Transition in the dense granular flow down an inclinded plane.

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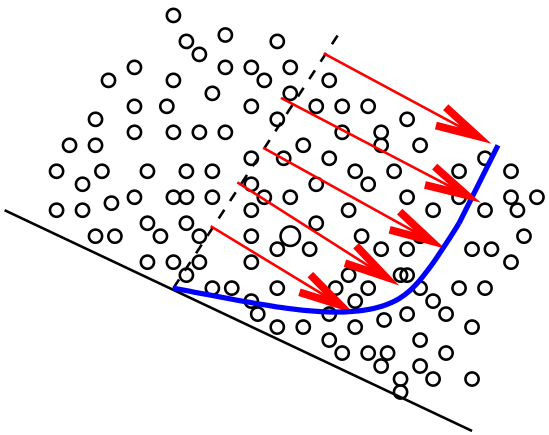
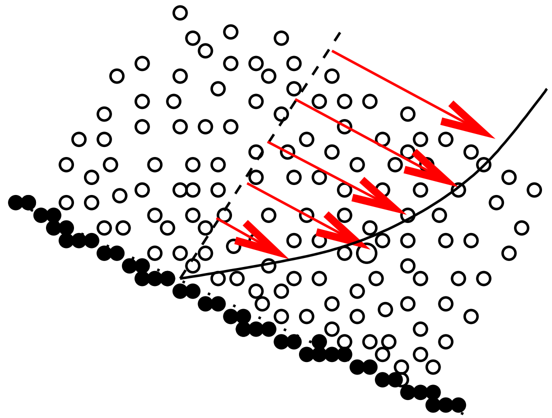
Flow regimes:

Flat frictional:



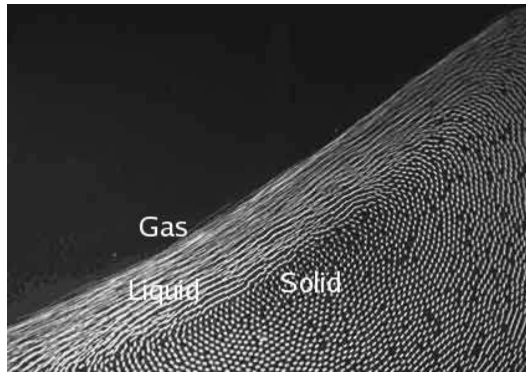
Plug + Shear layer.

Bumpy:



Homogeneous shearing.

Erodible:



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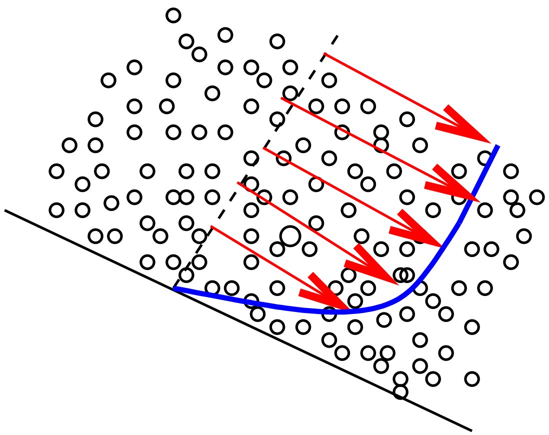

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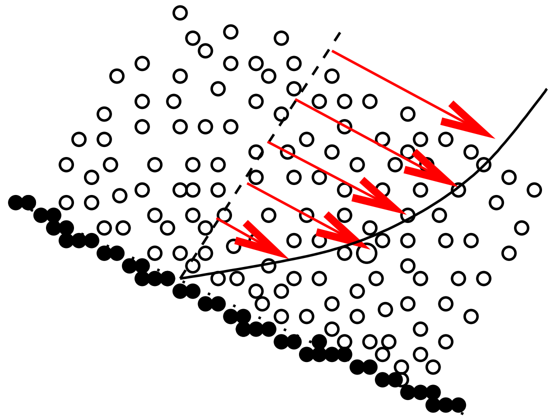

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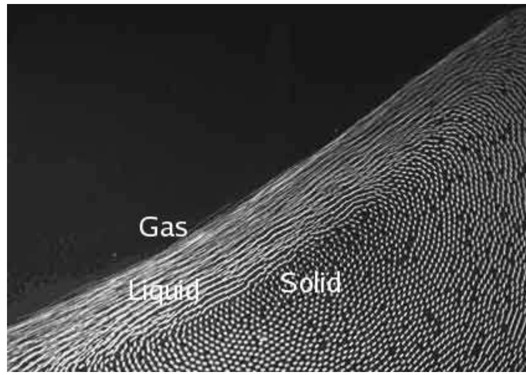
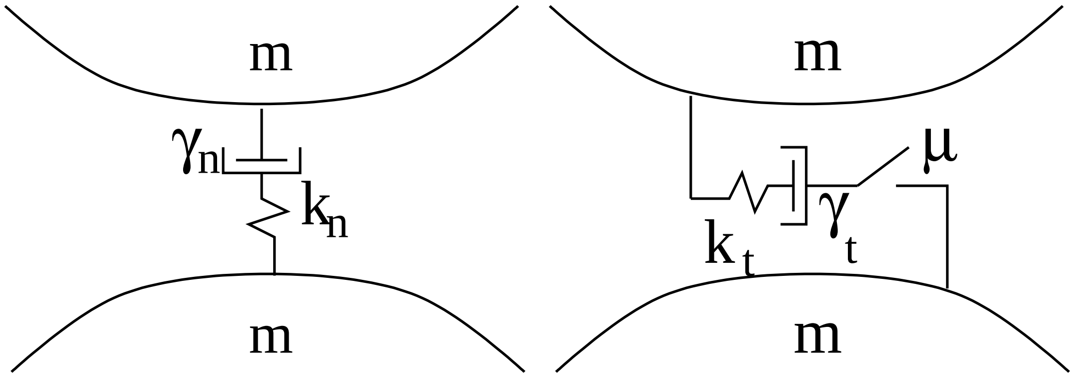


Flow down inclined plane (DEM simulations):

Particle interaction: Linear Spring-dashpot-slider model



$$\mathbf{F}_{ij}^{n} = k_n \delta_{ij} \mathbf{n}_{ij} - \gamma_n m_{eff} \mathbf{v}_{ij}^{n}$$

$$\mathbf{F}_{ij}^{t} = \mathrm{Min}\left(k_t \mathbf{u}_{\mathbf{t}_{\mathbf{ij}}} - \gamma_t m_{eff} \mathbf{v}_{ij}^{t}, \mu F_{ij}^{n} \right)$$

$t_{col} = (\pi / \sqrt{2k_n/m - \gamma_n^2/4})$,

Restitution coeff, $e = \exp\left(-\gamma_n t_{col}/2\right)$.

Simulations: $(k_n/(mg/d)) = 10^6$, $e = 0.6$, $\mu = 0.4$

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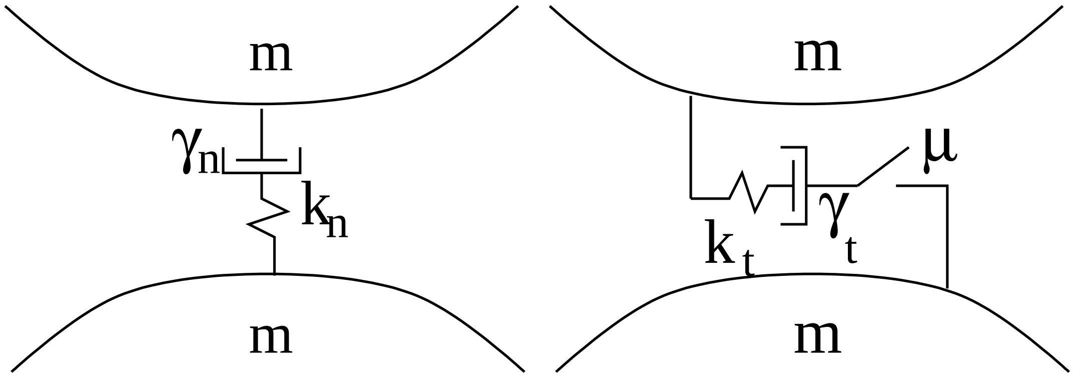
Flow down inclined plane (DEM simulations):

Microscopic contact models: Hertzian spring-dashpot:



$$F_n = \sqrt{(\delta/d)}(-k_n\delta - \gamma_n m_{eff} v_n)$$

$$\mathbf{F}^t_{ij} = \mathbf{Min}\left(k_t\sqrt{\delta_{ij}/d)}\mathbf{u_{t_{ij}}} - \gamma_t m_{eff}\mathbf{v}^t_{ij}, \mu F^n_{ij}\right)$$

Simulations: $(k_n/(mg/d^{1/2})) = 10^6$, $\gamma_n/\sqrt{g/d} = 454$, $\mu = 0.4$

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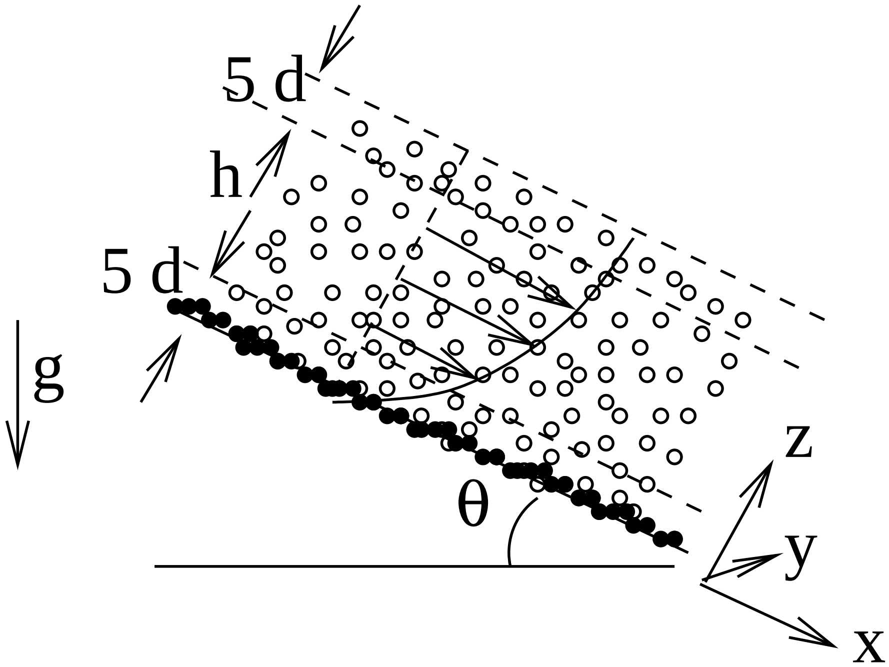
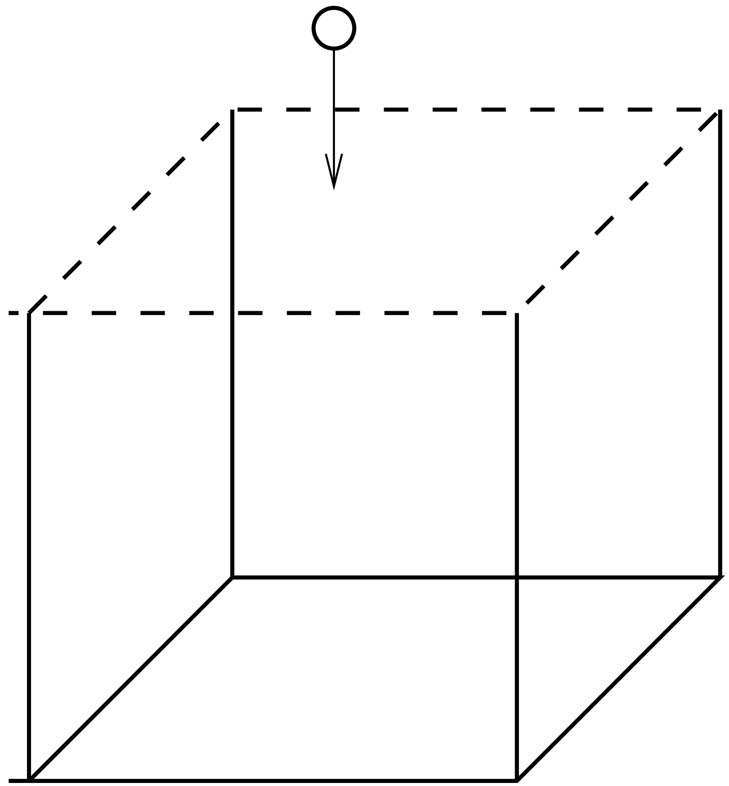
Flow down inclined plane: Configuration.

- Simulation box
  $x : y = 40d : 20d$; $80d : 40d$;
  $160d : 80d$.
- Periodic streamwise (x)
  and spanwise (y).
- Bottom rough base, top free
  surface.
- Two system sizes
  $32000, 64000$ particles
  ($\approx 35d, 70d$ height)
- Random base configuration.



Flow down inclined plane: Initial state.

- Vertical box of desired base dimensions.
- Sequentially drop particles into box to desired height.



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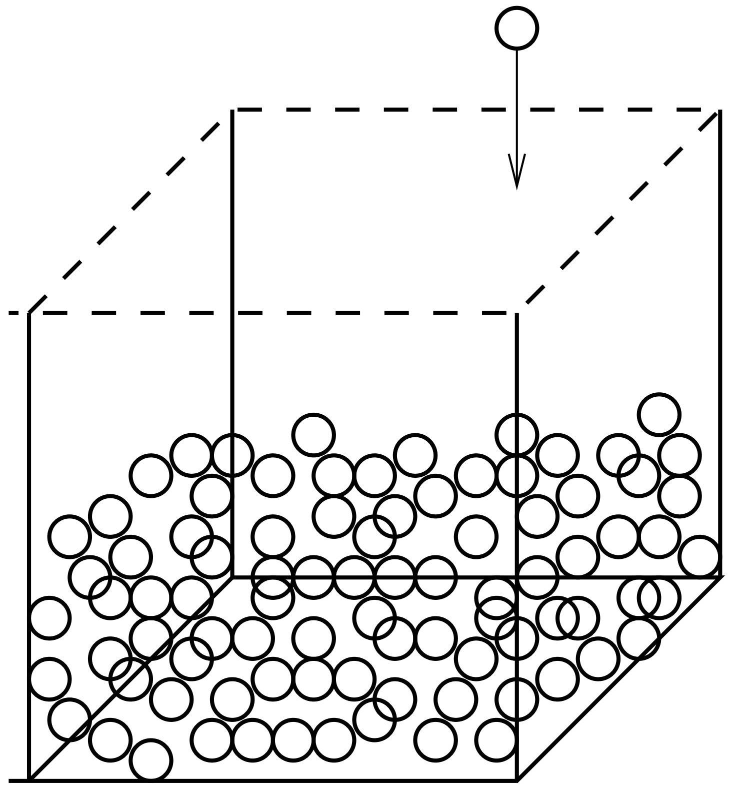
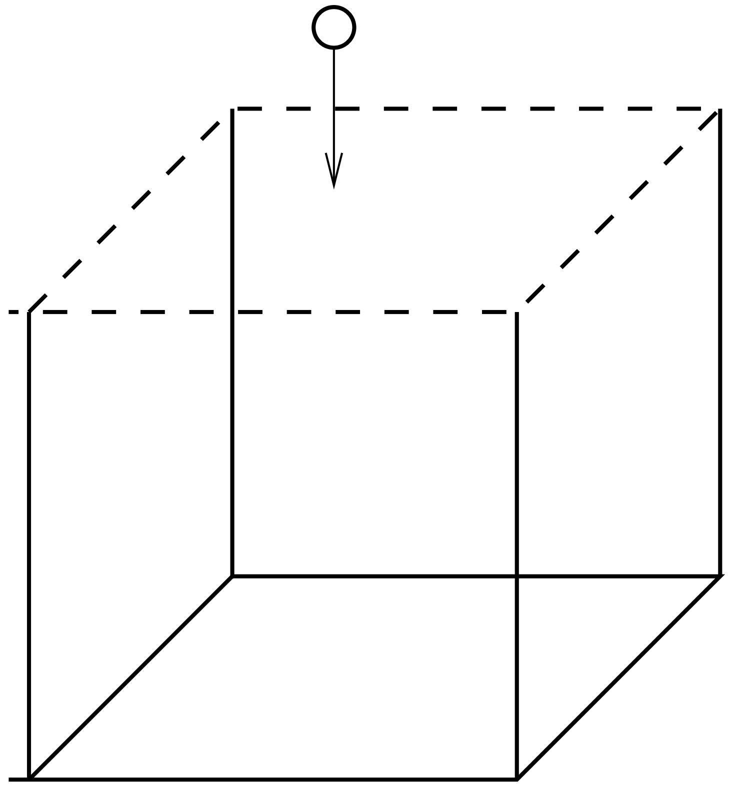


Flow down inclined plane: Base particle roughness.

- Vertical box of desired base dimensions.
- Sequentially drop particles into box.
- Select a layer of particles with centers within height d_b.
- Use this as frozen base.
- Vary ratio of diameters of base and moving particles, (d_b/d_f).



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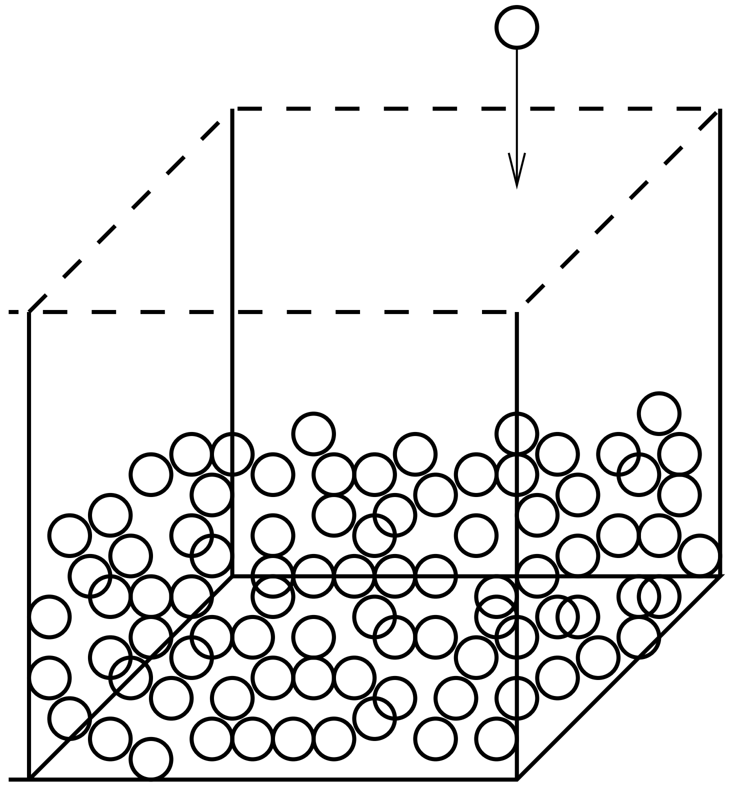

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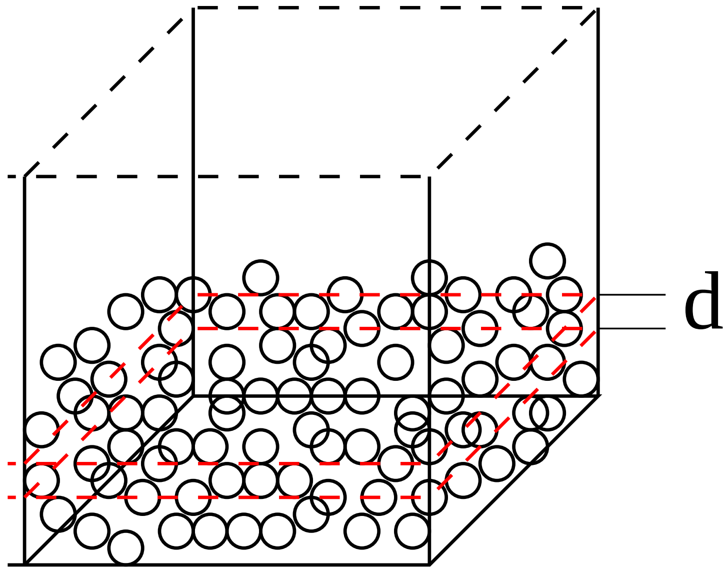

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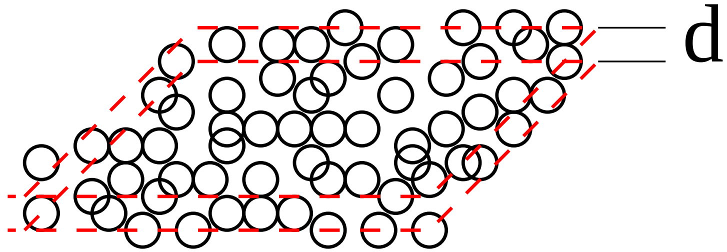

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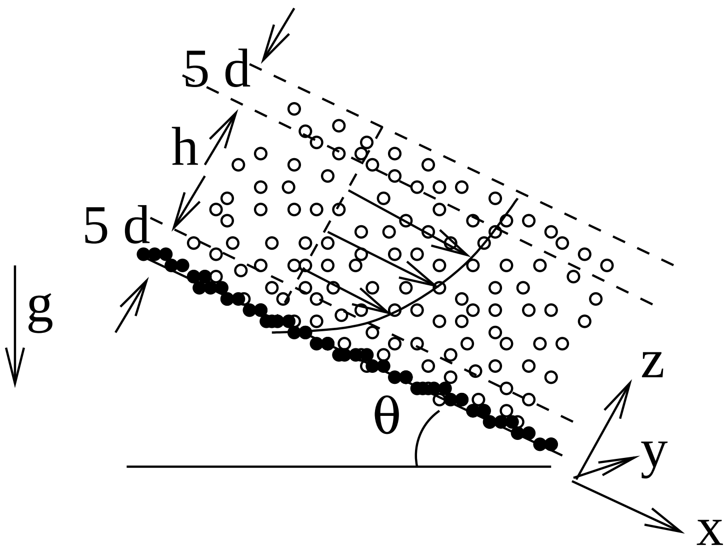

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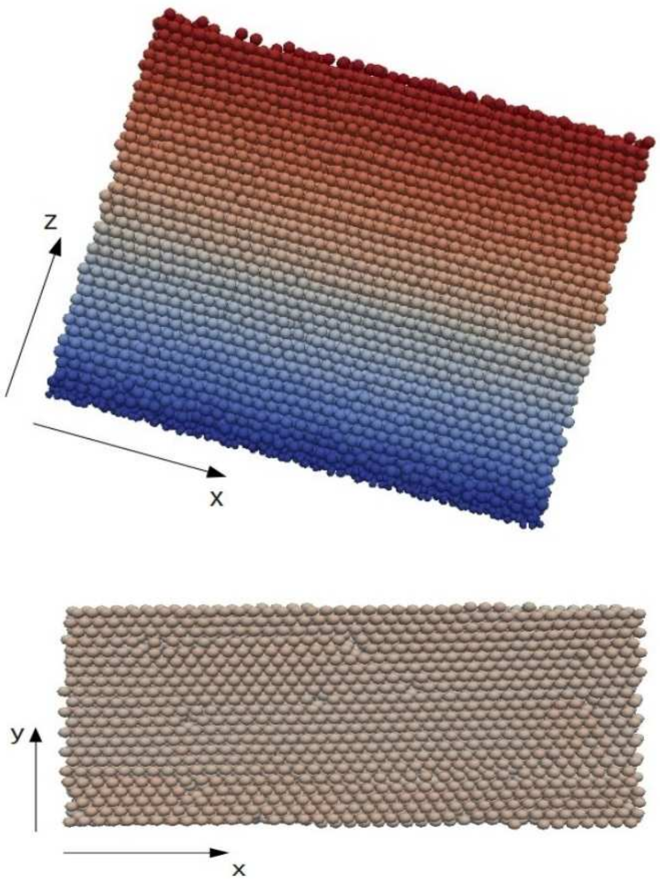
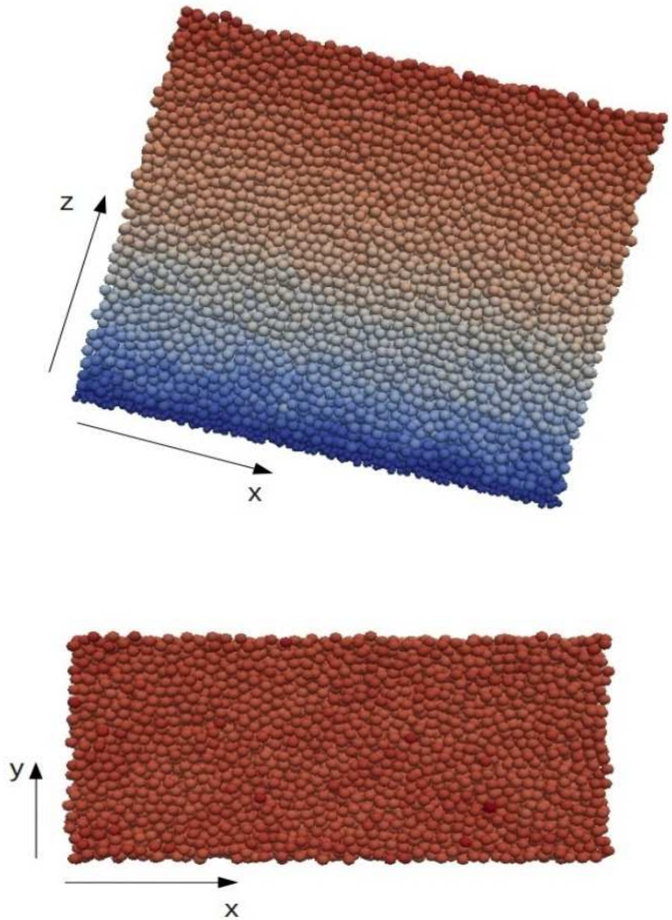
Flow down inclined plane: Flow regimes.



$$(d_b/d_f) = 0.61$$



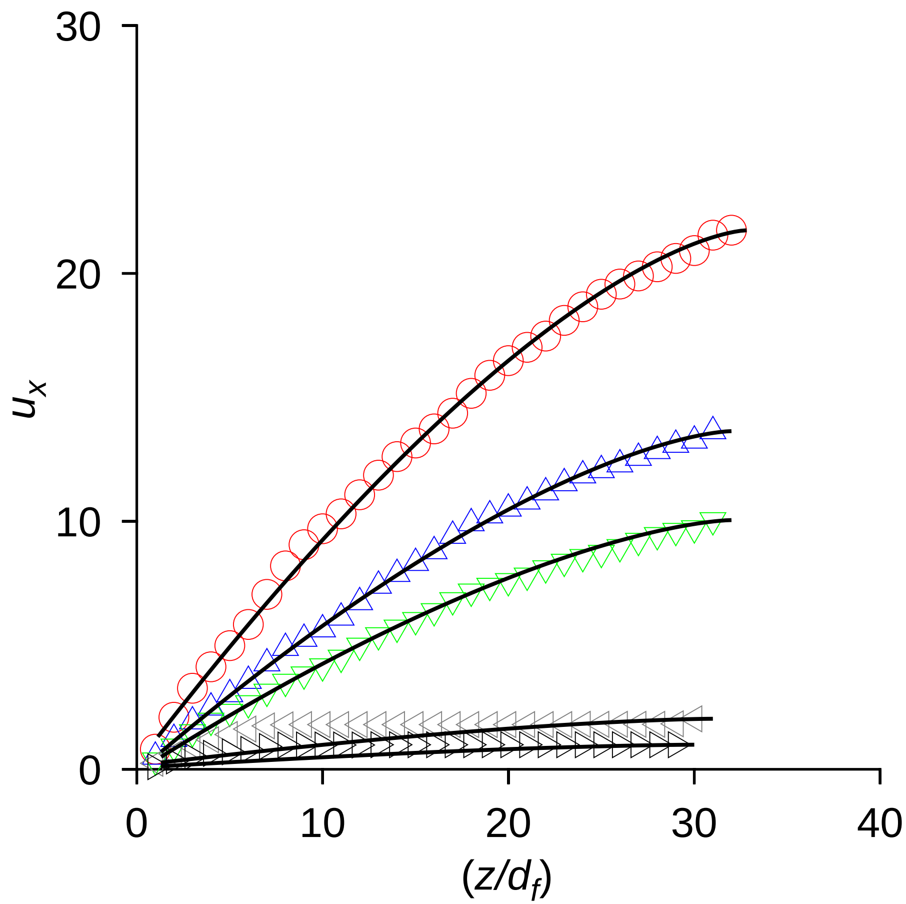
$$(d_b/d_f) = 0.62$$

Flow down inclined plane: Order

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous

Flow down inclined plane: Inclination for flow

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous



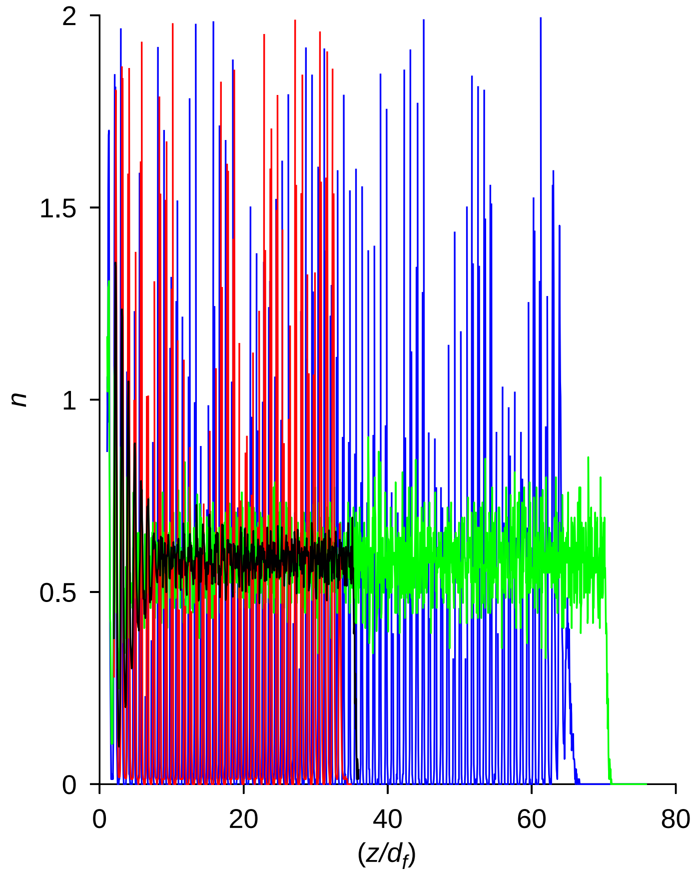
$(d_b/d) = 0.5$

$\theta = 18^o, 16^o, 15^o, 14^o, 13^o.$

Flow down inclined plane: Layering

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous

$n(z) A \Delta z$ = (# of particles with centers in Δz).



(d_b/d) = 0.61, (32000); 0.62, (32000)

(d_b/d) = 0.62, (64000) 0.63, 64000

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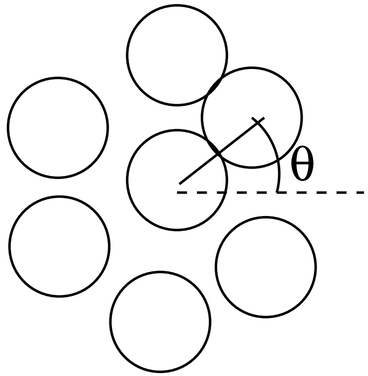
Flow down inclined plane: In-layer order

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous



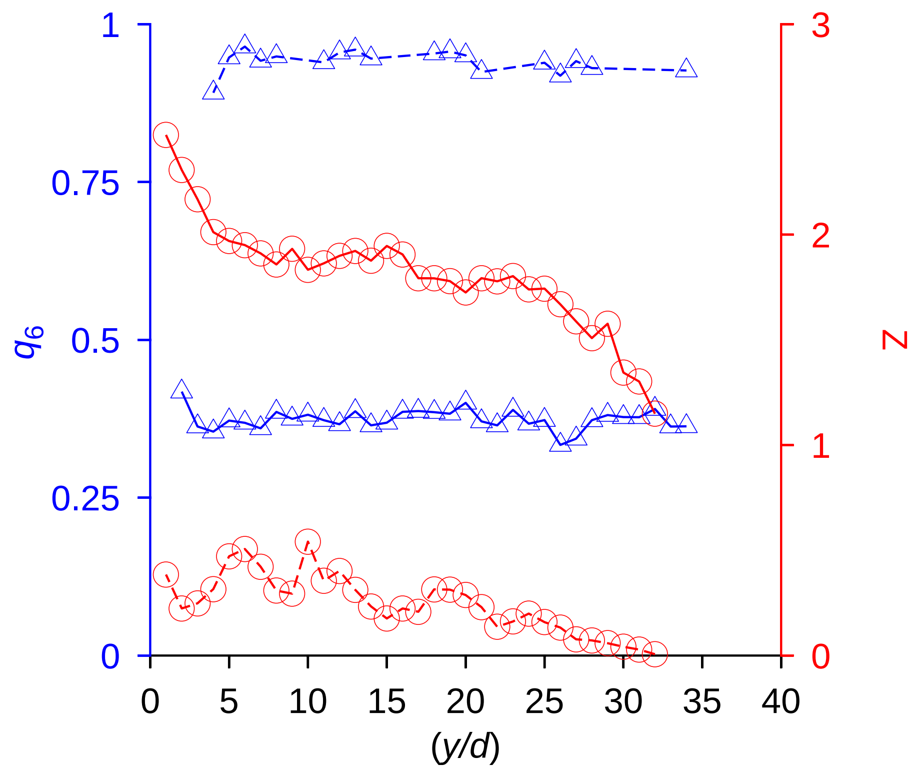
- 2D: $q_6 = \sum_{i=1}^{N} \exp\left(6\imath\theta\right)$
- $q_6 = 1$ hexagonal packing.
- 3D Icosahedral order parameter:

$$Q_l = \left(\frac{2l+1}{4\pi} \sum_{m=-l}^{l} |\langle Y_{lm}(\theta,\phi)\rangle|^2 \right)^{1/2}$$

- $Q_6 = 0.6$ for FCC/HCP.

Flow down inclined plane: In-layer order

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous



Angle = 22^o,
$(d_b/d) = 0.5$ (dashed line).
$(d_b/d) = 1.0$ (solid line).

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Flow down inclined plane: In-layer order

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
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Polydispersity	
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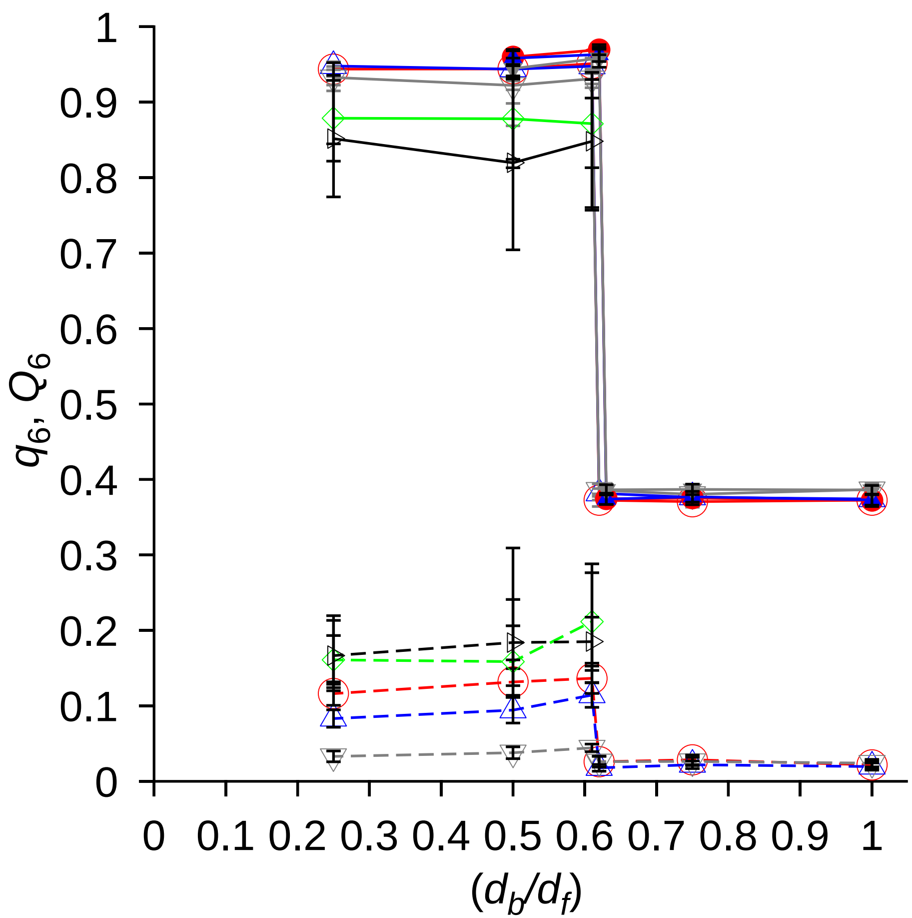
Plug	Homogeneous



Angle = 15^o, 18^o, 20^o, 22^o, 25^o.

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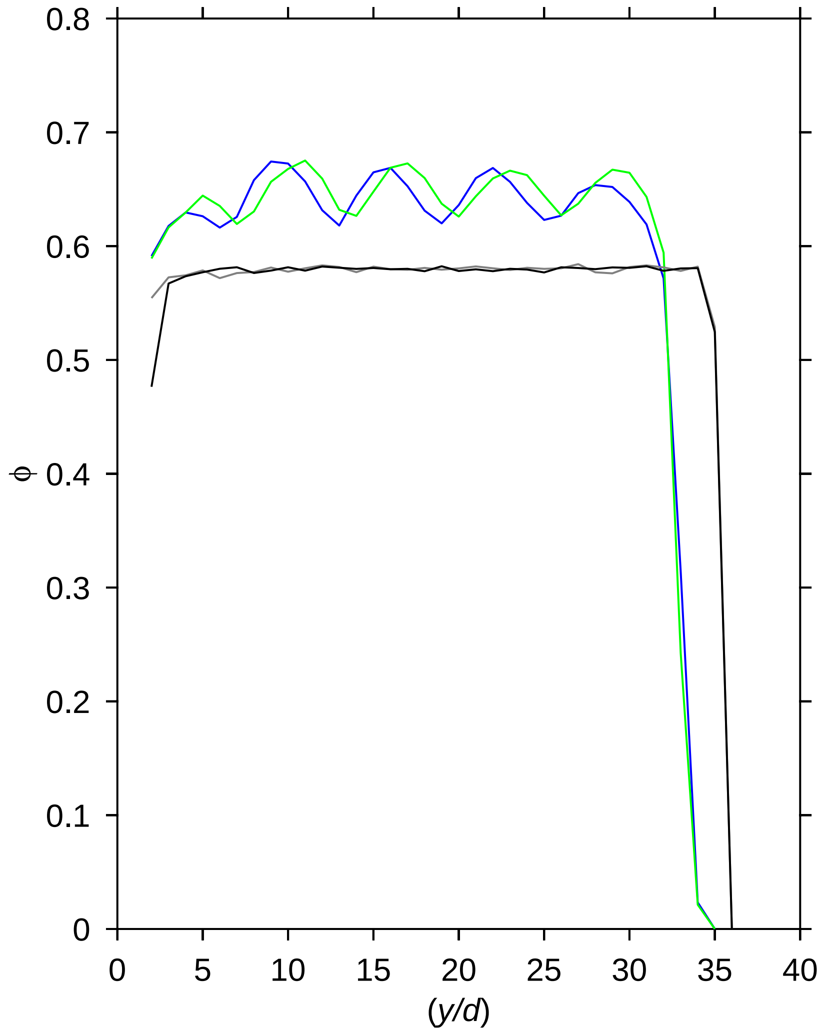
Flow down inclined plane: Volume fraction

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous



Angle = 22^o $h \approx 35$,
$(d_b/d) = 0.5, 0.61, 0.62, 1.00.$

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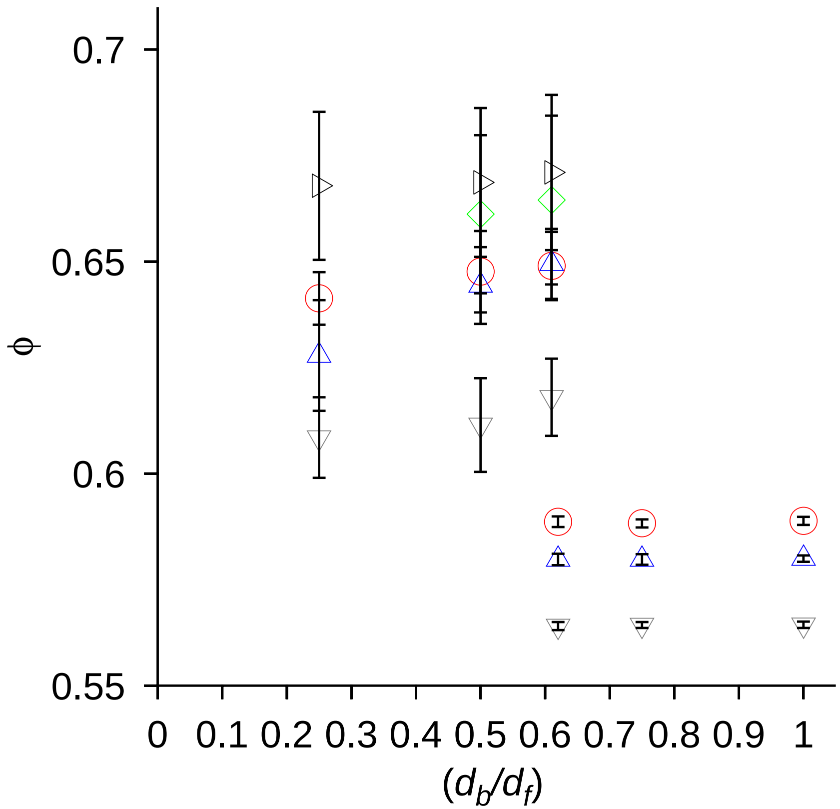
Flow down inclined plane: Volume fraction

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	**Less dense**
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous



Angle = 15^o, 18^o, 20^o, 22^o, 25^o.

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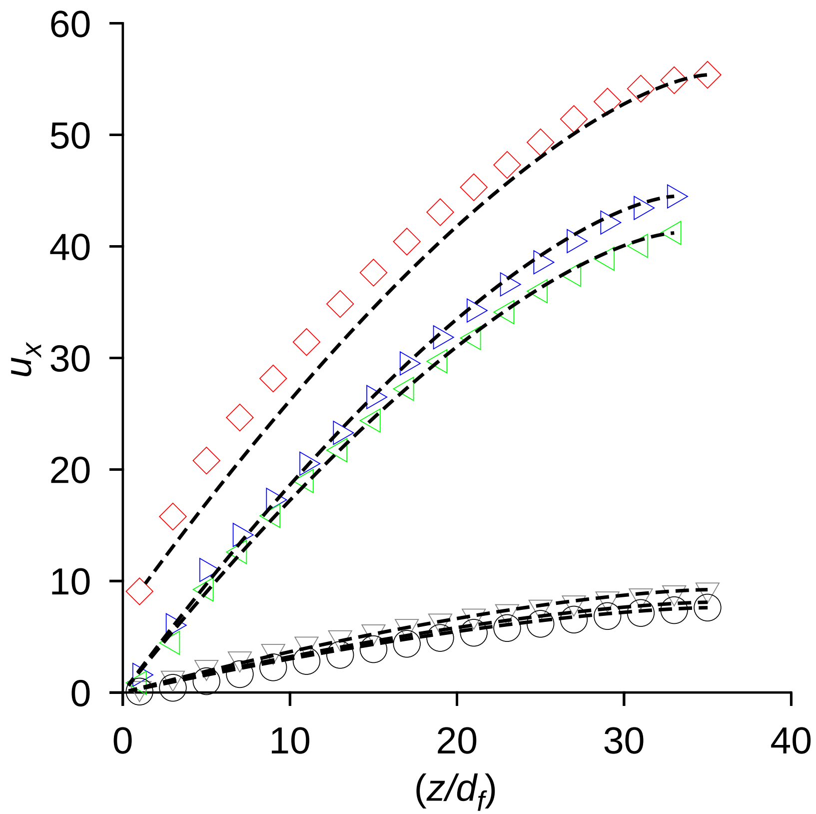
Flow down inclined plane: Bagnold law

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous



$(d_b/d) = 0.25, \ 0.5, \ 0.61, \ 0.62,$ 1.00.

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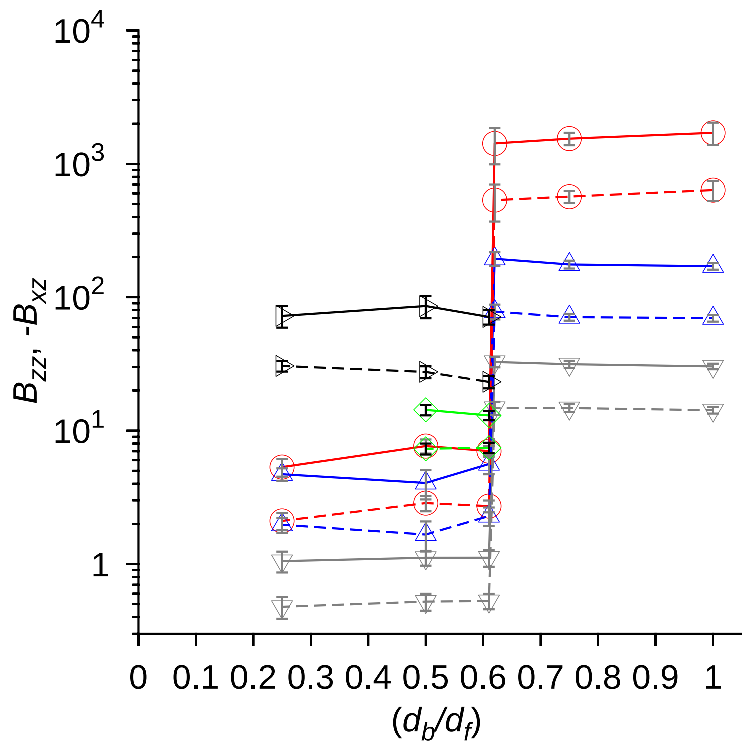
Flow down inclined plane: Bagnold coefficients

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous



B_{zz} —, $-B_{xz}$ − −−

Angle = 15^o, 18^o, 20^o, 22^o, 25^o.

0.61, 0.62, 1.00.

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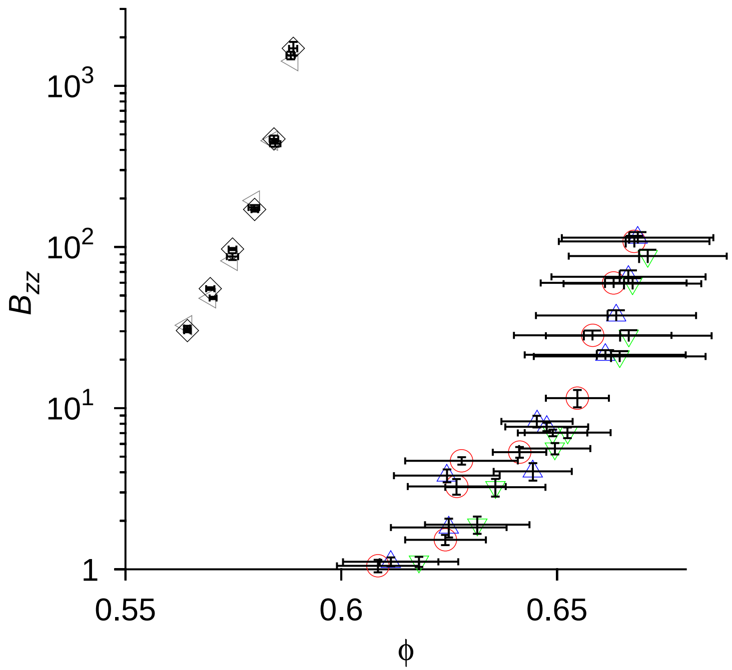
Flow down inclined plane: Bagnold coefficients

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous

Bagnold coeff vs. ϕ



$(d_b/d) = 0.25, 0.5, 0.61, 0.62, 1.00.$

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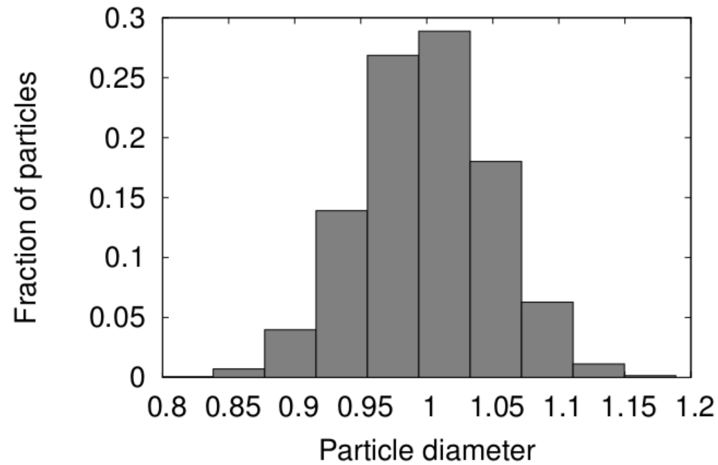
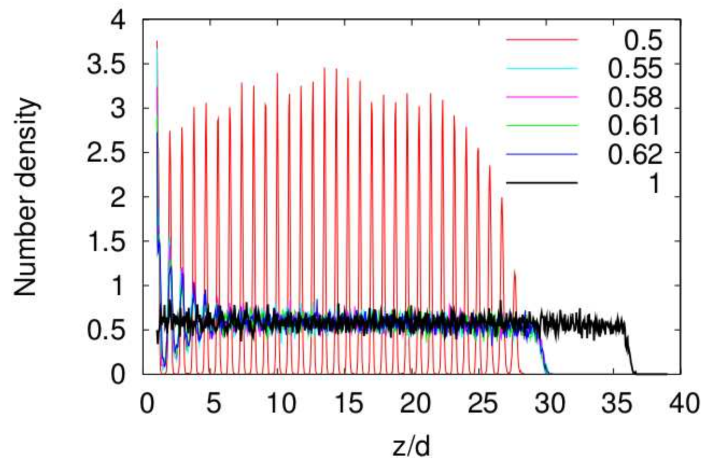
Flow down inclined plane: Polydispersity

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous

5% Polydispersity:





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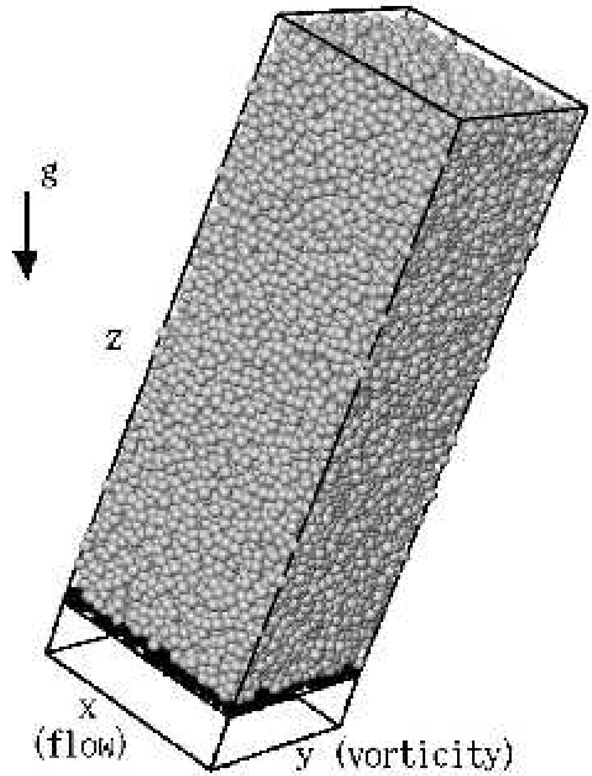
Flow down inclined plane: Developing flow

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous

Velocity evolution
(Zero initial velocity)



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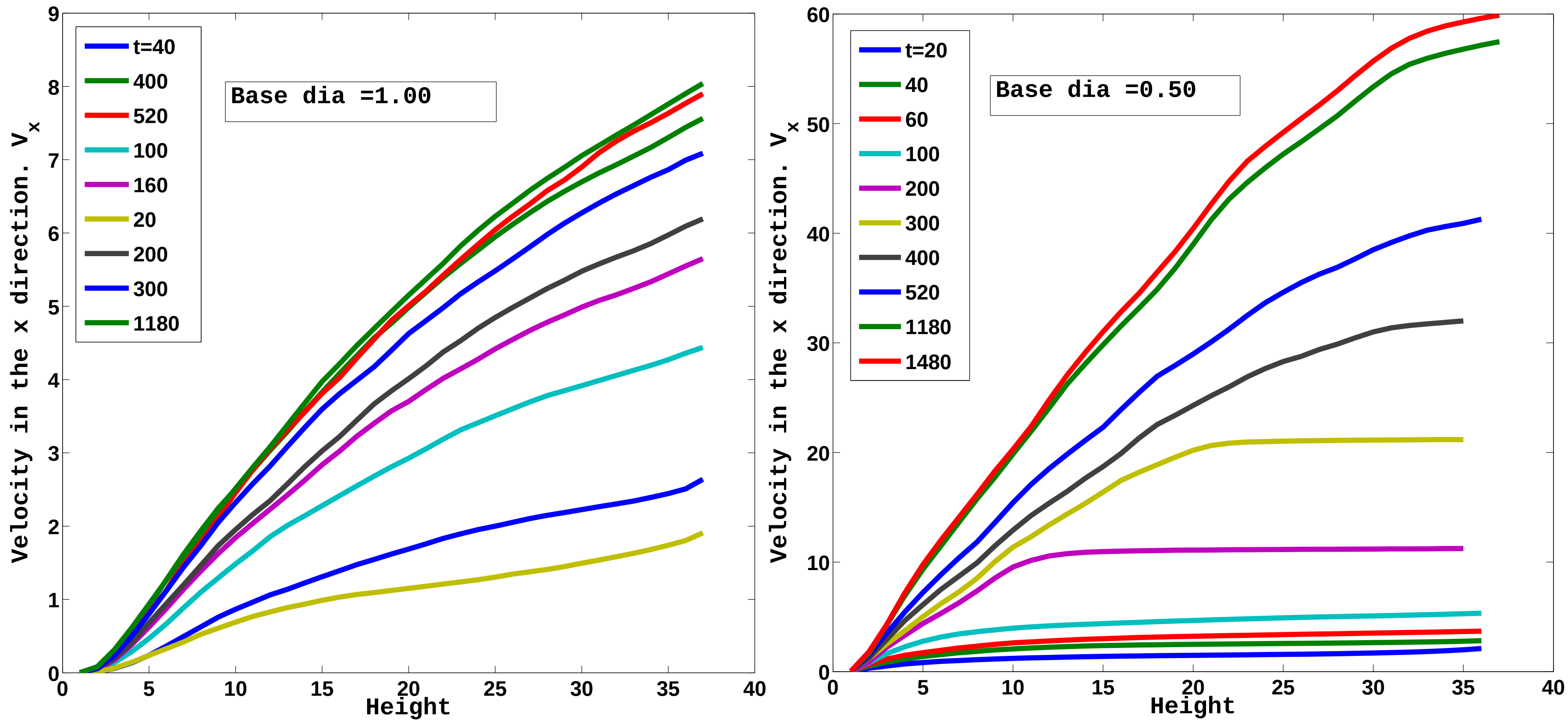
Flow down inclined plane: Developing flow



Angle = 22^o.

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Flow down inclined plane: Developing flow

Momentum equations:

$$\rho \frac{\partial u_x}{\partial t} = \frac{\partial \sigma_{xz}}{\partial y} + \rho g_x$$

$$0 = \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z$$

Bagnold law:

$$\sigma_{xz} = B_{xz}(\phi) \left(\frac{du_x}{dz} \right)^2$$

$$\sigma_{zz} = B_{zz}(\phi) \left(\frac{du_x}{dz} \right)^2$$

Steady solution:

$$\bar{u}_x = (5\bar{u}/3) \left(1 - (1 - (z/h))^{3/2}\right)$$

$$\bar{u} = \frac{2h^{3/2}}{5} \left(\frac{\rho g_x}{B_{xz}} \right)^{1/2}$$

Unsteady flow:

$u_x = \bar{u}_x(z) + u'_x(z,t)$

$\phi = \bar{\phi}(z) + \phi'(z,t)$

$B_{ij}(\phi) = \bar{B}_{ij} + B_{\phi ij}\phi'(z,t)$

Linearise in u'_x, ϕ'.

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Flow down inclined plane: Disordered flow

$$\rho\frac{\partial u_x'}{\partial t} = \frac{\partial}{\partial z}\left(B_{\phi xz}\phi'\left(\frac{\partial \bar{u}_x}{\partial z}\right)^2 + \bar{B}_{xz}\left(2\frac{\partial \bar{u}_x}{\partial z}\frac{\partial u_x'}{\partial z} + \left(\frac{\partial u_x'}{\partial z}\right)^2\right)\right)$$

$$0 = \frac{\partial}{\partial z}\left(B_{\phi zz}\phi'\left(\frac{\partial \bar{u}_x}{\partial z}\right)^2 + \bar{B}_{zz}\left(2\frac{\partial \bar{u}_x}{\partial z}\frac{\partial u_x'}{\partial z} + \left(\frac{\partial u_x'}{\partial z}\right)^2\right)\right)$$

Combine to give:

$$\tau\frac{\partial u_x'}{\partial t} = h^2\frac{\partial}{\partial z}\left(\sqrt{1-(z/h)}\frac{\partial u_x'}{\partial z}\right) \quad \Bigg| \quad \tau = \left(\frac{5\bar{u}}{\rho h^3}\left(\bar{B}_{xz} - \frac{B_{\phi xz}\bar{B}_{zz}}{B_{\phi zz}}\right)\right)^{-1}$$

Can show $\tau > 0$ if $\phi \downarrow$ as $\theta \uparrow$.

Flow down inclined plane: Disordered flow

Solution for u'_x:

$$u'_x = \sum C_n e^{(-\alpha_n t/\tau)} (1 - (z/h))^{1/4} \times J_{-1/3}((4\sqrt{\alpha_n}(1 - (z/h))^{3/4}/3)$$

Discrete eigenvalues α_n:

$$\begin{aligned} u'_x &= 0 \text{ at } z = 0 \\ \frac{du'_x}{dz} &= 0 \text{ at } z = h \end{aligned}$$

Orthogonality relation C_n from initial condition $u'_x = -\bar{u}_x$.

n	1	2	3	4
α_n	1.95934	13.9943	37.1371	71.363
$(C_n/\bar{u})$	-2.32322	0.200244	0.059188	0.026166

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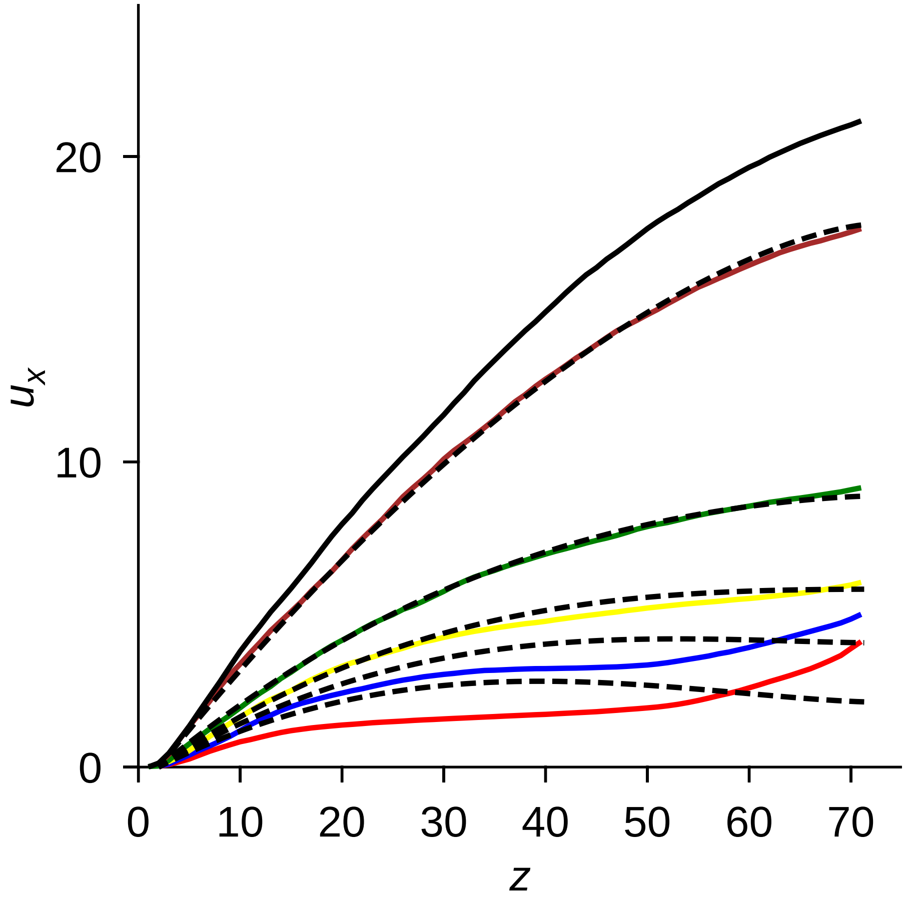
Flow down inclined plane: Disordered flow

Rough base, 22^o, $d_b = 1$:



- Unsteady momentum equation; Bagnold law.
- Linearise & use separation of variables.
- Retain first term in the expansion.

— t=40; — t=80; — t=120; — t=200; — t=600; — t=1000.

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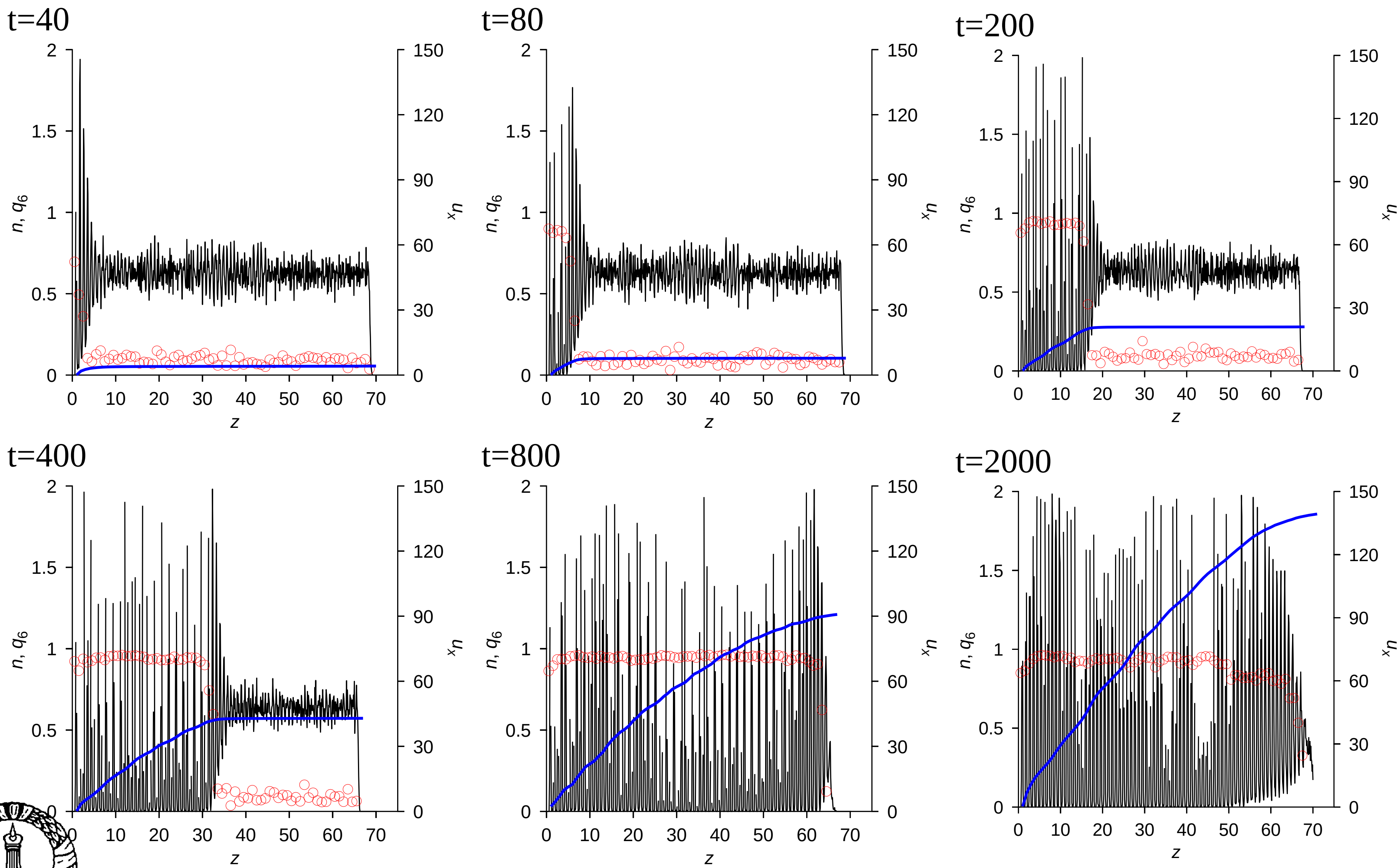
Flow down inclined plane: Ordered flow: n, q_6 u_x



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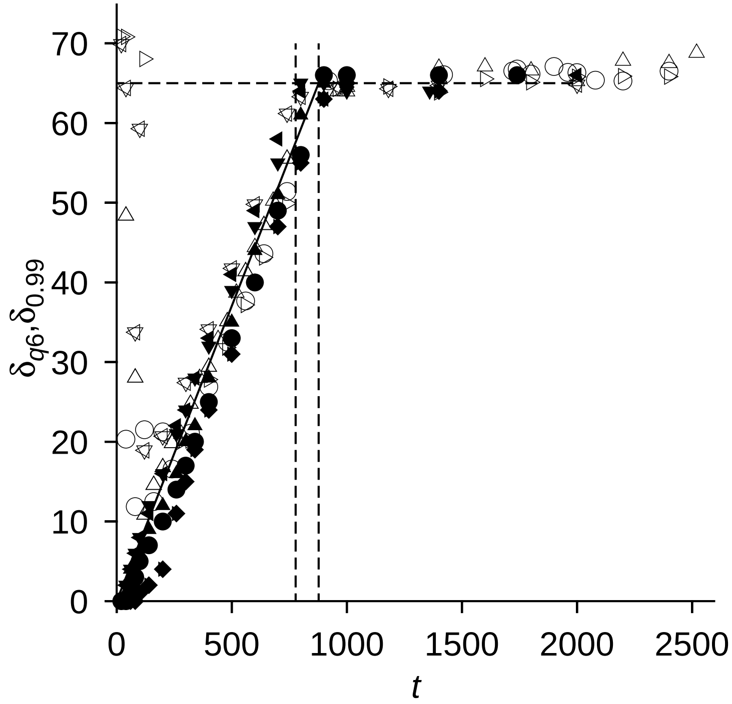
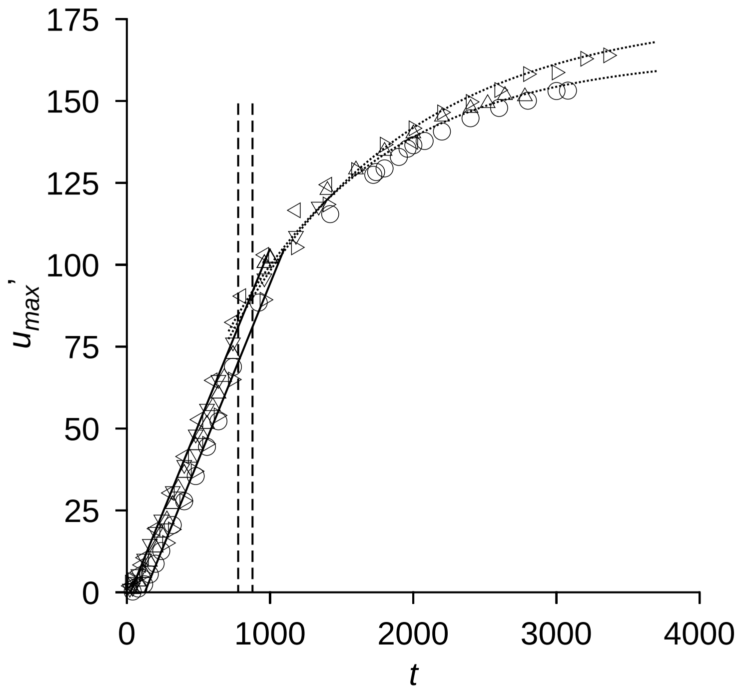
Flow down inclined plane: Ordered flow:

Shear layer height $\propto$ t:



Max velocity $\propto t$ (shear layer); Exp (homogeneous flow).



$d_b = 0.15$ ($\triangle$), 0.25 (∇), 0.35 ($\triangleleft$), 0.50 ($\triangleright$)

Independent of d_b, height. Depends only on θ.

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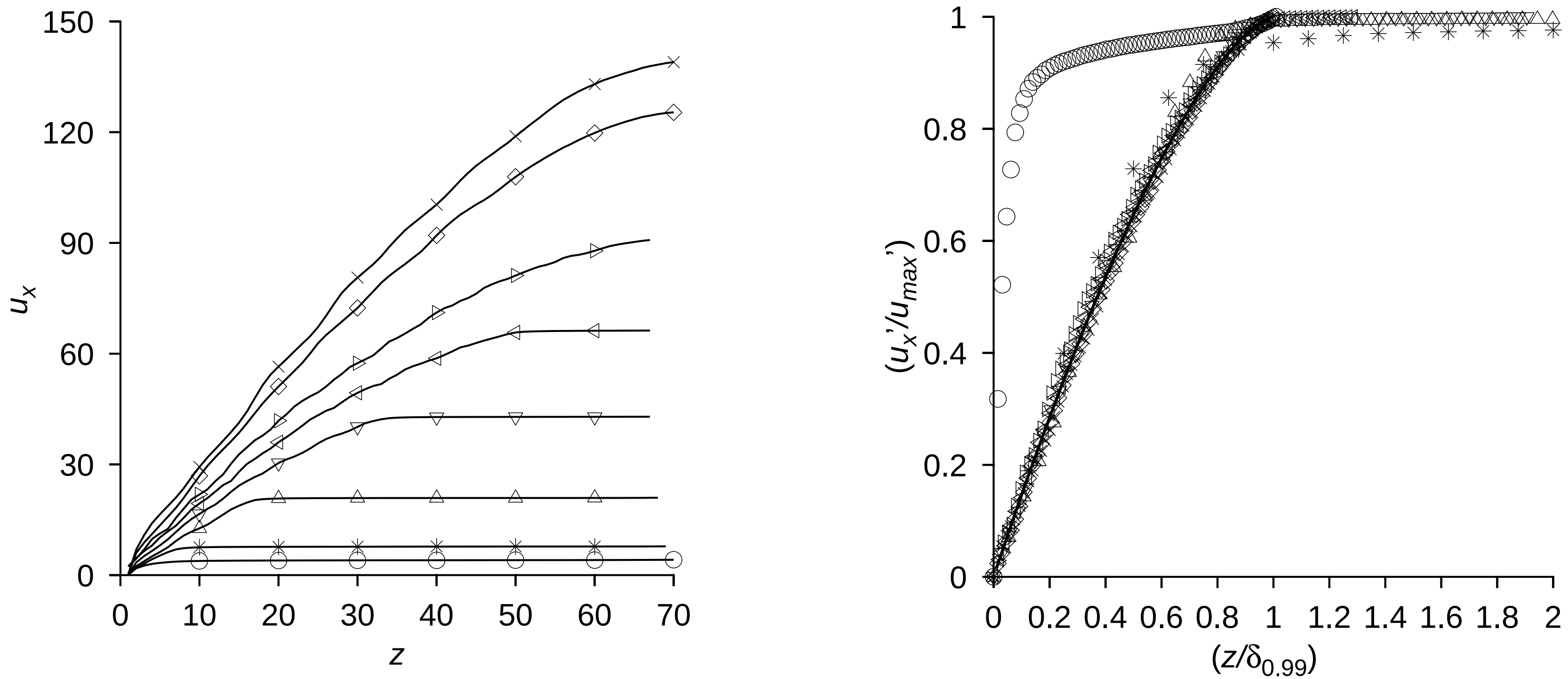
Flow down inclined plane: Ordered flow:

$\theta = 22^o$, $(d_b/d) = 0.35$, $u'_x = u_x - u_{slip}$, $u'_{max} = u_{max}/u_{slip}$.



$t = 40$ ($\circ$), 80 ($*$), 200 ($\triangle$), 400 (∇), 600 ($\triangleleft$), 800 ($\triangleright$), 1400 ($\diamond$), 2000 ($\times$).

Velocity in shearing zone well described by Bagnold law!

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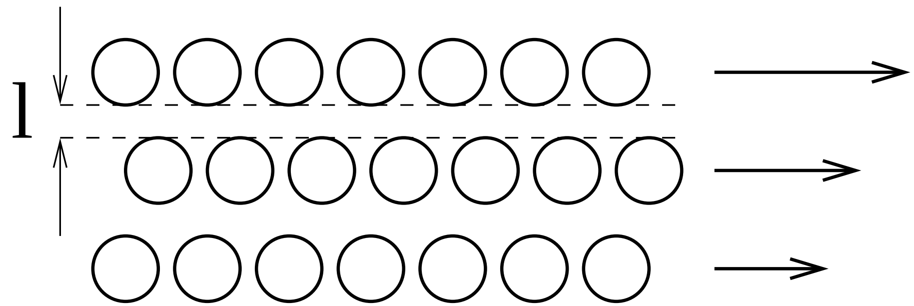
Layered fluid:

- Layered fluid with spacing l between particles.
- Spacing maintained by interactions with particles above and below.
- Granular temperature T nearly constant.
- Collision frequency $\nu \propto l^{-1} T^{1/2}$.
- Average force ($\nu \times$ impulse) $\propto m T l^{-1}$.



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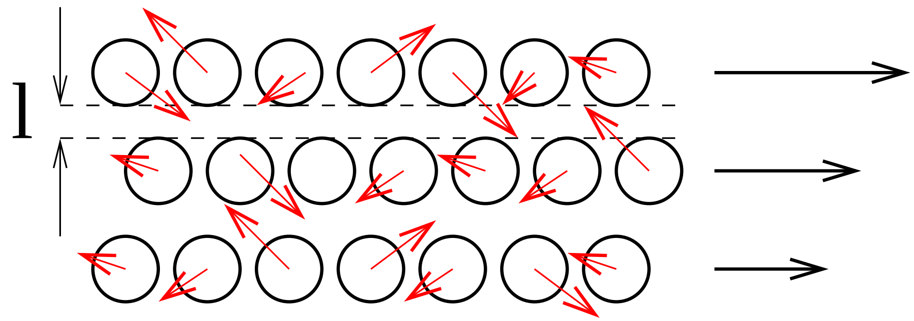

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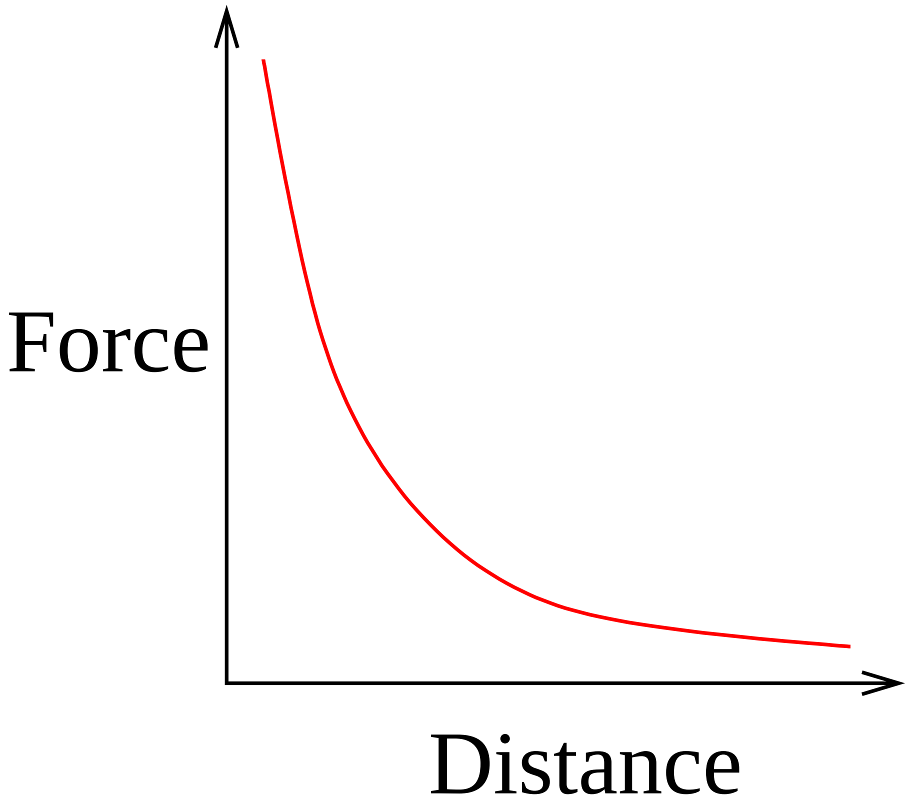

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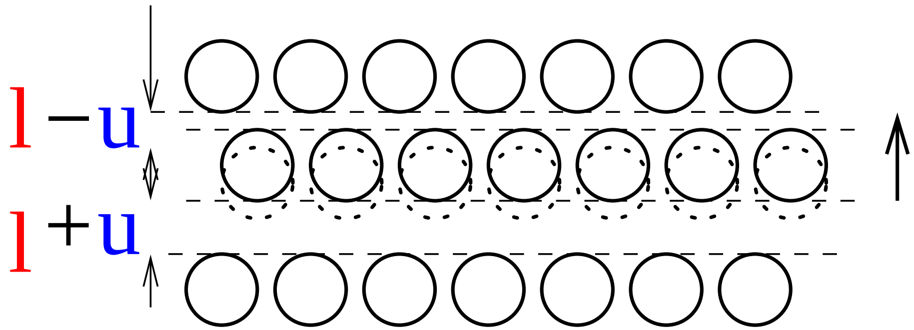
Layered fluid:

- Layer displacement u.
- Force from above $mT/(l-u)$
- Force from below $mT/(l+u)$.
- Net restoring force $mT((l-u)^{-1}-(l+u)^{-1})$ $\sim (2mTu/l^2)$. $\sim 2mT\frac{d^2u}{dz^2}$.



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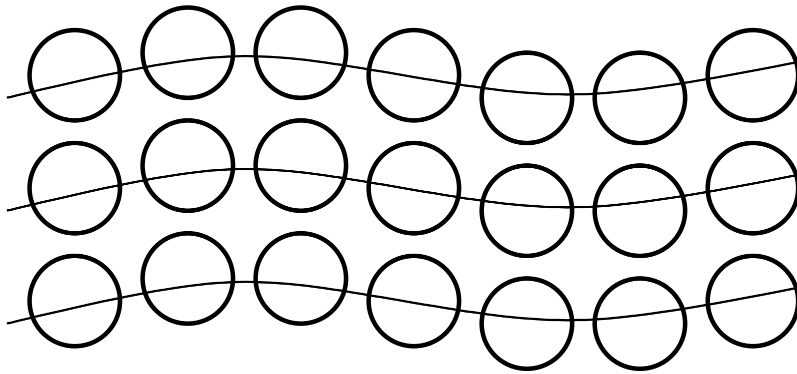
Layered fluid:

- Layer bending u.
- Zero tension: No restoring force proportional to u
- Change in lateral pressure due to expansion: $F \propto (\partial u/\partial x)^2$.
-



$$\frac{du}{dt} = B\frac{\partial^2 u}{\partial z^2} + K\left(\frac{\partial^4 u}{\partial x^4}\right) + A\left(\frac{\partial u}{\partial x}\right)^2$$

Linear stability analysis — always stable.
Sub-critical instability when perturbation amplitude exceeds critical value.

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Flow down inclined plane: Summary

Ordered	Disordered
$(d_b/d) < d_c$	$(d_b/d) > d_c$
Flow $\theta \geq 14^o$	Flow $\theta \geq 21^o$
Layering	No layering
In-layer order	No order
More dense	Less dense
Bagnold law valid	
Faster	Slower
Polydispersity	
Developing flow	
Plug	Homogeneous

- Distinct flow regimes with universal properties (no variation with layer height, base roughness) in each regime.
- Discontinuous transition at specific base roughness.
- Transient disordered flow well described by Bagnold law.
- Sub-critical instability for a layered fluid.
- Shearing layer & plug flow for smooth base seem to be due to flow development!