Asymptotic Behavior: \( Q_{eq} \propto \frac{\rho_{air}}{g} u^* \) (Bagnold 1941)
Experimental Scaling laws

- Equilibrium Flux:

\[ Q_{eq} = \rho_{air} l_s \frac{(u^* - u_c^*)}{(u_0\downarrow - u_0\uparrow)} \]

- Experimental Outcomes:
  - Erodible bed
    \[ \begin{align*}
    l_s &= \text{const} \\
    u_0\downarrow, u_0\uparrow &= \text{const}
    \end{align*} \Rightarrow Q_{eq} \propto (u^* - u_c^*) \]
  - Rigid bed
    \[ \begin{align*}
    l_s &\sim u^* \\
    u_0\downarrow, u_0\uparrow &\sim (u^* - u_c^*)
    \end{align*} \Rightarrow Q_{eq} \propto (u^* - u_c^*)(u^* - u_c^*) \]
Splash Process

- Model Collision Experiment (Beladjine et al, PRE 2006)
Experimental outcomes *(Beladjine et al, PRE 2007)*

\[
\xi' = e \xi = (A - B \sin \theta) \xi \\
\xi'_y = -e_y \xi_y = -(A_y / \sin \theta - B_y) \xi_y
\]
Splash process: Ejected particles

- Number of ejected particles \((Beladjine\ et\ al,\ PRE\ 2007)\)

\[
N_{ej} = N_0 (1 - e^2) (\xi/\xi_0 - 1) \quad \text{for} \quad \xi > \xi_0
\]

- \(\xi_0\): Threshold velocity for ejection \((\xi_0 \approx 40 \sqrt{gd})\)
Importance of the splash process

- Balance equations at the bed (Creyssels et al., JFM 2009)
  - Mass: \( n_{ej} \approx 0 \Rightarrow u_{0\downarrow} \approx \xi_0 \)
  - Particle Vertical momentum:
    \[ \nu_{0\uparrow}/\nu_{0\downarrow} \gtrsim 1 \Rightarrow e_z(\theta_i) \gtrsim 1 \Rightarrow \theta_i \approx 10^\circ \]
  - Particle horizontal momentum:
    \[ s_0/p_0 = \mu(e_z, e) \approx 0.6 \]
    with \( s_0 \propto \nu_0 \nu_{0\downarrow}(u_{0\downarrow} - u_{0\uparrow}) \) and \( p_0 \propto \nu_0 \nu_{0\downarrow} \)
    \[ \Rightarrow (u_{0\downarrow} - u_{0\uparrow}) \approx \mu \nu_{0\downarrow} \]
    \[ \Rightarrow \nu_0 \approx (S^* - S_0)/\rho_{air}v_{0\downarrow}^2 \]

- Consequences
  - The particle velocity is completely controlled by the splash process
  - The particle concentration is driven by the wind strength
Conclusion

Erodible bed:
- Saltation layer height, saltation hopping length and particle velocity in the saltation layer are controlled by the Splash process.
- These quantities are invariant with the wind strength.
- The transport rate scales quadratically with the shear velocity.

Rigid bed:
- Saltation layer height, saltation hopping length and particle velocity in the saltation layer are driven by the wind strength.
- The maximum transport rate scales as the cubic power of the shear velocity.