

# A Thermal Quench Induces Spatial Inhomogeneities in a Holographic Superconductor

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INI, Cambridge, 17/09/2013

Coauthor: Antonio M. García-García & Hua-Bi Zeng, arXiv:1308.5398



# Introduction

- **Static homogeneous** holographic superconductor (HSC)
- **Dynamical homogeneous** HSC
- **Static inhomogeneous** HSC

- **Static homogeneous HSC:**

Gubser, Phys. Rev. D 78 (2008) 065034

Hartnoll, Herzog, & Horowitz, Phys.Rev.Lett. 101 (2008) 031601

- ✓ Einstein-Maxwell-charged scalar action with negative cosmological constant

- ✓  $T > T_c$  (or  $\mu < \mu_c$ , since  $T \sim 1/\mu$ ):  $\psi = 0$ ;

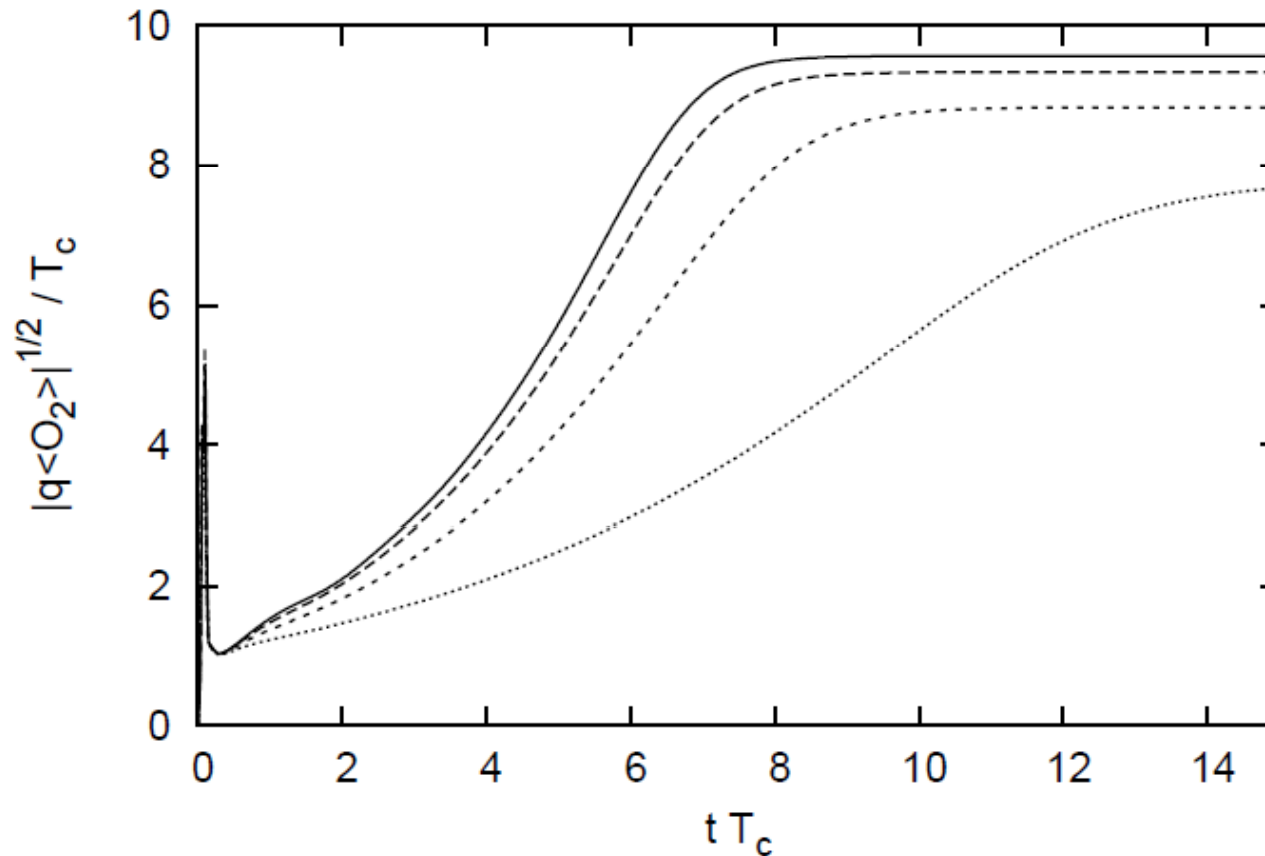
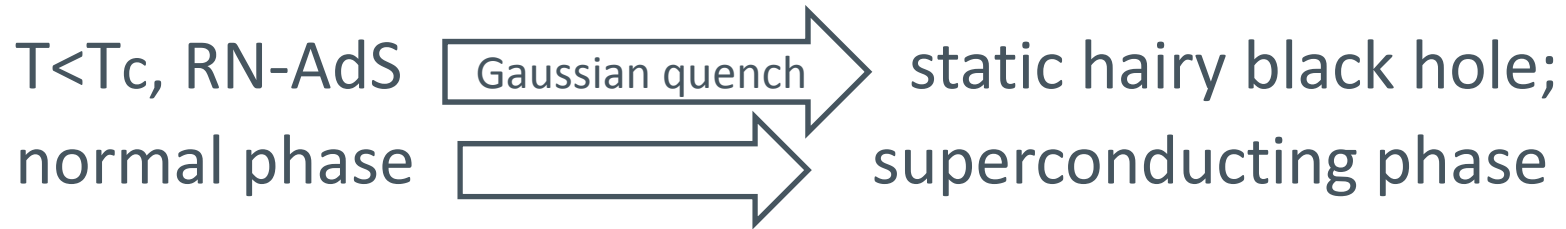
$T < T_c$  : two available solutions

$\psi = 0$        $>$        $\psi \neq 0$       superconducting solution

free energy

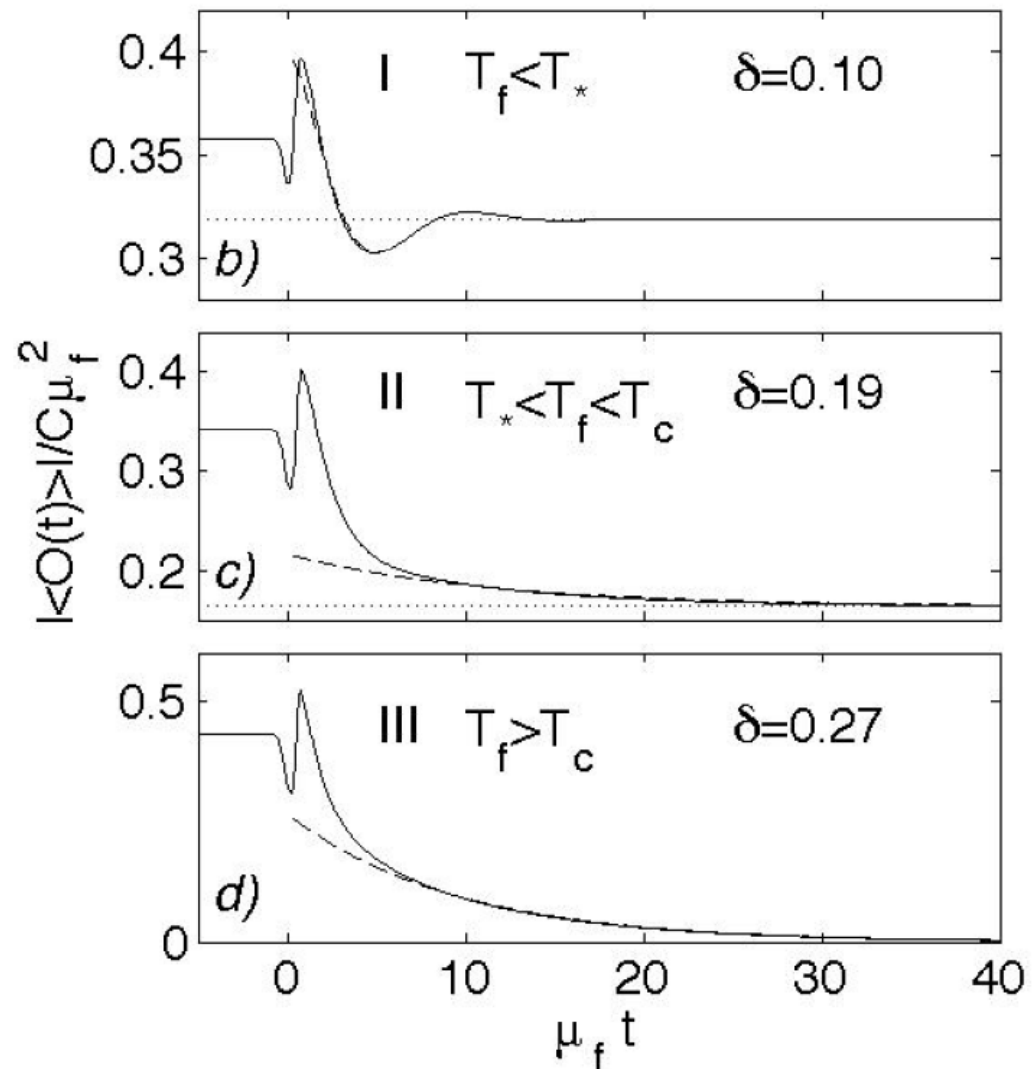
- **Dynamical homogeneous HSC after a quench:**

✓ [Murata, Kinoshita & Tanahashi, JHEP 1007 \(2010\) 050](#)



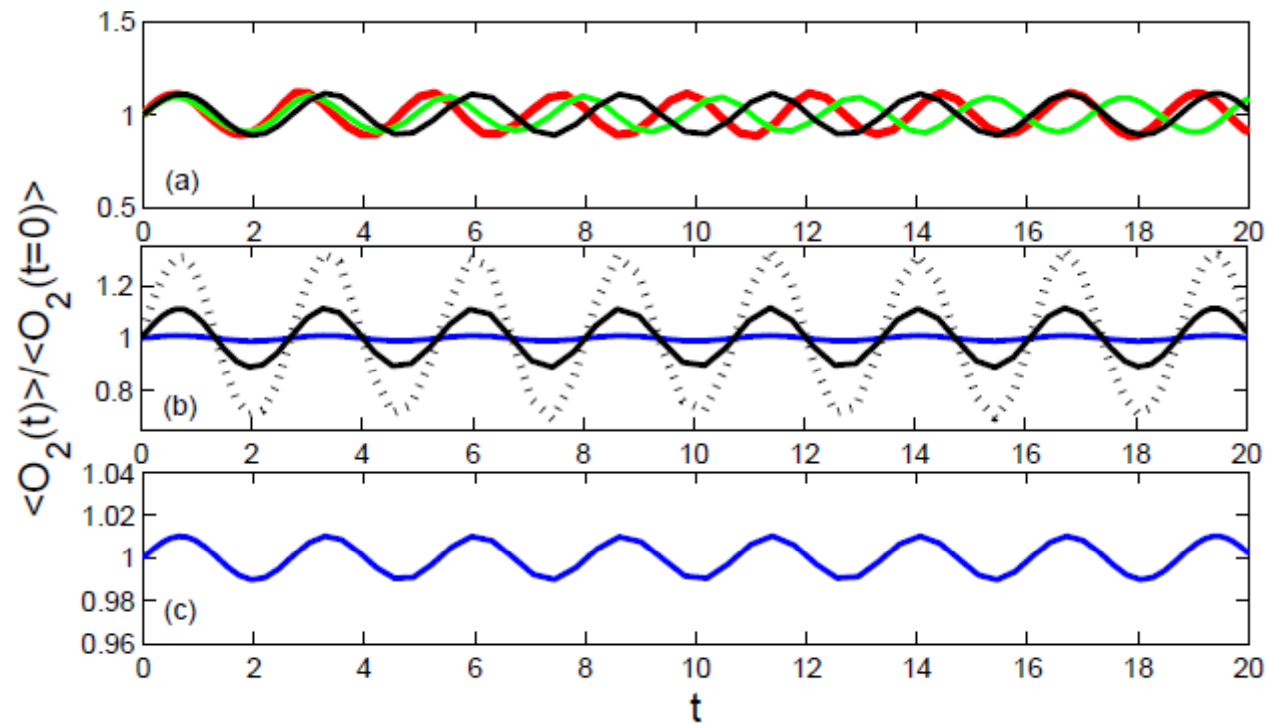
✓ Bhaseen, Gauntlett, Simons, Sonner & Wiseman, Phys.Rev.Lett. 110 (2013) 015301

three kinds of decays are found, consistent with QNMs



✓ Gao, Garcia-Garcia, Zeng & Zhang: arXiv:1212.1049

AdS soliton background, undamped oscillation modes are found; lack of thermalization in CFT



- ✓ [Polkovnikov, Sengupta, Silva, Vengalattore, Rev.Mod.Phys. 83 \(2011\) 863](#)

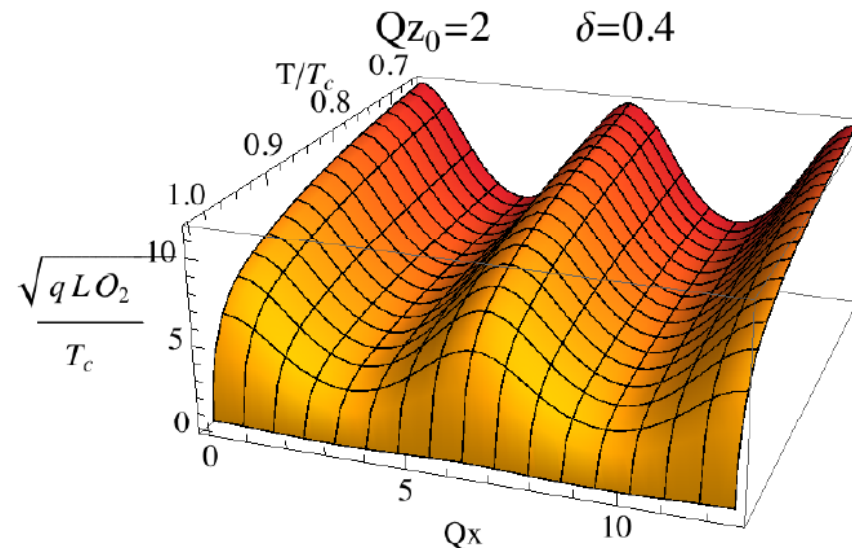
CMT: lack of thermalization comes from integrability.

Question: what is the relation between a gravity without horizon and the integrable field theory on boundary?

- **Static inhomogeneous HSC:**

Flauger, Pajer & Papanikolaou, Phys.Rev. D83 (2011) 064009

Einstein- Maxwell-scalar action, modulated chemical potential, striped superconductor, *explicitly* break translational symmetry, free energy is lower than the homogeneous case

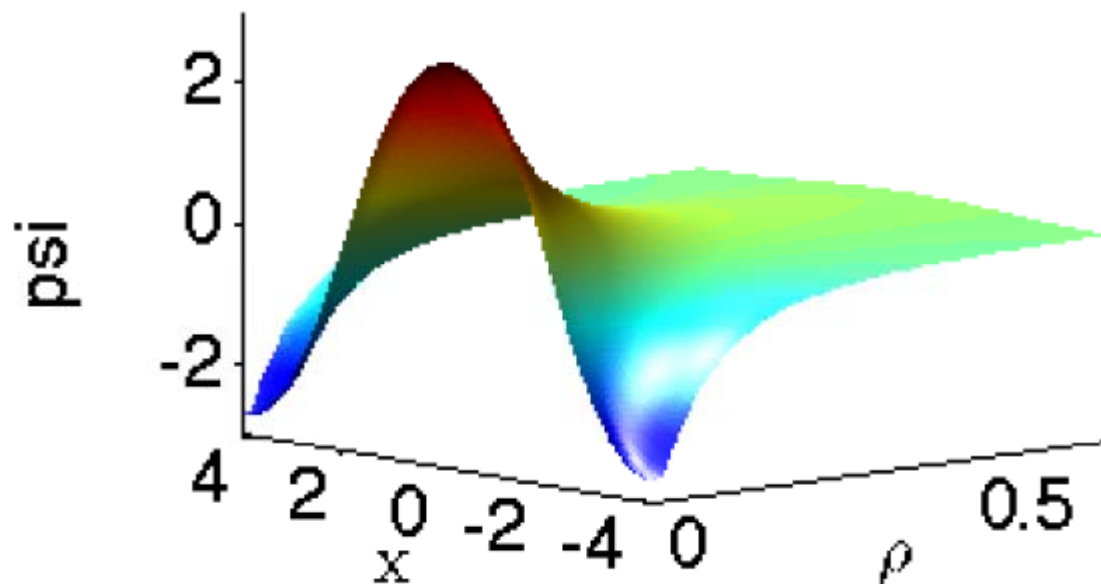




Donos & Gauntlett, JHEP 1108, 140 (2011)

Rozali, Smyth, Sorkin & Stang, Phys.Rev.Lett. 110, (2013) 201603

Einstein-Maxwell-axion action, *spontaneously*  
break the translational symmetry, striped  
order parameter



Liu, Ooguri, Stoica & Yunes, Phys. Rev. Lett. 110, (2013) 211601

Chern-Simons term , *spontaneously*  
generate angular momentum

- Other references:

- ✓ dynamical studies :

I. Amado, M. Kaminski & K. Landsteiner, JHEP, 0905021 (2009)

S. Bhattacharyya, and S. Minwalla, JHEP 0909:034, (2009)

P. Bizon, and A. Rostworowski, Phys. Rev. Lett. 107, 031102 (2011)

O. Dias, G. Horowitz, D. Marolf and J. Santos, Class. Quant. Grav. 29, 235019 (2012)

P. Basu, D. Das, S. Das and T. Nishioka, arXiv:1211.7076

...

- ✓ inhomogeneous studies:

S. Nakamura, H. Ooguri and C. -S. Park, Phys. Rev. D 81, 044018 (2010)

G. T. Horowitz, J. E. Santos and D. Tong, JHEP 1211, 102 (2012)

...

# Motivation to Dynamical Inhomogeneous HSC

- Kibble-Zurek mechanisms:

Zel'dovich, Kobzarev & Okun, Zh. eksp. teor. Fiz. 67, 3 (1974);  
Soviet Phys. JETP 67, 401 (1975)

Kibble, J. Phys. A 9, 1387 (1976)

Zurek, Nature 317, 505 (1985)

topological defects generated from a sudden  
quench

Dzero, Yuzbashyan & Altshuler, Eur. Phys. Lett. 85 (2009) 20004

system size larger than superconducting  
coherence length, quench can excite finite  
momentum states, results in spatial  
inhomogeneities

- From holography:

Donos & Gauntlett, JHEP 1108, 140 (2011)

Rozali, Smyth, Sorkin & Stang, Phys.Rev.Lett. 110, (2013) 201603

- ✓ axion fields, spontaneously break the translational symmetry.

Liu, Ooguri, Stoica & Yunes, Phys. Rev. Lett. 110, (2013) 211601

- ✓ angular momentum generation from Chern-Simons terms.

- How about a quench depending on time?

# Dynamical inhomogeneous HSC

- Einstein-Maxwell-complex scalar action:

$$S = \int d^4x \sqrt{-g} \left[ R - 2\Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |\nabla\psi - iqA\psi|^2 - m^2|\psi|^2 \right].$$

in which  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

- AdS-Schwarzschild black hole:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$$

with

$$f(r) = r^2/L^2(1 - r_0^3/r^3)$$

✓ Fields depend on  $t, r$  &  $x$ .

✓ Ansatz:

$$\psi(t, r, x) = \rho(t, r, x)e^{i\varphi(t, r, x)}, \quad \psi^*(t, r, x) = \rho(t, r, x)e^{-i\varphi(t, r, x)}.$$

$$A_i(t, r, x) = M_i(t, r, x) + \partial_i\varphi(t, r, x), \quad i = t, r, x.$$

✓ 4 independent EoMs, 1 constraint equation

✓ Boundary conditions:

✓ At horizon:  $M_t(t, x)|_{rh}=0$ ;

✓ Other fields are finite



✓ For  $m^2 = -2$ , expansions near boundary:

$$\rho = \rho_1(t, x)/r + \rho_2(t, x)/r^2 + \dots,$$

$$M_t = \mu(t, x) - \tilde{\rho}(t, x)/r + \dots,$$

$$M_r = M_r^{(2)}(t, x)/r^2 + \dots,$$

$$M_x = v(t, x) + J(t, x)/r + \dots$$

$\tilde{\rho}(t, x)$ -charge density;

$\mu(t, x)$ -chemical potential;

source  $\rho_1 = 0$  ;

order parameter  $\langle \mathcal{O}(x, t) \rangle \equiv \rho_2(x, t)$

$v(t, x)$  - superfluid velocity;

supercurrent  $J = 0$ .

- Thermal quench, since  $T \sim 1/\mu$ , it is equal to quench chemical potential.

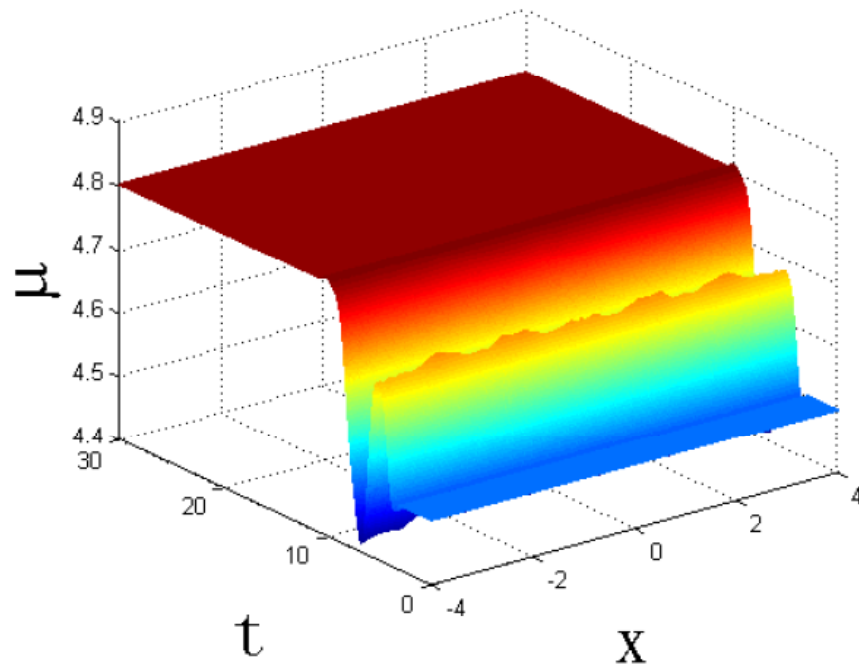
- Quench A:

$\mu_i=4.5,$        $\mu_f=4.8,$       corresponding to

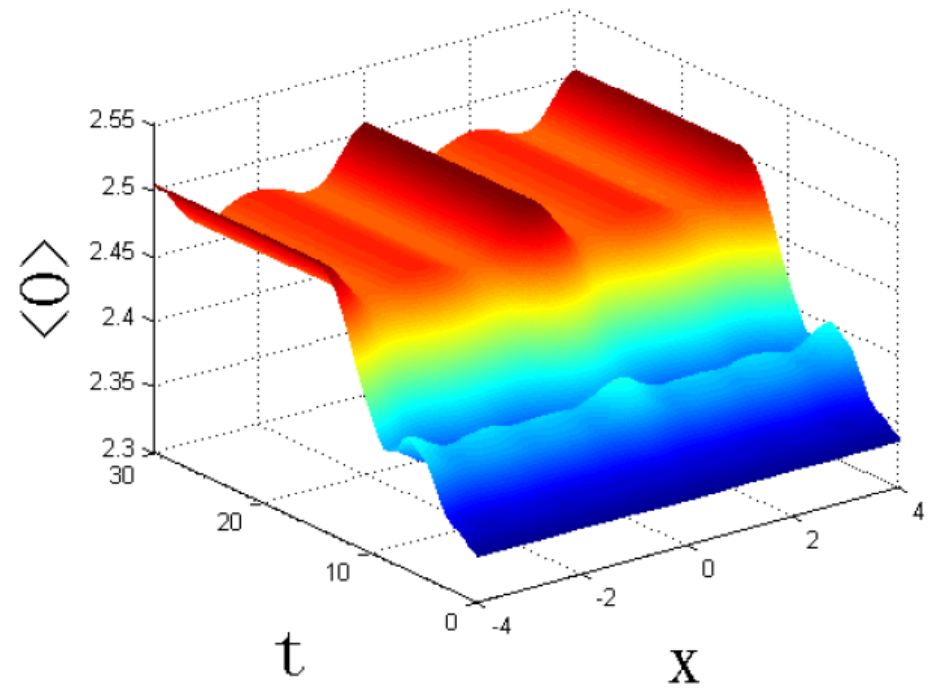
$T_i=0.903T_c,$        $T_f=0.846T_c$

since for homogeneous case  $\mu_c=4.063$  ( $m^2=-2$ )

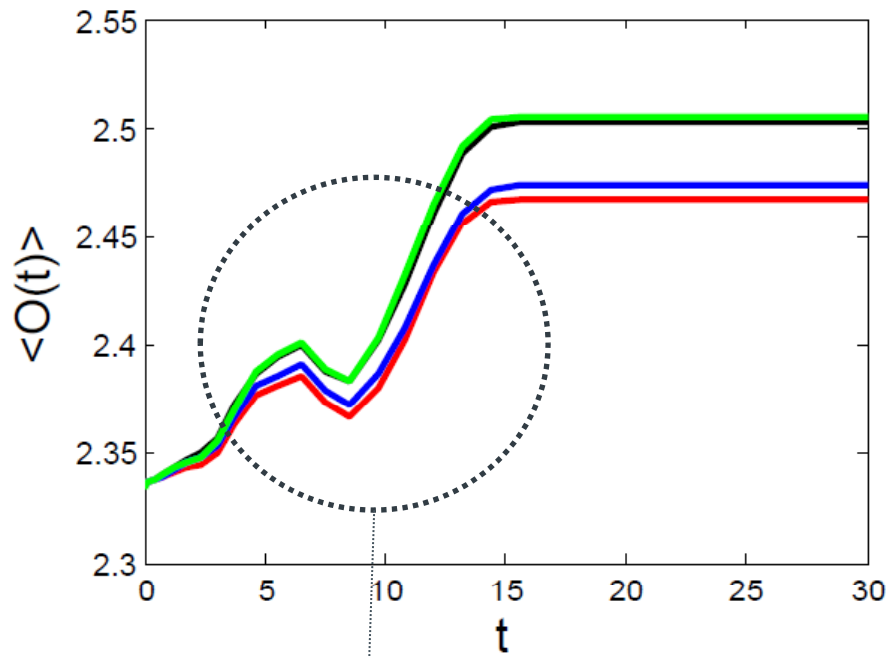
[Hartnoll, Herzog, & Horowitz, Phys.Rev.Lett. 101 \(2008\) 031601](#)



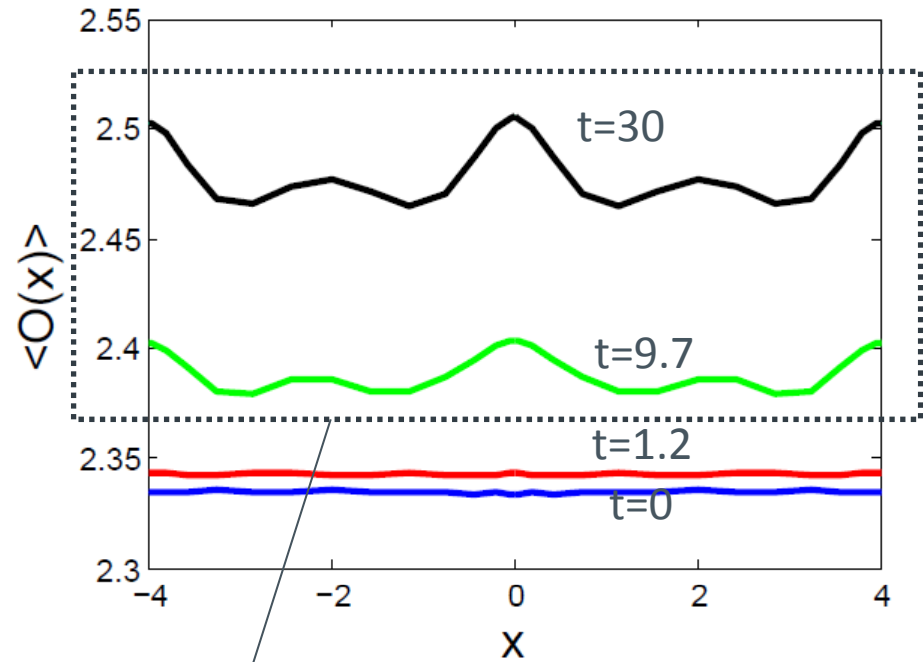
chemical potential



expectation value



oscillations in time



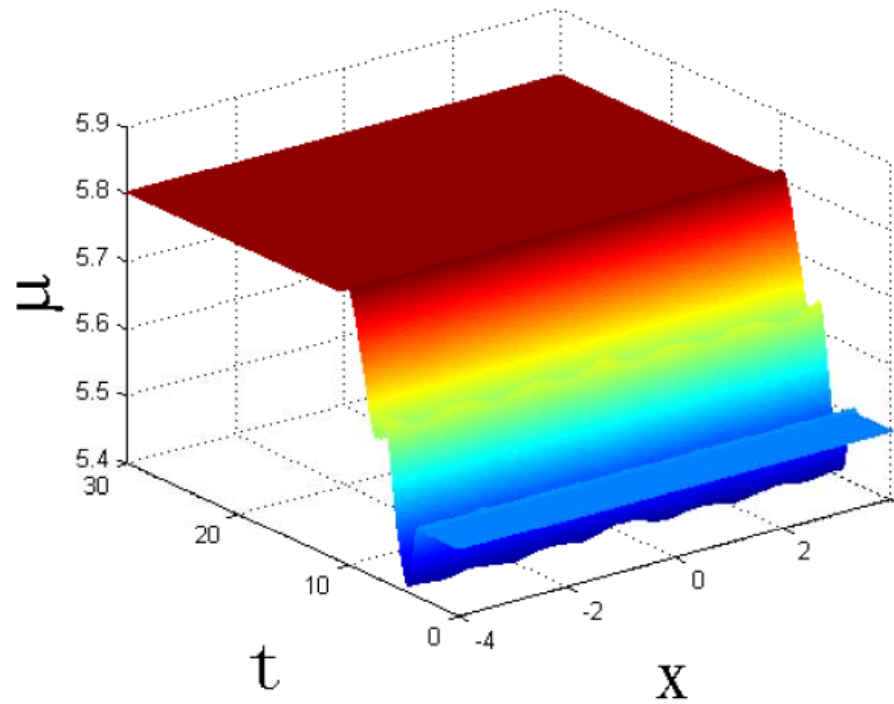
inhomogeneous order parameter

Physical meaning: quench will excite finite momentum

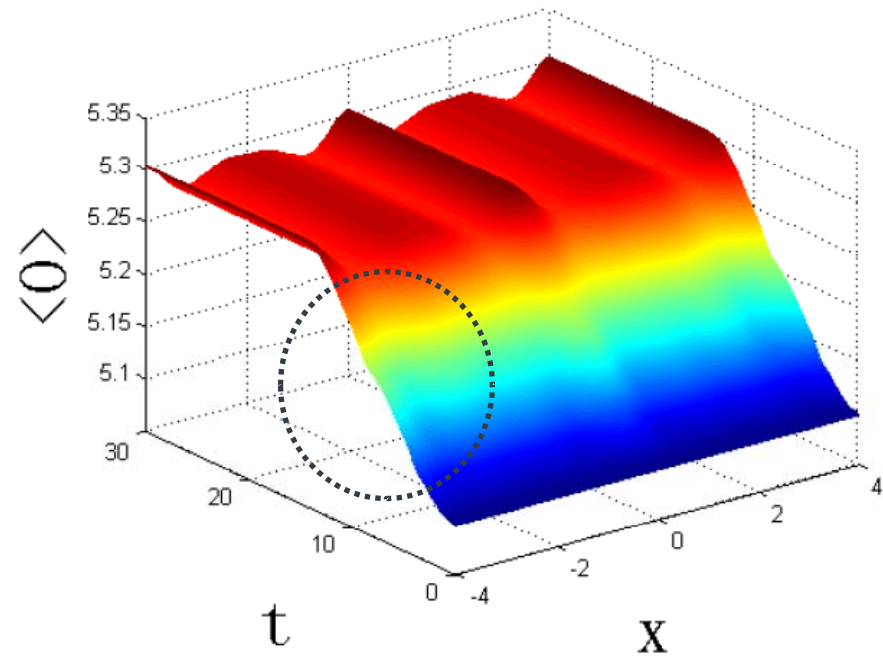
- Quench B:

$\mu_i=5.5$ ,  $\mu_f=5.8$ , corresponding to

$T_i=0.738 T_c$ ,  $T_f=0.700 T_c$ .



chemical potential



expectation value

- Free energy, grand canonical ensemble,

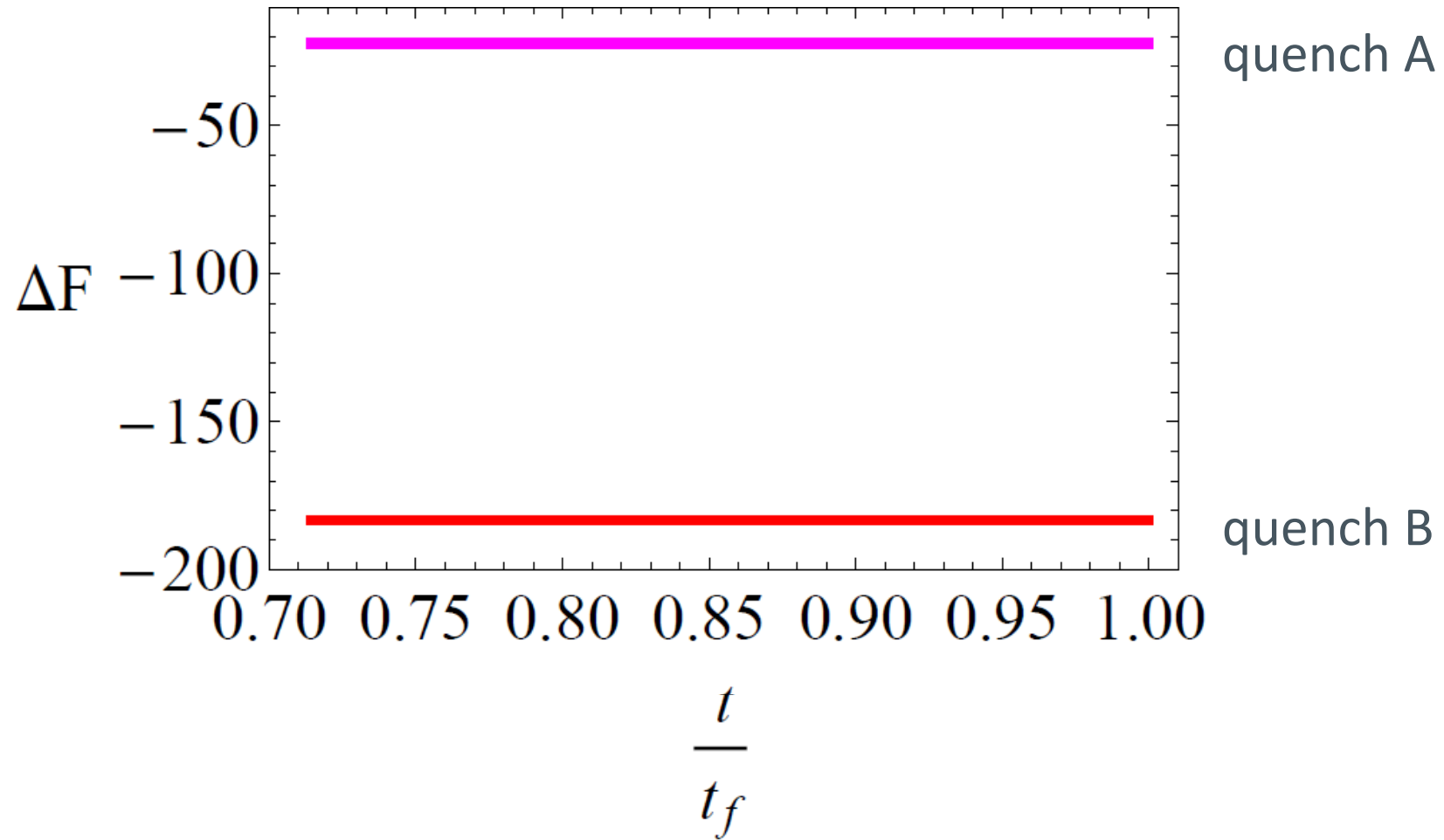
$$F = -TS_{os}$$

$$S_{os} = \int_{z=0} d^3x \left[ \frac{h}{z^2} \psi \partial_z \psi - M_t \partial_z M_t + h M_x \partial_z M_x + h^2 M_r \partial_z M_r \right] \\ - \int d^4x \left[ \frac{M_t^2}{hz^2} \psi - \frac{M_x^2 \psi^2}{z^2} + \frac{h}{z^2} M_r^2 \psi^2 \right]$$

where  $z=1/r$ ,  $h=1-z^3$

$$F \propto \int_{z=0} d^3x M_t \partial_z M_t + \int d^4x \left[ \frac{M_t^2}{hz^2} \psi - \frac{M_x^2 \psi^2}{z^2} + \frac{h}{z^2} M_r^2 \psi^2 \right]$$

- at late times



# Summaries

- Developments in HSC: static homogeneous, dynamical homogeneous, static inhomogeneous.
- Motivation: spontaneous symmetry breaking
- Dynamical inhomogeneous HSC, a quench induces translational symmetry breaking



Thanks for your attention!