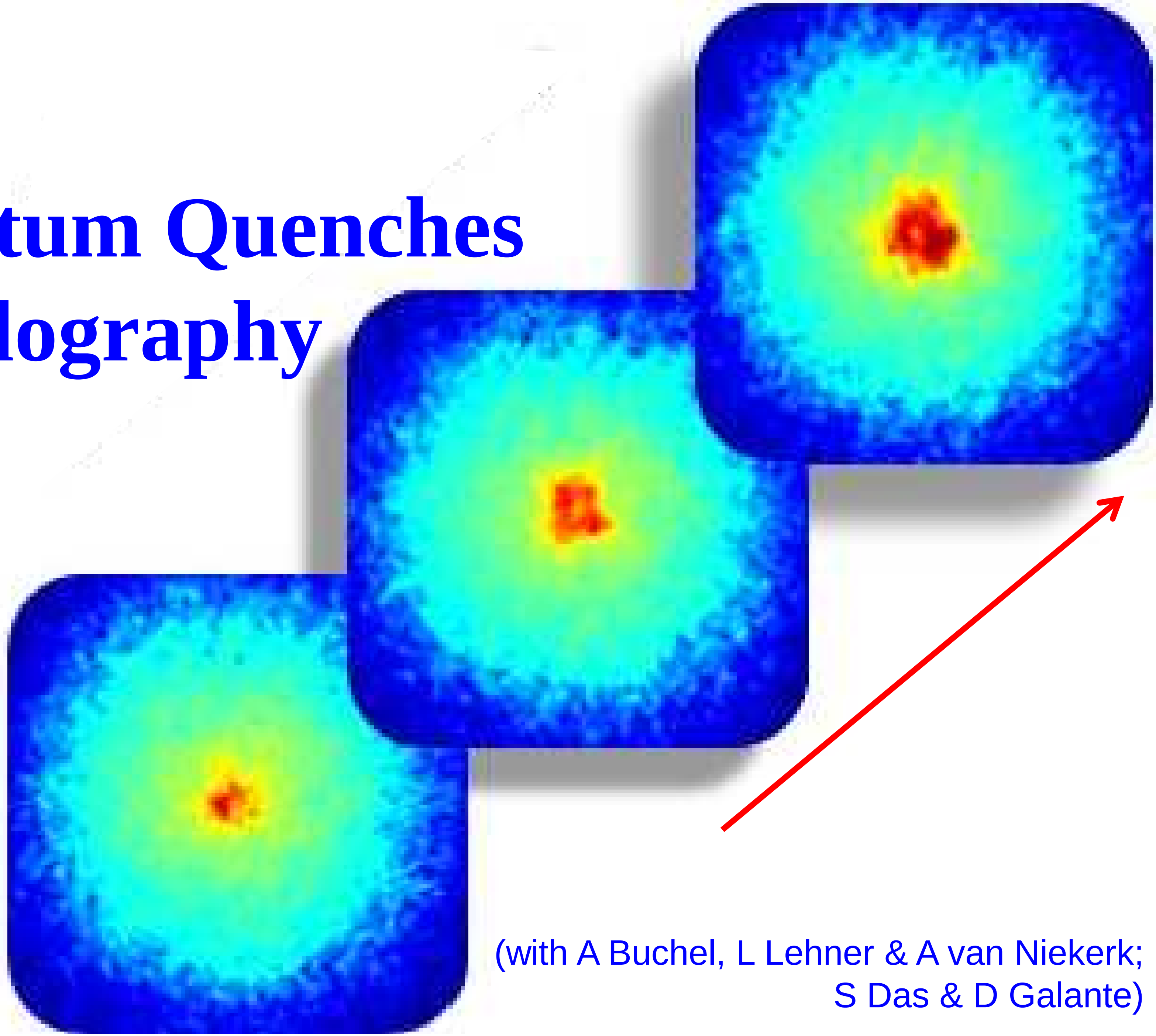
Quantum Quenches & Holography



(with A Buchel, L Lehner & A van Niekerk; S Das & D Galante)

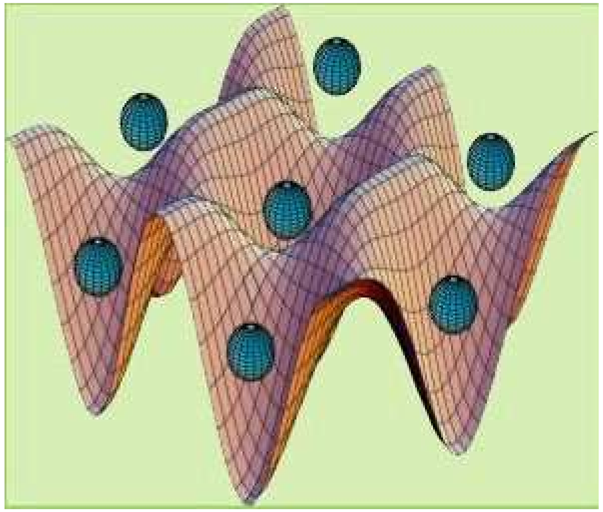
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Quantum Quenches:

- consider quantum system with Hamiltonian:

$$H = H_0 + \lambda(t)\,\delta H$$

- prepare system in eigenstate $|\psi_0\rangle$ of Hamiltonian H_0
- abruptly turn on λ ; system evolves *unitarily* according to H
- Question: How do observables, eg, expectation values and correlation functions, evolve in time?
- for most systems, coupling to environment is unavoidable ⟶ decoherence, dissipation
- effects minimized for, eg, cold atoms in optical lattice



⟶ is there "universal" behaviour?

Quantum Quenches & Holography:

→ is there “universal” behaviour?

what are organizing principles for out-of-equilibrium systems?

- theoretical progress made for variety systems: d=2 CFT, (nearly) free fields, integrable models, ...
- still seeking broadly applicable and efficient techniques
- what can AdS/CFT correspondence offer?
  - → strongly coupled field theories
  - → real-time analysis
  - → finite temperature (if desired)
  - → general spacetime dimension
- perhaps re-organization of problem will lead to new insights

Quantum Quenches & Holography:

- AdS/CFT allows us to study quantum quenches for strongly coupled field theories in any number of dimensions

- there has been a great deal of interest in the past few years

Chesler, Yaffe; Das, Nishioka, Takayanagi, Basu; Bhattacharyya, Minwalla; Abajo-Arrastia, Aparicio, Lopez; Albash, Johnson; Ebrahim, Headrick; Balasubramanian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Mueller, Schafer, Shigemori, Staessens, Galli; Allias, Tonni; Keranen, Keski-Vakkuri, Thorlacius; Galante, Schvellinger; Carceres, Kundu; Wu; Garfinkle, Pando Zayas, Reichmann; Bhaseen, Gauntlett, Simons, Sonner, Wiseman; . . . . . .

- much was work aimed at “thermalization”

- C&Y initiated application of “numerical relativity” techniques to study far-from-equilibrium physics

Quantum Quenches & Holography:

- AdS/CFT allows us to study quantum quenches for strongly coupled field theories in any number of dimensions

Where are control parameters in AdS/CFT framework?

AdS/CFT dictionary:

gravity fields $\longleftrightarrow$ boundary operators

Φ $\qquad\qquad$ δH

eg, consider some scalar field in AdS:

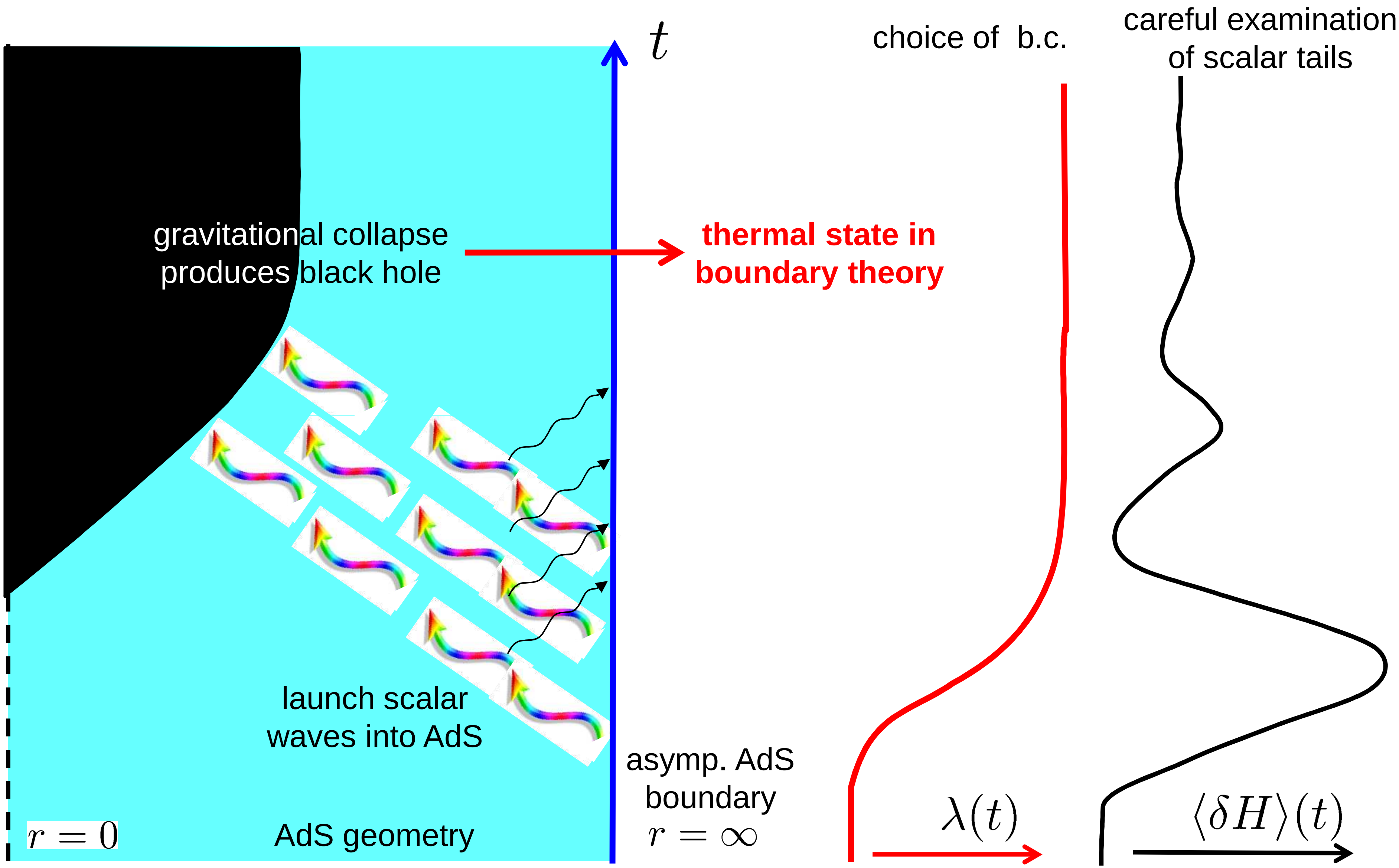
equation of motion: $(\nabla^2 - m^2)\Phi + \cdots = 0$

asymptotic solutions: $\Phi \sim \dfrac{\lambda}{r^{d-\Delta}} + \dfrac{\langle \delta H \rangle}{r^{\Delta}} + \cdots$

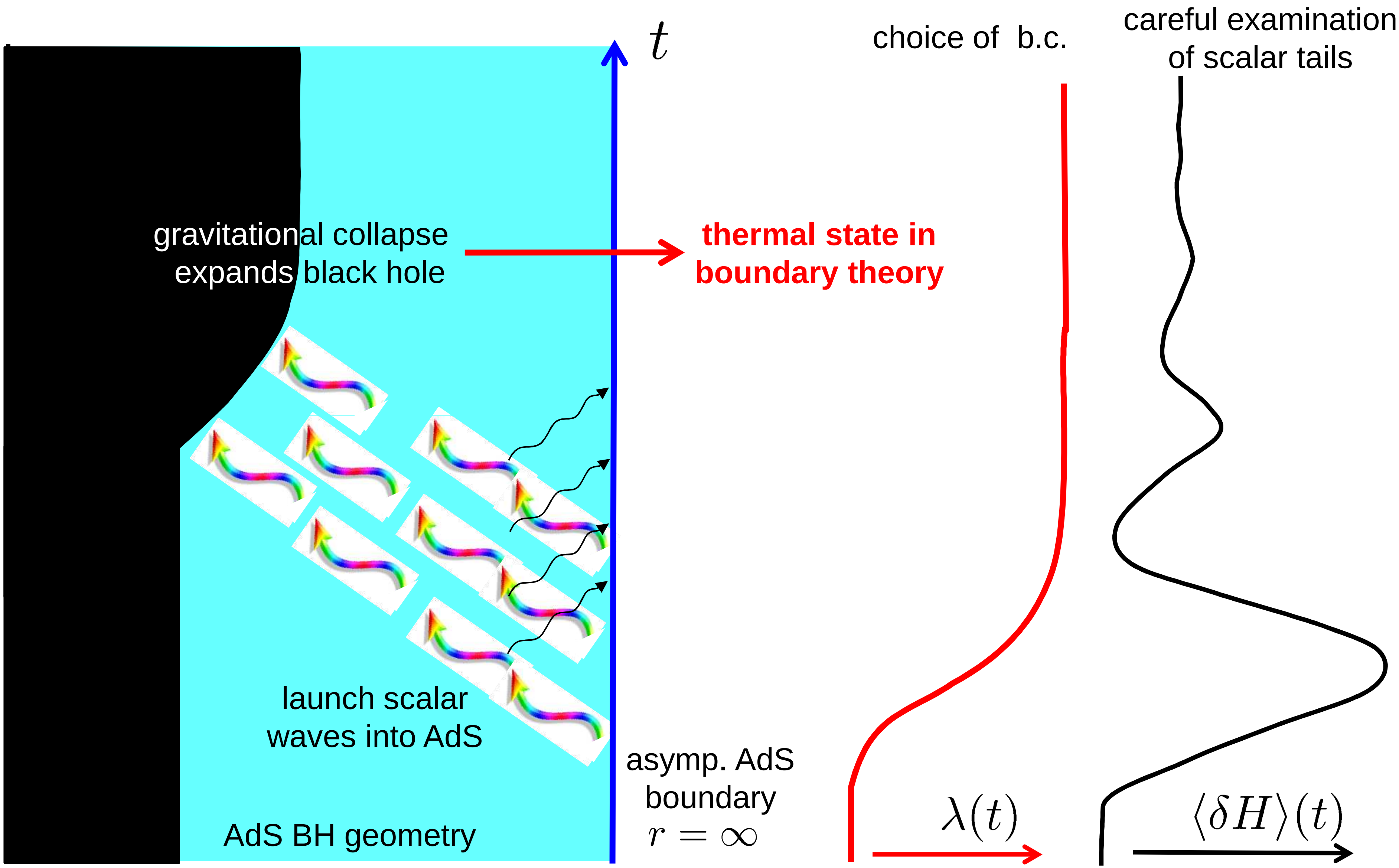
$\longrightarrow$ integration constants become coupling and expectation value

recall conformal dimension: $\Delta = \dfrac{d}{2} + \sqrt{\dfrac{d^2}{4} + m^2 L^2}$

Holographic Quantum Quench (cartoon): (cf. Chesler & Yaffe)

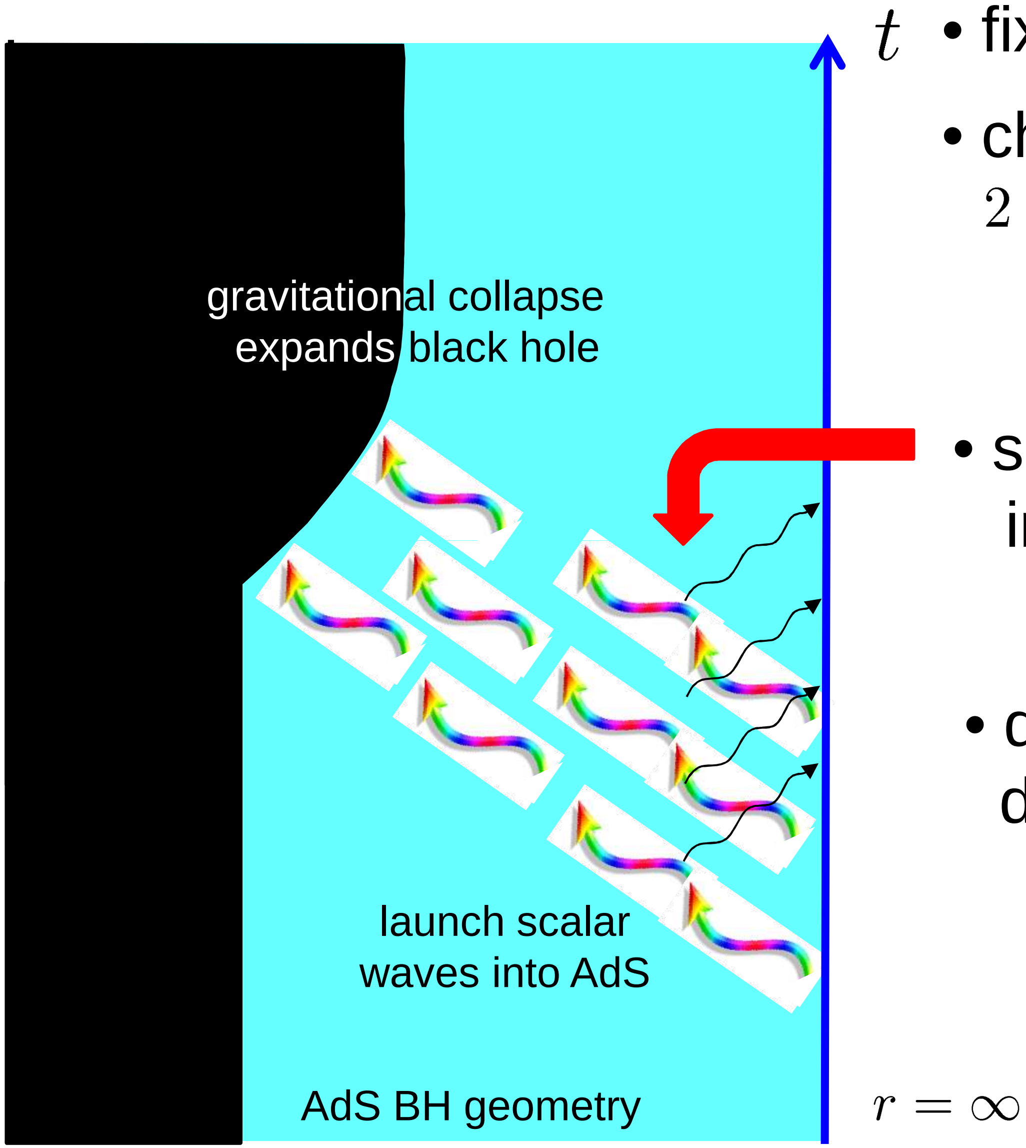


Holographic Quantum Quench (cartoon):



- **"thermal quench"**: quantum quench at finite temperature

Holographic Thermal Quench:



- fix boundary dimension: $d = 4$
- choose conformal dimension, $2 \le \Delta \le 4$ and profile

$$\lambda(t) = \frac{\Delta\lambda}{2}\left[1 + \tanh(t/\Delta t)\right]$$

- solve linearized scalar eom in fixed BH geometry

  ⟶ determines $\langle \mathcal{O}_\Delta \rangle (t)$

- determine "BH mass" $\mathcal{E}(t)$ with diffeomorphism Ward identity*:

$$\partial^i \langle T_{ij} \rangle \;=\; \langle \mathcal{O}_\Delta \rangle \; \partial_j \lambda$$

⟶ integrate for $\mathcal{E}(t)$, ie,

$$\partial_t \mathcal{E} = -\langle \mathcal{O}_\Delta \rangle \; \partial_t \lambda$$

* boundary constraint from Einstein eq's

Holographic Thermal Quench:

- lessons learned:

  1. Renormalization of (strongly coupled) boundary QFT with time-dependent couplings works in a straightforward way

- holography gives well-defined approach to renormalize bdry QFT
- bdry theory has new divergences: (Λ = UV cut-off scale)

$$I_{ct} \simeq \int d^4x\sqrt{-g}\Big(\Lambda^4 + \Lambda^{2\Delta-4}\,\lambda^2(t) + \cdots$$
$$+\Lambda^{2\Delta-6}g^{ij}\,\partial_i\lambda\,\partial_j\lambda + \Lambda^{2\Delta-6}\mathcal{R}(g)\,\lambda^2 + \cdots\Big)$$

- familiar in the context of QFT in curved backgrounds
- new log divergences lead to new scheme dependent ambiguities

Holographic Thermal Quench:

- lessons learned:

  2. Response to fast quenches exhibits universal scaling

- for example: $\max\langle\mathcal{O}_\Delta\rangle \sim \dfrac{\Delta\lambda}{(\Delta t)^{2\Delta-4}}$

$$\Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}}$$

$\Big\}$ $\Delta t \to 0$ **yields physical divergence!!**

- seems to indicate instantaneous quench is problematic
  - ► physical problem?
  - ► simply an issue with perturbative expansion?

- compare to seminal work of Cardy & Calabrese

⟶ "instantaneous quench" is basic starting point

Holographic Thermal Quench:

- lessons learned:

2. Response to fast quenches exhibits universal scaling

- for example: $$\max\langle \mathcal{O}_\Delta \rangle \sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-4}}$$

$$\Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}}$$

$$\Delta t \to 0$$

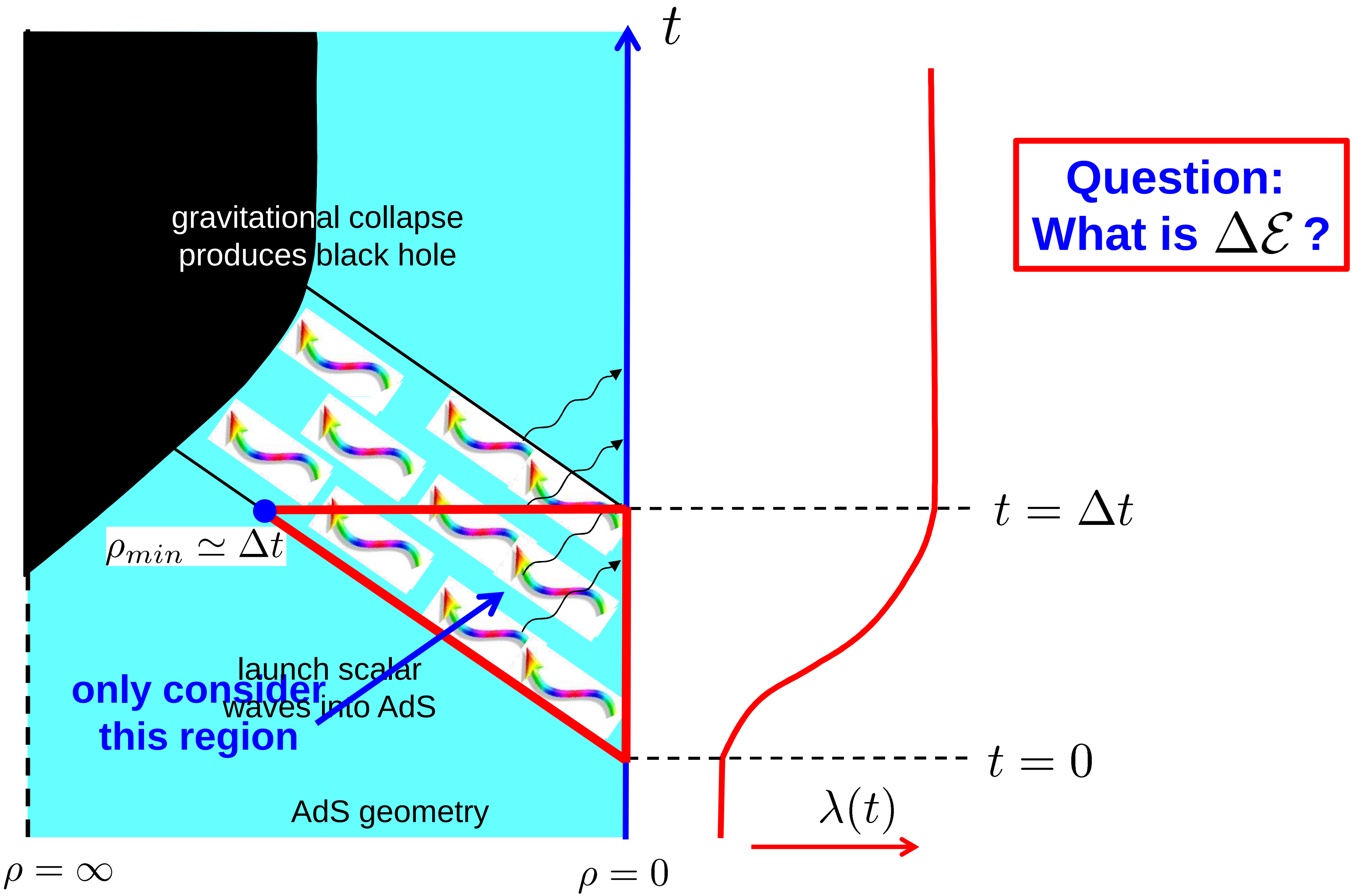
yields physical divergence!!

- seems to indicate instantaneous quench is problematic
  - ► physical problem?
  - ► simply an issue with perturbative expansion?

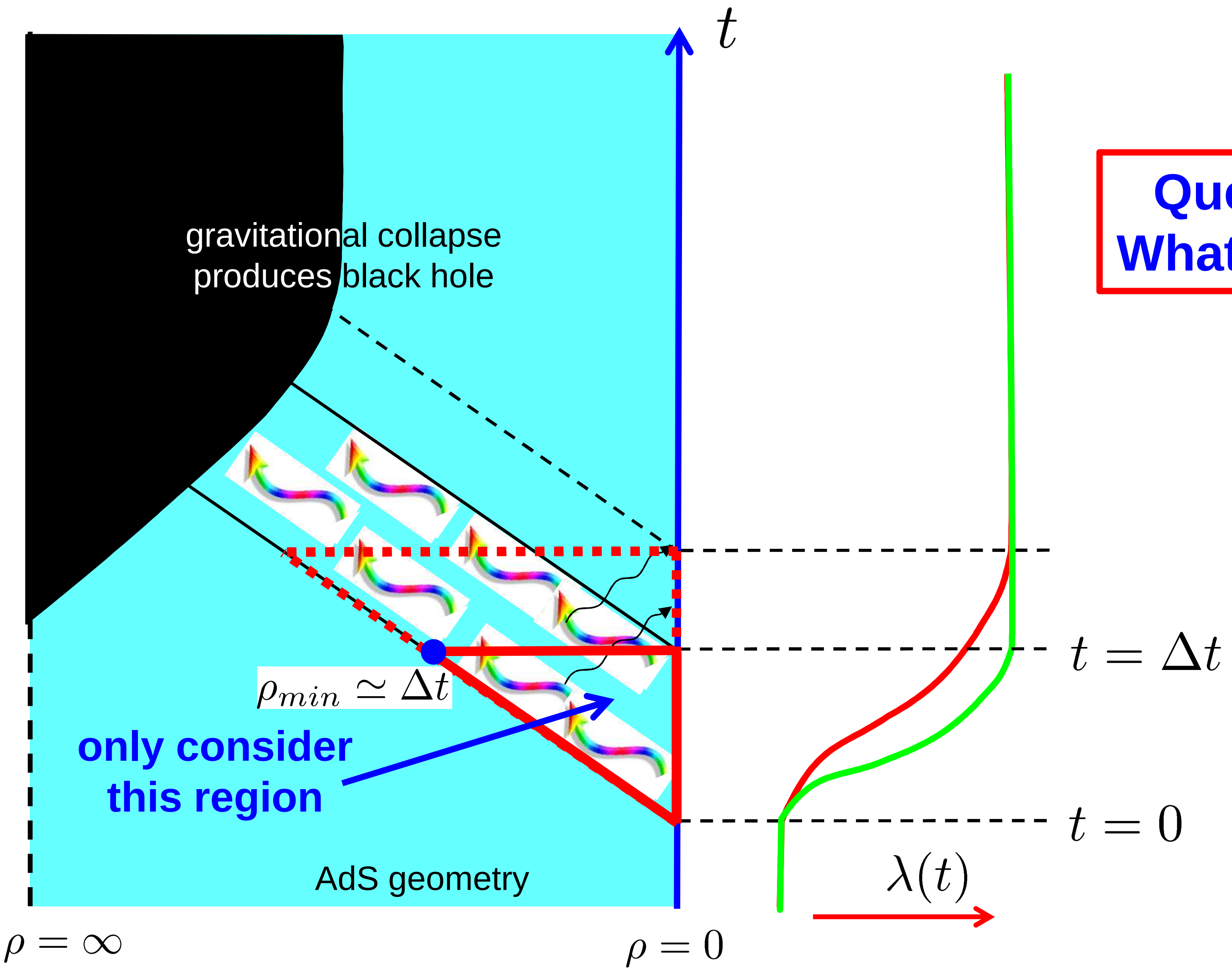
- same limit produces divergence in Chesler & Yaffe:

$$\Delta\mathcal{E} \propto \Delta t^{-4}$$ (dimensional analysis)

Holographic Quantum Quench (cartoon):

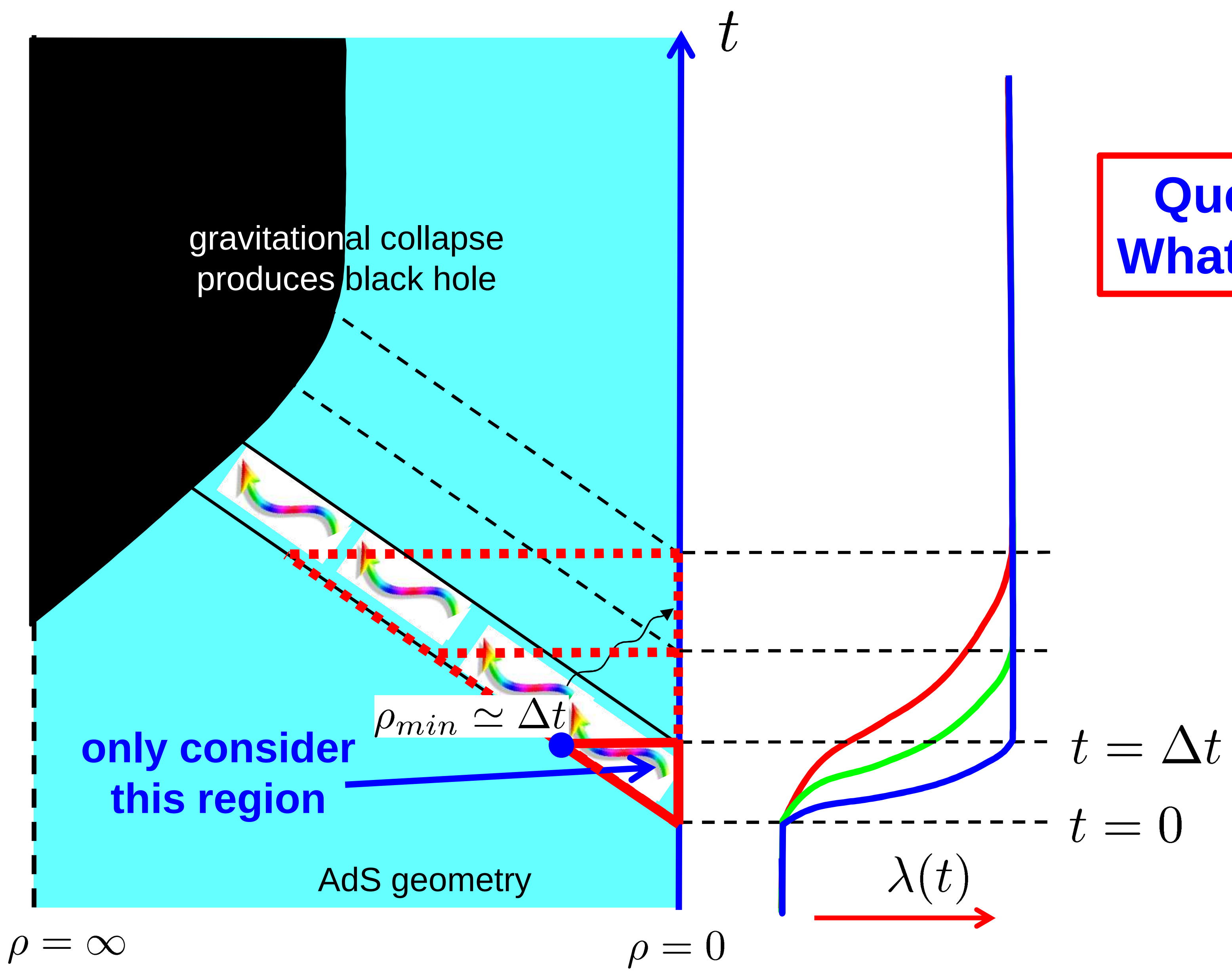


Holographic Quantum Quench (cartoon):



Question:
What is $\Delta\mathcal{E}$?

Holographic Quantum Quench (cartoon):



Question:
What is $\Delta\mathcal{E}$?

Generalizing "Fast" Quenches:

→ **Question: What is $\Delta\mathcal{E}$?**

- as we scale $\Delta t \to 0$, only "tiny" region of solution in asymptotic AdS relevant **for this question**

  → certainly full numerical simulation is not needed

  → perhaps solvable with purely analytic approaches

- given $\Delta\mathcal{E}$, properties of final state are fixed but full details of evolution, eg, approach to final state, are not determined

Generalizing "Fast" Quenches:

→ **Question: What is $\Delta\mathcal{E}$?**

- solve full bulk equations of motion perturbatively for $\rho \ll 1$

$$
\begin{aligned}
0 &= -\frac{2(d-3)}{(d-1)A}u(\phi) + \frac{2d(d-3)}{A} + \rho^4\left(\phi'\right)^2 - \frac{d-3}{(d-1)A}m^2\phi^2 - \left(\frac{\dot\phi}{A}\right)^2 \\
&\quad + 2(d-2)(d-1)\left[\left(\frac{\dot\Sigma}{A\Sigma}\right)^2 - \left(\frac{\rho^2\Sigma'}{\Sigma}\right)^2\right] + \frac{2\rho^2\left(\rho^2 A'\right)'}{A} - 4\left(\frac{\dot A}{A^2}\right)^2 + 2\frac{\ddot A}{A^3} \\
0 &= d - \frac{u(\phi)}{(d-1)} - \frac{m^2\phi^2}{2(d-1)} + \frac{\rho^4 A}{2(d-1)}\left(\phi'\right)^2 + \frac{\dot\phi^2}{A} - \rho^4\frac{A'\Sigma'}{\Sigma} - (d-2)\rho^4 A\frac{(\Sigma')^2}{\Sigma^2} \\
&\quad + \frac{2\ddot\Sigma}{A\Sigma} - \frac{\dot A\dot\Sigma}{A^2\Sigma} + (d-2)\frac{\dot\Sigma^2}{A\Sigma^2} \\
0 &= \frac{\left(\phi'\right)^2}{2(d-1)} + \frac{1}{2(d-1)}\left(\frac{\dot\phi}{\rho^2 A}\right)^2 + \frac{\Sigma''}{\Sigma} + \frac{2\Sigma'}{\rho\Sigma} + \frac{\ddot\Sigma}{\rho^4 A^2\Sigma} \\
0 &= \frac{\phi'\dot\phi}{d-1} + \frac{\dot A\Sigma'}{A\Sigma} - \frac{A'\dot\Sigma}{A\Sigma} + 2\frac{\dot\Sigma'}{\Sigma} \\
0 &= -\frac{\delta u(\phi)}{\delta\,\phi} - m^2\phi + \rho^4 A\phi'' + 2\rho^3 A\phi' + \rho^4 A'\phi' + \frac{(d-1)\rho^4 A\Sigma'\phi'}{\Sigma} + \frac{\dot A\dot\phi}{A^2} - \frac{(d-1)\dot\Sigma\dot\phi}{A\Sigma} - \frac{\ddot\phi}{A}
\end{aligned}
$$

Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
- solve full bulk equations of motion perturbatively for $\rho \ll 1$

$$\begin{aligned}\phi \quad = \quad & \rho^{d-\Delta}\left(p_0(t) + a_1\,\rho\,p_0'(t) + a_2\,\rho^2\,p_0''(t) + \cdots\right)\\ & +c_1\rho^{2(d-\Delta)}p_0^2(t) + c_2\rho^{2(d-\Delta)+1}p_0(t)p_0'(t) + \cdots\\ & +d_1\rho^{3(d-\Delta)}p_0^3(t) + \cdots\\ & +\rho^{\Delta}\left(b_1 p_2(t) + b_2\,\rho\,p_2'(t) + b_3\,\rho^2\,p_2''(t) + \cdots\right)\\ & +e_1\rho^{d}p_0(t)p_2(t) + e_2\rho^{d+1}p_0(t)p_2'(t) + \cdots\\ & +\cdots\end{aligned}$$

nonlinearities in eom

recall $\lambda(t) \sim p_0(t)$ and $\langle \mathcal{O}_\Delta \rangle(t) \sim p_2(t)$

Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
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$$
\begin{aligned}
\phi \;=\; & \rho^{d-\Delta}\left(p_0(t) + a_1\,\rho\, p_0'(t) + a_2\,\rho^2\, p_0''(t) + \cdots\right) \\
& +c_1\rho^{2(d-\Delta)} p_0^2(t) + c_2\rho^{2(d-\Delta)+1} p_0(t)p_0'(t) + \cdots \\
& +d_1\rho^{3(d-\Delta)} p_0^3(t) + \cdots \\
& +\rho^{\Delta}\left(b_1 p_2(t) + b_2\,\rho\, p_2'(t) + b_3\,\rho^2\, p_2''(t) + \cdots\right) \\
& +e_1\rho^{d} p_0(t)p_2(t) + e_2\rho^{d+1} p_0(t)p_2'(t) + \cdots \\
& + \cdots
\end{aligned}
$$

nonlinearities in eom (arrows point to the c, d, e and final $+\cdots$ terms)

- set $\Delta t = \alpha\,\widehat{\Delta t}$ and take limit $\alpha \to 0$ (while $p_0(t/\Delta t)$ kept fixed)

→ natural to scale coordinates: $t = \alpha\,\hat{t}\,, \quad \rho = \alpha\hat{\rho}$

eg, $p_0(t/\Delta t) = p_0(\hat{t}/\widehat{\Delta t})$

Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
- solve full bulk equations of motion perturbatively for $\rho \ll 1$

$$\begin{aligned}\phi \;=\; & \underline{\alpha^{d-\Delta}}\hat\rho^{d-\Delta}\left(p_0(\hat t) + a_1\,\hat\rho\, p_0'(\hat t) + a_2\,\hat\rho^2\, p_0''(\hat t) + \cdots\right) \\ & +\underline{\alpha^{2(d-\Delta)}}\Big(c_1\hat\rho^{2(d-\Delta)}p_0^2(\hat t) + c_2\hat\rho^{2(d-\Delta)+1}p_0(\hat t)p_0'(\hat t) + \cdots \\ & +\underline{\alpha^{3(d-\Delta)}}d_1\hat\rho^{3(d-\Delta)}p_0^3(\hat t) + \cdots \\ & +\underline{\alpha^{\Delta+\delta}}\hat\rho^{\Delta}\left(b_1\hat p_2(\hat t) + b_2\,\hat\rho\,\hat p_2'(\hat t) + b_3\,\hat\rho^2\,\hat p_2''(\hat t) + \cdots\right) \\ & +\underline{\alpha^{d+\delta}}\left(e_1\hat\rho^{d}p_0(\hat t)\hat p_2(\hat t) + e_2\hat\rho^{d+1}p_0(\hat t)\hat p_2'(\hat t) + \cdots\right) \\ & +\cdots\end{aligned}$$

- set $\Delta t = \alpha\,\widehat{\Delta t}$ and take limit $\alpha \to 0$ (while $p_0(t/\Delta t)$ kept fixed)

⟶ natural to scale coordinates: $t = \alpha\,\hat t\,, \quad \rho = \alpha\hat\rho$

⟶ add: $\underline{p_2(t) = \alpha^\delta\,\hat p_2(t)}$

Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
- solve full bulk equations of motion perturbatively for $\rho \ll 1$

$$
\begin{aligned}
\phi \;=\; & \alpha^{d-\Delta}\hat\rho^{d-\Delta}\left(p_0(\hat t)+a_1\,\hat\rho\,p_0'(\hat t)+a_2\,\hat\rho^2\,p_0''(\hat t)+\cdots\right)\\
& +\alpha^{2(d-\Delta)}\left(c_1\hat\rho^{2(d-\Delta)}p_0^2(\hat t)+c_2\hat\rho^{2(d-\Delta)+1}p_0(\hat t)p_0'(\hat t)+\cdots\right.\\
& +\alpha^{3(d-\Delta)}d_1\hat\rho^{3(d-\Delta)}p_0^3(\hat t)+\cdots\\
& +\alpha^{\Delta+\delta}\hat\rho^{\Delta}\left(b_1\hat p_2(\hat t)+b_2\,\hat\rho\,\hat p_2'(\hat t)+b_3\,\hat\rho^2\,\hat p_2''(\hat t)+\cdots\right)\\
& +\alpha^{d+\delta}\left(e_1\hat\rho^{d}p_0(\hat t)\hat p_2(\hat t)+e_2\hat\rho^{d+1}p_0(\hat t)\hat p_2'(\hat t)+\cdots\right)\\
& +
\end{aligned}
$$

- set $\Delta t = \alpha\,\widehat{\Delta t}$ and take limit $\alpha \to 0$ (while $p_0(t/\Delta t)$ kept fixed)

⟶ natural to scale coordinates: $t=\alpha\,\hat t\,,\quad \rho=\alpha\hat\rho$

⟶ add: $p_2(t)=\alpha^{\delta}\,\hat p_2(t)$ ⟶ "matching bc": $\delta=-(2\Delta-d)$

Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
- solve full bulk equations of motion perturbatively for $\rho \ll 1$

$$\begin{aligned}\phi &= \alpha^{d-\Delta}\hat\rho^{d-\Delta}\left(p_0(\hat t) + a_1\,\hat\rho\, p_0'(\hat t) + a_2\,\hat\rho^2\, p_0''(\hat t) + \cdots\right) \\ &\quad +\alpha^{d-\Delta}\hat\rho^{\Delta}\left(b_1 \hat p_2(\hat t) + b_2\,\hat\rho\,\hat p_2'(\hat t) + b_3\,\hat\rho^2\,\hat p_2''(\hat t) + \cdots\right)\end{aligned}$$

- set $\Delta t = \alpha\,\widehat{\Delta t}$ and take limit $\alpha \to 0$ (while $p_0(t/\Delta t)$ kept fixed)

⟶ natural to scale coordinates: $t = \alpha\,\hat t\,,\quad \rho = \alpha\hat\rho$

⟶ need: $p_2(t) = \alpha^{-(2\Delta-d)}\,\hat p_2(t)$ ⟶ $\phi \to \alpha^{d-\Delta}\phi$

- similar scaling arguments yield:

$$\Sigma = 1/\rho\,, \qquad \Sigma \to \Sigma/\alpha; \qquad A = 1/\rho^2\,, \qquad A \to A/\alpha^2$$

- **relevant solution = linearized scalar solution in AdS space!**

* require $\Delta < d$ (**not** $\Delta = d$)

Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
- **relevant solution = linearized scalar solution in AdS space!**
- combine scaling $p_2 = \alpha^{-(2\Delta - d)}\,\hat{p}_2$ with holographic dict., eg,

$$\Delta\mathcal{E} \simeq \int_0^{\Delta t} dt\,\langle \mathcal{O}_\Delta \rangle\;\partial_t \lambda \simeq \int_0^{\Delta t} dt\,p_2(t)\,p_0'(t)$$

$$\Rightarrow \quad \max\langle \mathcal{O}_\Delta \rangle \sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta - d}}\;; \quad \Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta - d}}$$

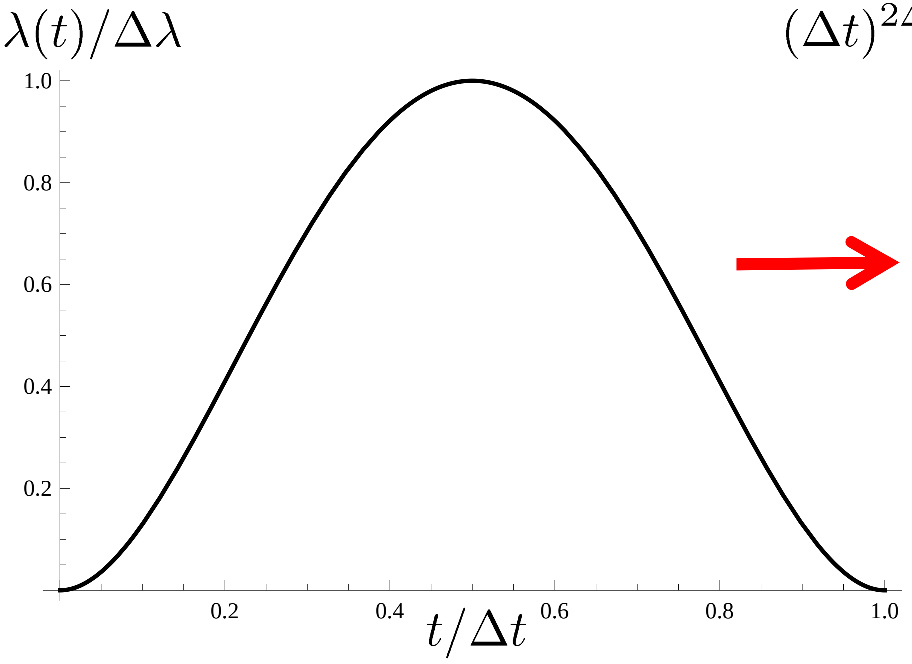
- matches previous perturbative analysis (for d=4)
- result here applies for any full nonlinear solution!!
    - ► physical ~~problem?~~ effect
    - ~~► simply an issue with perturbative expansion?~~

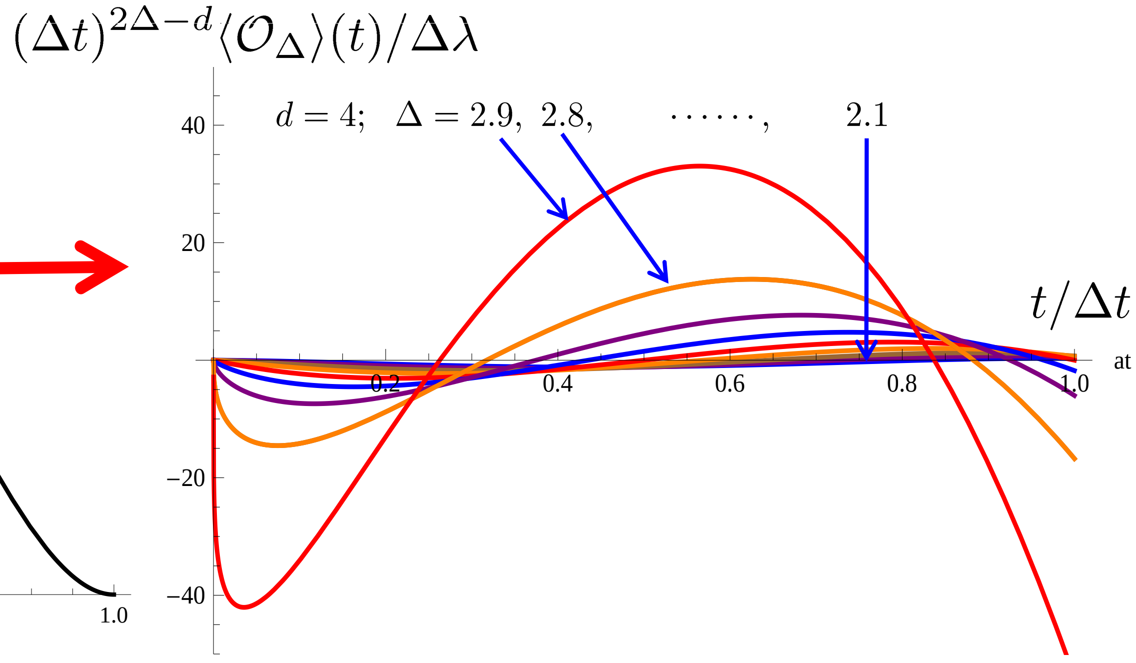
Generalizing "Fast" Quenches:

- analytic solutions, eg: $\lambda(t) = 16\Delta\lambda\big((t/\Delta t)^2 - 2(t/\Delta t)^3 + (t/\Delta t)^4\big)$

$$\rightarrow \langle \mathcal{O}_\Delta \rangle(t) = \frac{16\,\Delta\lambda}{(\Delta t)^{2\Delta-d}} \left(\frac{t}{\Delta t}\right)^{d-2\Delta} \left(b_2 \left(\frac{t}{\Delta t}\right)^2 - 2b_3 \left(\frac{t}{\Delta t}\right)^3 + b_4 \left(\frac{t}{\Delta t}\right)^4 \right)$$

where $b_\kappa = -\dfrac{2^{d-2\Delta}\,\Gamma(\kappa+1)\,\Gamma(\frac{d+2}{2}-\Delta)}{\Gamma(d+1+\kappa-2\Delta)\,\Gamma(\Delta-\frac{d-2}{2})}$





Generalizing "Fast" Quenches:

$$\Delta\mathcal{E} \simeq \frac{\Delta p_0^2}{\alpha^{2\Delta-d}}$$

- $\alpha \to 0$ yields physical divergence for $\frac{d}{2} < \Delta < d$

⟶ "instantaneous" quench seems problematic!?!

- maintain finite $\Delta\mathcal{E}$ by simultaneously scaling $\Delta p_0 = \alpha^{\Delta-d/2}\widehat{\Delta p_0}$

⟶ response still divergent! ie, $p_2 \propto \frac{\widehat{\Delta p_0}}{\alpha^{\Delta-d/2}}$

("gravity waves" or $\frac{d}{2} - 1 \leq \Delta < \frac{d}{2}$ probably okay)

⟶ "instantaneous" quench still seems problematic!?!

("physical quench" versus "far-from-equilibrium initial data")

Generalizing "Fast" Quenches:

- compare directly to C&C, ie, quench mass of a free scalar:

$$\lambda = m^2 \;; \qquad \mathcal{O}_\Delta = \phi^2 \;; \qquad \Delta = d-2$$

- quench with finite Δt and examine limit $\Delta t \to 0$

eq. of motion: $\left[\nabla^2 - \frac{m^2}{2}\,(1+\tanh(t/\Delta t))\right]\phi = 0$

- a worked example in Birrell & Davies

"in" modes: $$f_k(t) = \frac{1}{\sqrt{4\pi k}} \exp\left[-i(\omega_+ + \omega_- \Delta t \log(2\cosh(t/\Delta t))\right] \times {}_2F_1\left(1+i\omega_-\Delta t, i\omega_-\Delta t, 1-ik\Delta t; \frac{1}{2}(1+\tanh(t/\Delta t)\right)$$

with $\omega_\pm = \frac{1}{2}\left(\pm k + \sqrt{k^2+m^2}\right)$

- in limit $\Delta t \to 0$, recover two-point correlator $G_k(t_1,t_2)$ of C&C

Generalizing "Fast" Quenches:

- consider response: $\langle\phi^2\rangle \simeq \int_0^{k_{max}} dk\, k^{d-3}\, |{}_2F_1|^2$

- UV divergences: eg, consider a constant mass

$$\langle\phi^2\rangle \simeq \int_0^{k_{max}} dk\, \frac{k^{d-2}}{\sqrt{k^2+m^2}} = \int_0^{k_{max}} dk \left[k^{d-3} - \frac{1}{2}m^2 k^{d-5} + \cdots\right]$$

$$= \frac{1}{d-2}k_{max}^{d-2} - \frac{m^2}{2(d-4)}k_{max}^{d-4} + \cdots$$

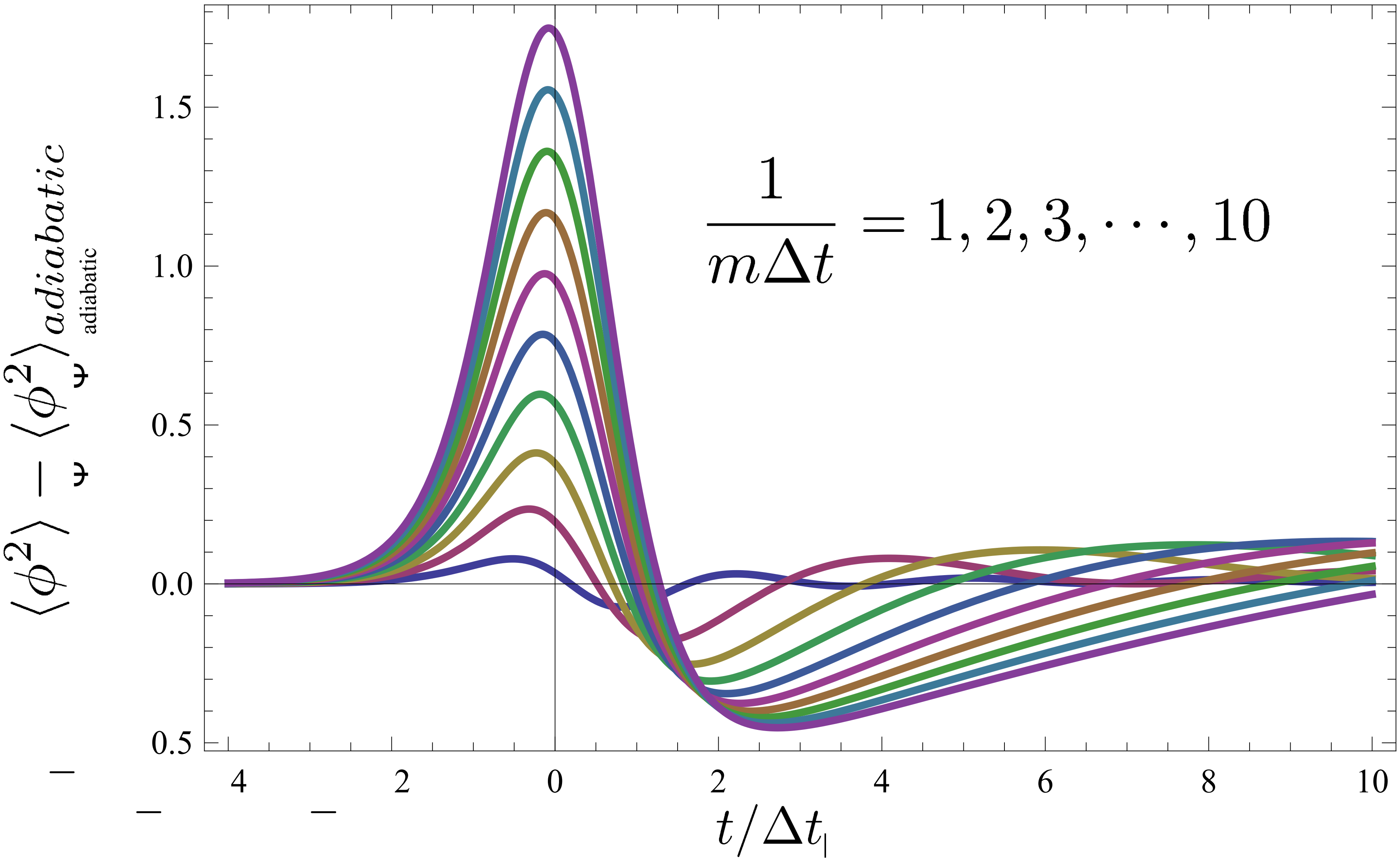
- regulated response (d=5):

$$\langle\phi^2\rangle \simeq \int_0^{\infty} dk \left[k^2\, |{}_2F_1|^2 - k^2 + \frac{1}{2}m^2(t)\right]$$

where $m^2(t) = \dfrac{m^2}{2}\left(1+\tanh(t/\Delta t)\right)$

Generalizing "Fast" Quenches:

- regulated response (d=5): $\langle\phi^2\rangle \simeq \int_0^\infty dk \left[k^2\,|{}_2F_1|^2 - k^2 - \frac{1}{2}m^2(t)\right]$



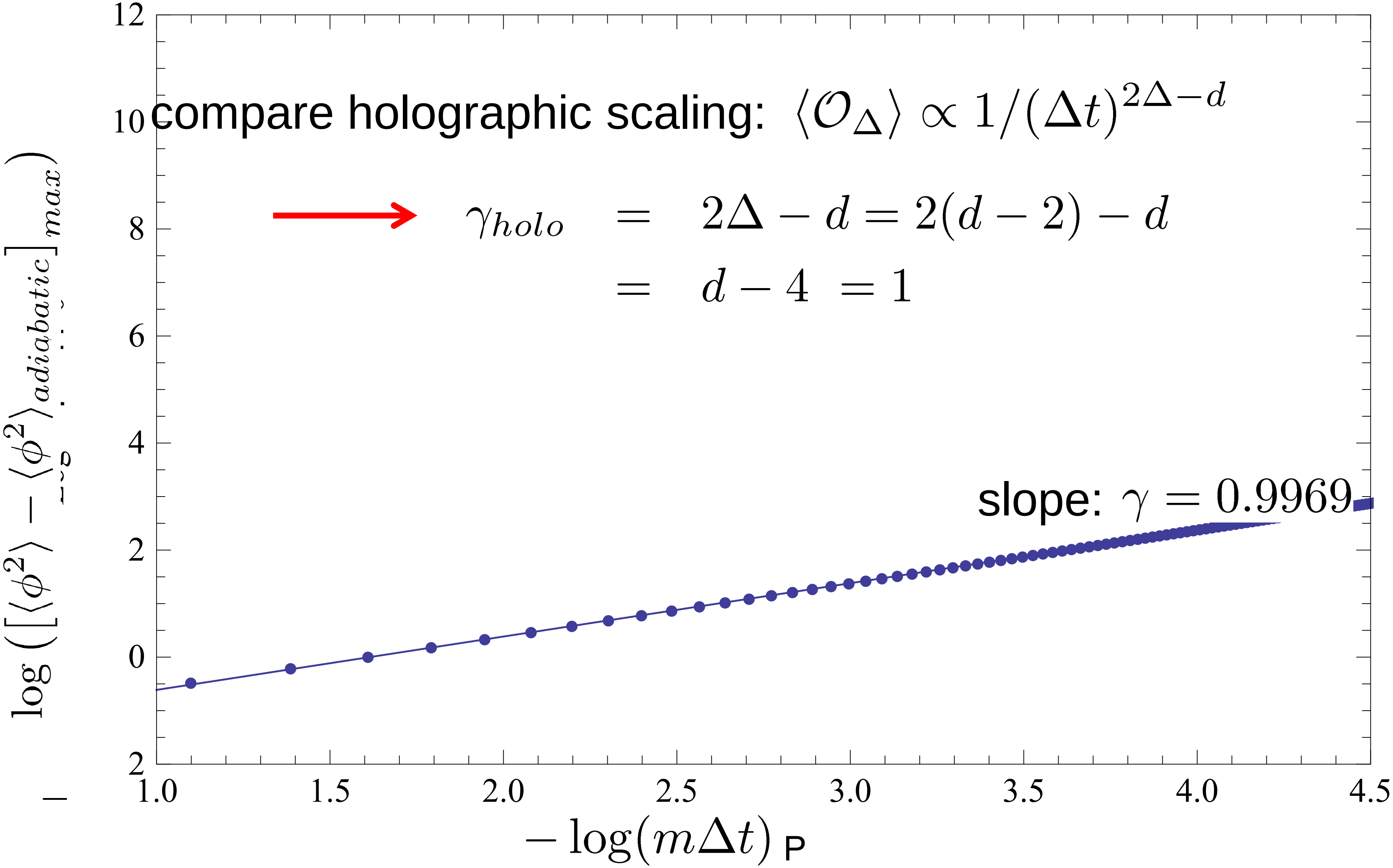
Generalizing "Fast" Quenches:

- regulated response (d=5):

compare holographic scaling: $\langle \mathcal{O}_\Delta \rangle \propto 1/(\Delta t)^{2\Delta - d}$

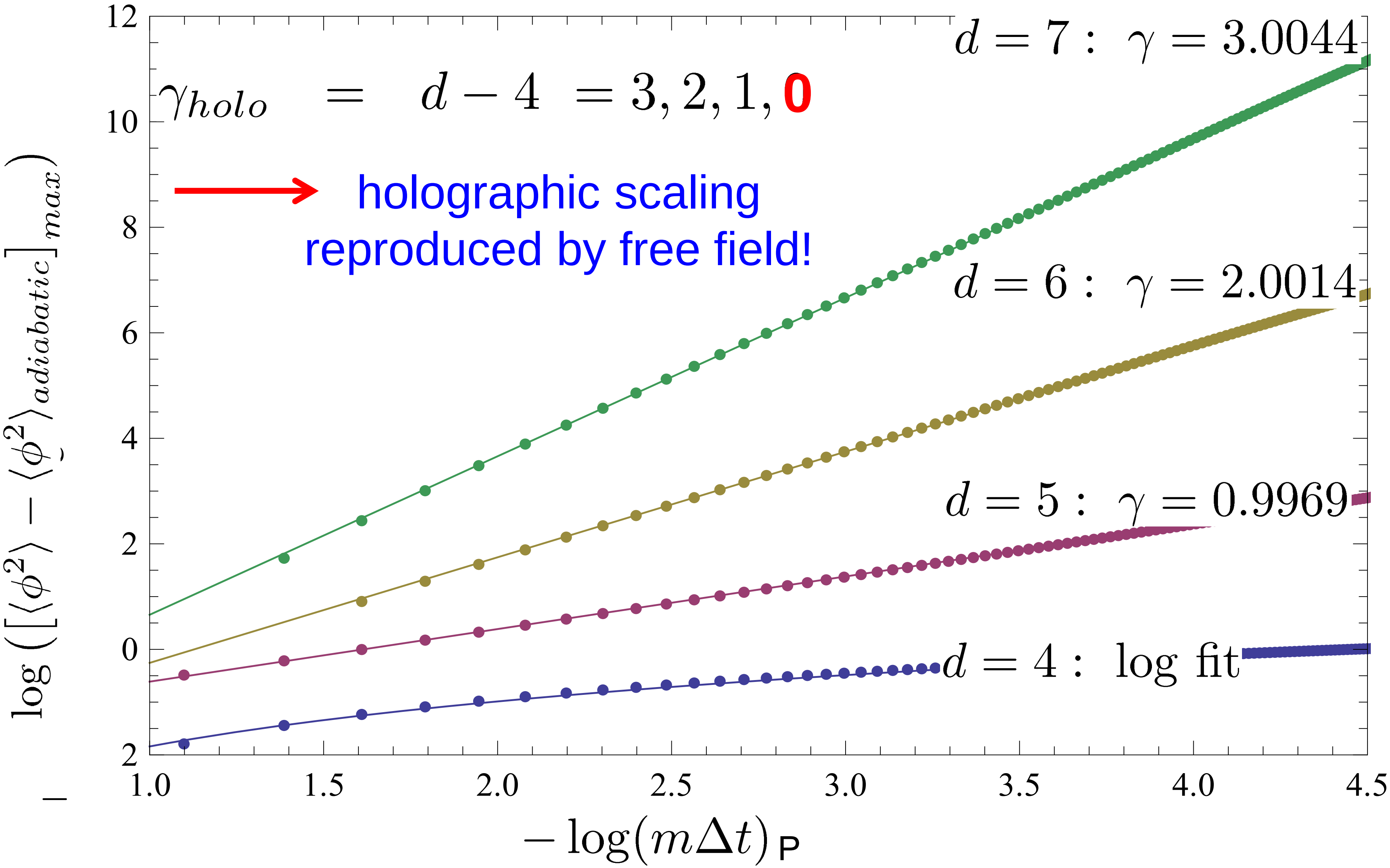
$$\gamma_{holo} = 2\Delta - d = 2(d-2) - d$$
$$= d - 4 = 1$$

slope: $\gamma = 0.9969$



Generalizing "Fast" Quenches:

- regulated response (d=7,6,5,4):



Generalizing "Fast" Quenches:

→ why is holographic scaling reproduced by free field?!?!?

- consider $S = S_{CFT} + \int d^dx\, \lambda(t)\, \mathcal{O}_\Delta(x)$ with $\lambda(t) = \Delta\lambda\ f(t/\Delta t)$

- apply conformal perturbation theory

$$\begin{aligned}
\langle\mathcal{O}_\Delta(0)\rangle &= \langle\mathcal{O}_\Delta(0)\,\exp[i\int d^dx\lambda(t)\mathcal{O}_\Delta(x)]\rangle_{\rm CFT} \\
&= \langle\mathcal{O}_\Delta(0)\rangle_{\rm CFT} + i\Delta\lambda\langle\mathcal{O}_\Delta(0)\int d^dx f(t/\Delta t)\mathcal{O}_\Delta(x)\rangle_{\rm CFT} \\
&\quad -\frac{\Delta\lambda^2}{2}\langle\mathcal{O}_\Delta(0)\int d^dx f(t/\Delta t)\mathcal{O}_\Delta(x)\int d^dx' f(t'/\Delta t)\mathcal{O}_\Delta(x')\rangle_{\rm CFT} + \cdots \\
&= b_1\,\frac{\Delta\lambda}{(\Delta t)^{2\Delta-d}} + b_2\,\frac{\Delta\lambda^2}{(\Delta t)^{3\Delta-2d}} + \cdots
\end{aligned}$$

Generalizing "Fast" Quenches:

→ why is holographic scaling reproduced by free field?!?!?

- consider $S = S_{CFT} + \int d^dx\, \lambda(t)\, \mathcal{O}_\Delta(x)$ with $\lambda(t) = \Delta\lambda\ f(t/\Delta t)$

- apply conformal perturbation theory

$$\begin{aligned}
\langle \mathcal{O}_\Delta(0) \rangle &= \langle \mathcal{O}_\Delta(0) \exp[i \int d^dx \lambda(t) \mathcal{O}_\Delta(x)] \rangle_{\rm CFT} \\
&= \langle \mathcal{O}_\Delta(0) \rangle_{\rm CFT} + i\Delta\lambda \langle \mathcal{O}_\Delta(0) \int d^dx f(t/\Delta t) \mathcal{O}_\Delta(x) \rangle_{\rm CFT} \\
&\quad - \frac{\Delta\lambda^2}{2} \langle \mathcal{O}_\Delta(0) \int d^dx f(t/\Delta t) \mathcal{O}_\Delta(x) \int d^dx' f(t'/\Delta t) \mathcal{O}_\Delta(x') \rangle_{\rm CFT} + \cdots \\
&= \frac{1}{(\Delta t)^\Delta} \left(b_1\, g + b_2\, g^2 + \cdots \right)
\end{aligned}$$

- organized with dimensionless effective coupling: $g = \Delta\lambda\, (\Delta t)^{d-\Delta}$

- in limit $\Delta\lambda$ fixed and $\Delta t \to 0$: $g \to 0$ **!!**

→ leading term dominates: $\langle \mathcal{O}_\Delta(0) \rangle \simeq b_1 \frac{\Delta\lambda}{(\Delta t)^{2\Delta - d}}$

Generalizing "Fast" Quenches:

- **holographic scaling should appear quite generally!!**

- for example:

$$\left.\begin{aligned} \max\langle\mathcal{O}_\Delta\rangle &\sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-4}} \\ \Delta\mathcal{E} &\sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}} \end{aligned}\right\} \quad \Delta t \to 0$$

Conclusions:

- quantum quenches: interesting arena for holographic study
- lessons learned:

1. Renormalization of (strongly coupled) boundary QFT with time-dependent couplings works in a straightforward way
2. Response to fast quenches exhibits universal scaling

► much of fast holographic quenches analytically accessible

► both lessons 1 & 2 apply beyond holographic arena!!

Lots to explore!