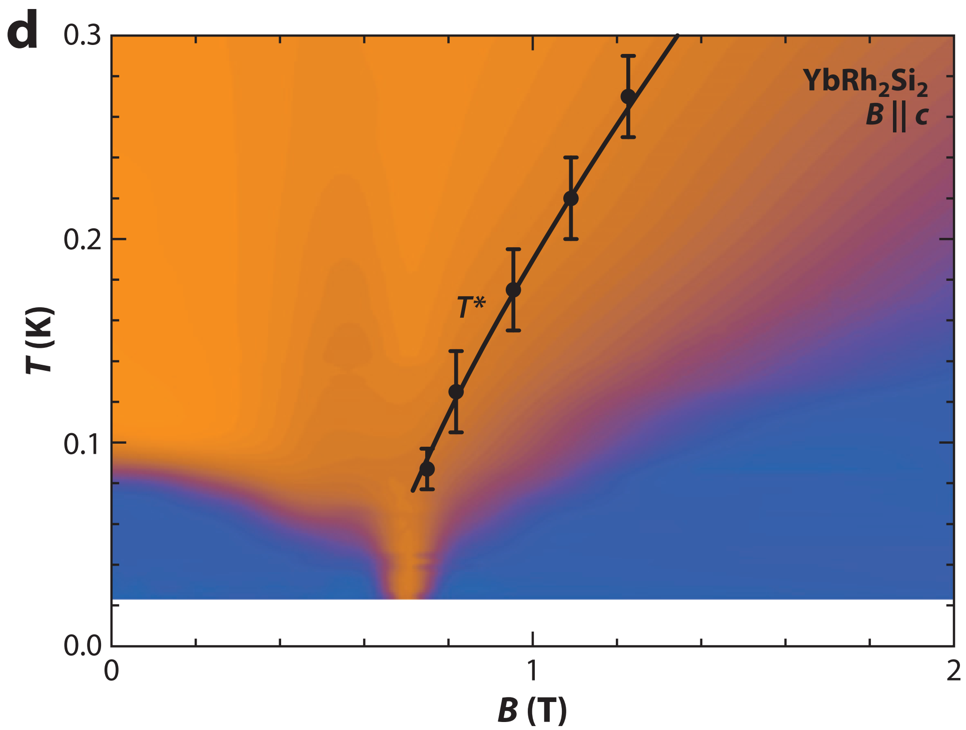
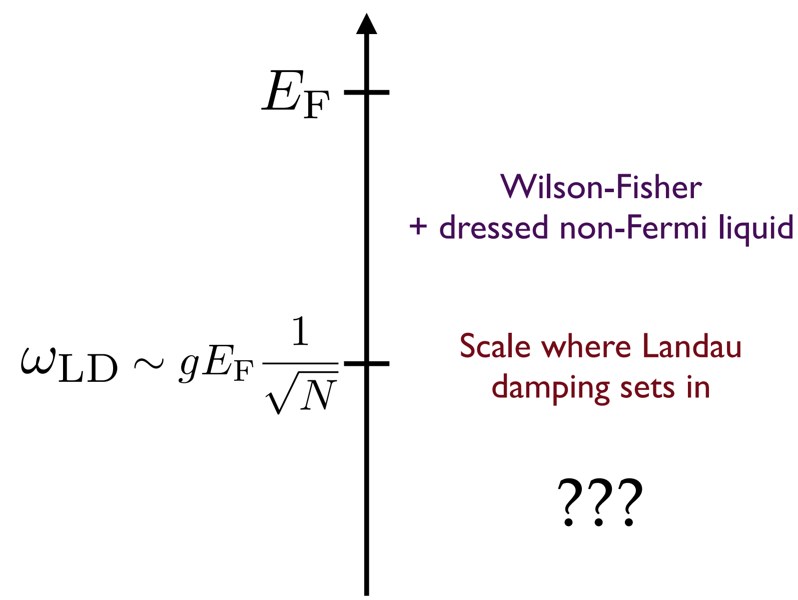
Wilsonian and large N approaches to non-Fermi liquids





Shamit Kachru (Stanford & SLAC)

Based in large part on this paper and work in progress...

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Non-Fermi-liquid fixed point in a Wilsonian theory of quantum critical metals

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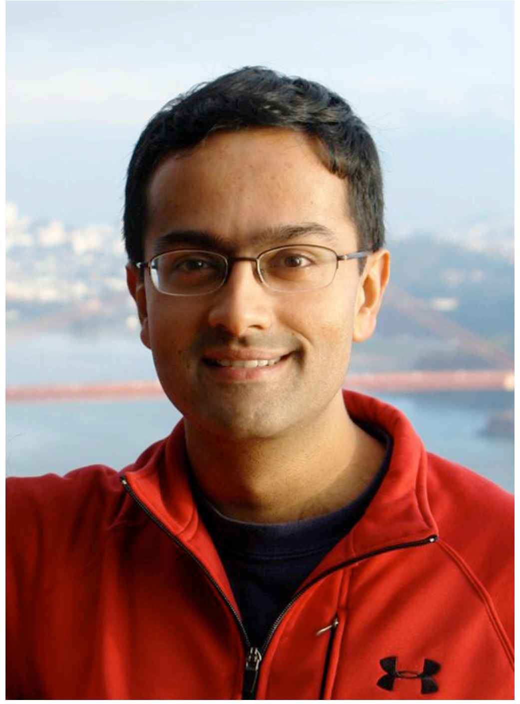

A few important references

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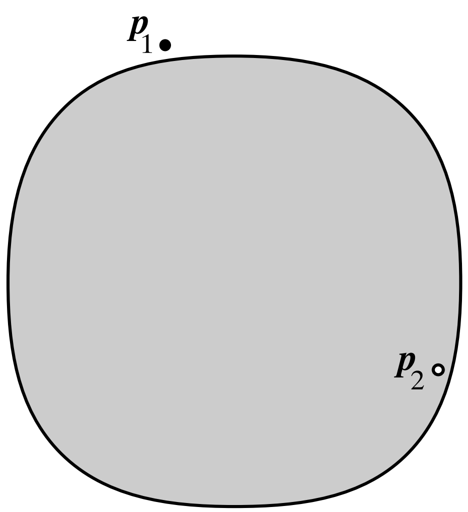
1. Introduction

The Fermi liquid fixed point is a stable IR fixed point (modulo the BCS instability).

Shankar,
Polchinski

Its starting point is the free fermion action:

$$\int dt\, d^3\mathbf{p} \left\{ i\psi^\dagger_\sigma(\mathbf{p})\partial_t\psi_\sigma(\mathbf{p}) - (\varepsilon(\mathbf{p}) - \varepsilon_{\mathrm{F}})\psi^\dagger_\sigma(\mathbf{p})\psi_\sigma(\mathbf{p}) \right\}.$$



The scaling which governs the Fermi liquid fixed point, determined by the free action, is:

$$\mathbf{p} = \mathbf{k} + \mathbf{l},$$

$$\varepsilon(\mathbf{p}) - \varepsilon_{\mathrm{F}} = l v_{\mathrm{F}}(\mathbf{k}) + O(l^2),$$

$$dt \to s^{-1} dt, \quad d\mathbf{k} \to d\mathbf{k}, \quad d\mathbf{l} \to s d\mathbf{l}, \quad \partial_t \to s\partial_t, \quad l \to sl,$$

so examining the action

$$\int dt\, d^2\mathbf{k}\, d\mathbf{l} \left\{ i\psi^\dagger_\sigma(\mathbf{p})\partial_t\psi_\sigma(\mathbf{p}) - l v_{\mathrm{F}}(\mathbf{k})\psi^\dagger_\sigma(\mathbf{p})\psi_\sigma(\mathbf{p}) \right\}$$

we see that the Fermi field should scale as:

$$\psi \to s^{-1/2}\psi \ .$$

The first interaction, which determines the (in)stability of this fixed point, is:

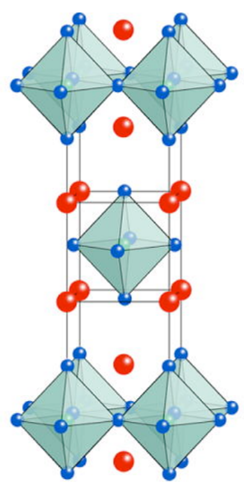
$$\int dt\, d^2\mathbf{k}_1\, d\mathbf{l}_1\, d^2\mathbf{k}_2\, d\mathbf{l}_2\, d^2\mathbf{k}_3\, d\mathbf{l}_3\, d^2\mathbf{k}_4\, d\mathbf{l}_4\, V(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$
$$\psi^\dagger_\sigma(\mathbf{p}_1)\psi_\sigma(\mathbf{p}_3)\psi^\dagger_{\sigma'}(\mathbf{p}_2)\psi_{\sigma'}(\mathbf{p}_4)\delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4).$$

It naively scales like s and is irrelevant, but for special kinematic configurations the delta function scales and we get marginal forward scattering and BCS couplings.

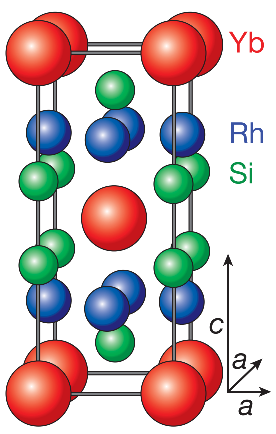
As it has no other obvious interesting perturbations, it is seemingly not a good starting point to describe the non-Fermi liquid physics which is apparently seen in many real systems...

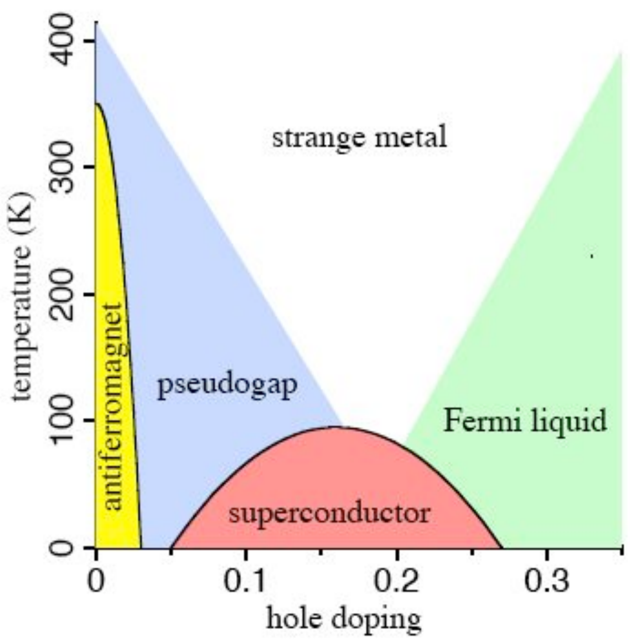
Some poster-child quantum critical metals:

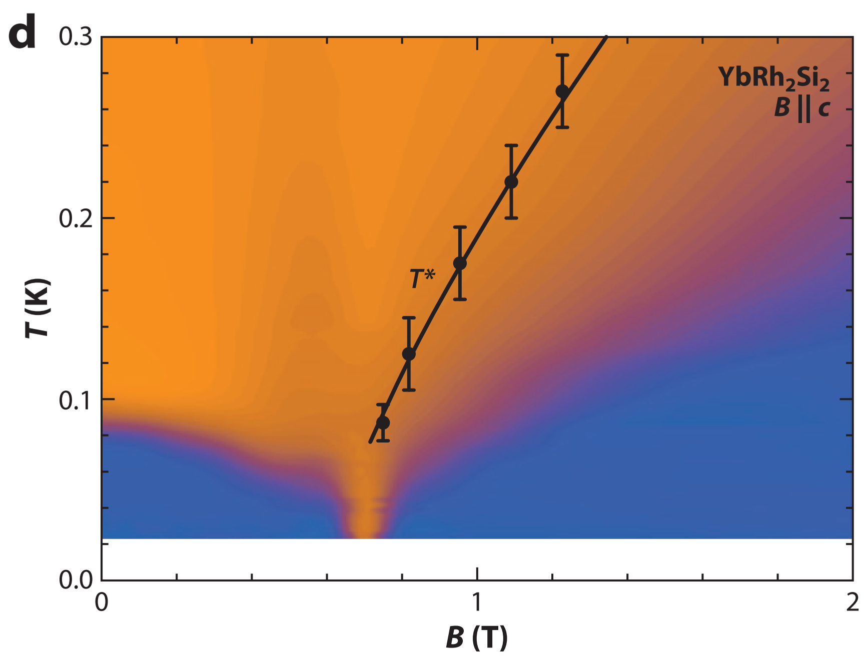
$La_{2-x}Sr_xCuO_4$



$YbRh_2Si_2$



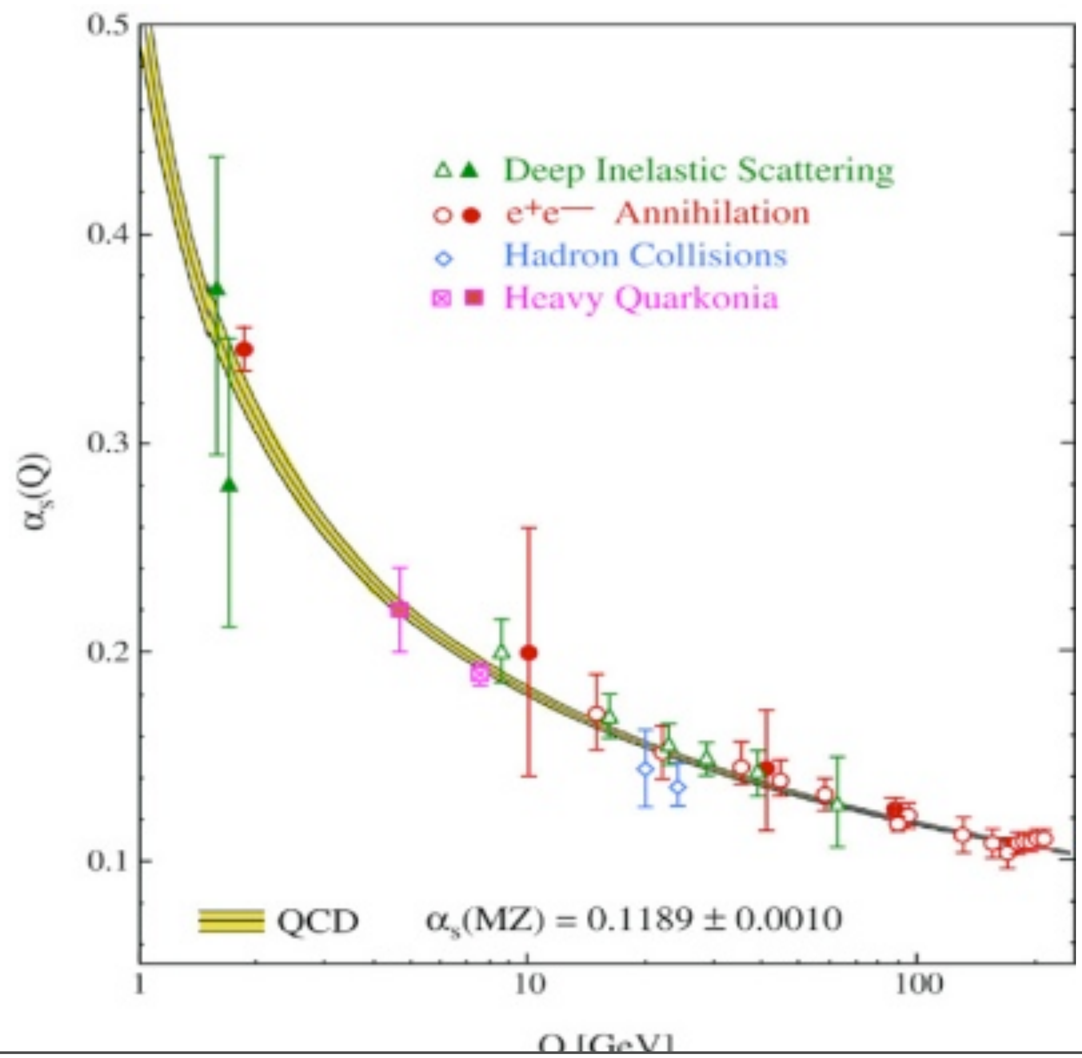




$$\rho(T) = \rho_0 + AT + \cdots$$

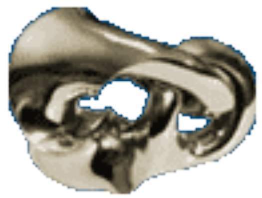
The theoretical description of this non-Fermi liquid behavior then remains a subject of active interest.

As a high energy theorist, I am going to borrow a lesson from our study of QCD.



The effective coupling runs strong in the IR. The emergent physics includes chiral symmetry breaking and confinement of quarks.

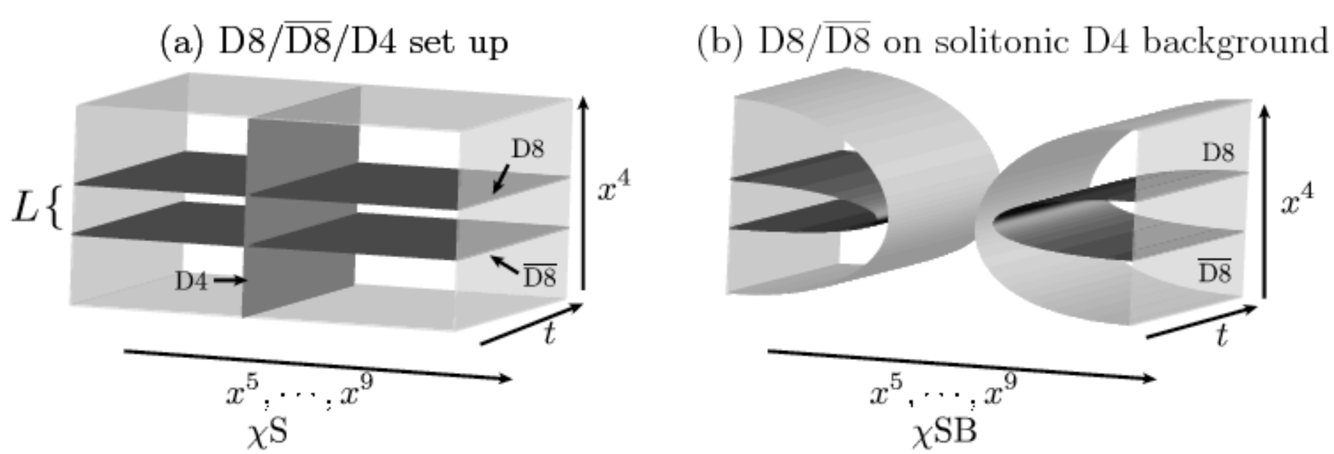
The brute force understanding of this behavior remains an open problem:



Clay Millenium Problem: prove mass gap in Yang-Mills theory.

However, through use of:

* Large N methods parametrized by (N_c, N_f)



* Study of supersymmetric toy models, similarly parametrized by (N_c, N_f)

.....

we have come to understand a rich set of possible behaviors of non-Abelian gauge theories, including:

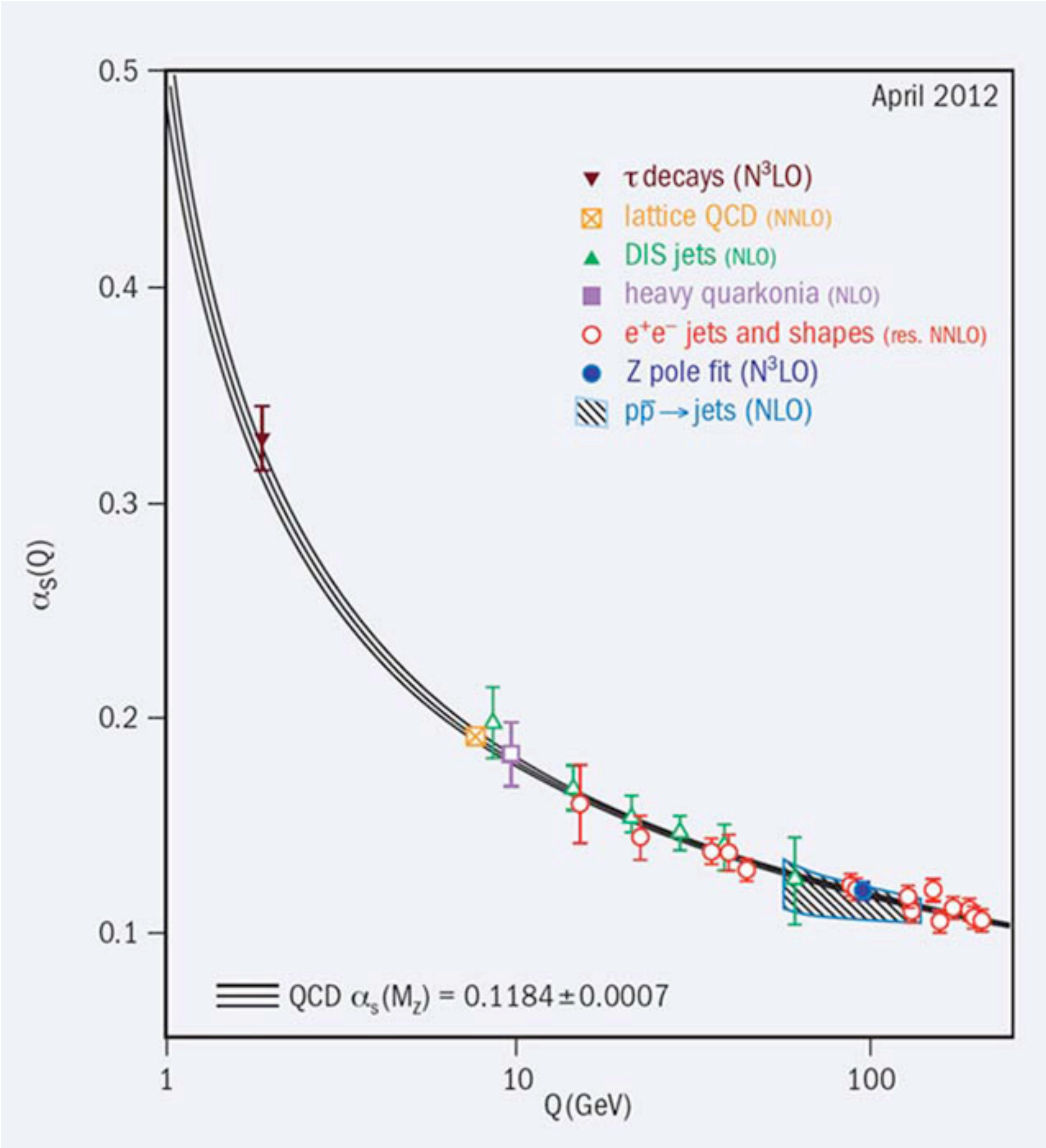
* strongly coupled theories with free magnetic duals

* confinement without chiral symmetry breaking

* Banks/Zaks and other conformal fixed points

...and of course, some phases with qualitatively similar behavior to real-world QCD.

A point which will be particularly important for me is that the study of asymptotic fixed points which are unstable but govern physics over wide energy ranges is often of interest.



e.g. the asymptotically free fixed point of QCD is unstable, but governs a wide range of energy scales in the phase diagram.

The lesson I will take from this is the following: it will be useful to find controlled approaches to non-Fermi liquid fixed points using toy models, even unrealistic toy models.

II. Basic setup and relations to other work

A. Our toy model

We will study the theory with UV action:

$$\mathcal{S} = \int d\tau \int d^d x \; \mathcal{L} = S_\psi + S_\phi + S_{\psi-\phi}$$

$$\mathcal{L}_\psi = \bar{\psi}_\sigma \left[\partial_\tau + \mu - \epsilon(i\nabla)\right] \psi_\sigma + \lambda_\psi \bar{\psi}_\sigma \bar{\psi}_{\sigma'} \psi_{\sigma'} \psi_\sigma$$

$$\mathcal{L}_\phi = m_\phi^2 \phi^2 + (\partial_\tau \phi)^2 + c^2 \left(\vec{\nabla}\phi\right)^2 + \frac{\lambda_\phi}{4!}\phi^4$$

$$S_{\psi,\phi} = \int \frac{d^{d+1}k d^{d+1}q}{(2\pi)^{2(d+1)}} g(k,q) \bar{\psi}(k) \psi(k+q) \phi(q),$$

i.e. a bosonic "order parameter field" coupled to a conventional Fermi liquid, in the tuned limit:

$$m_\phi \to 0 \ .$$

* We will do perturbative RG in the Yukawa and quartic couplings.

* We will introduce three additional control parameters:

$$\epsilon = 3 - d$$

$$N, N_f$$

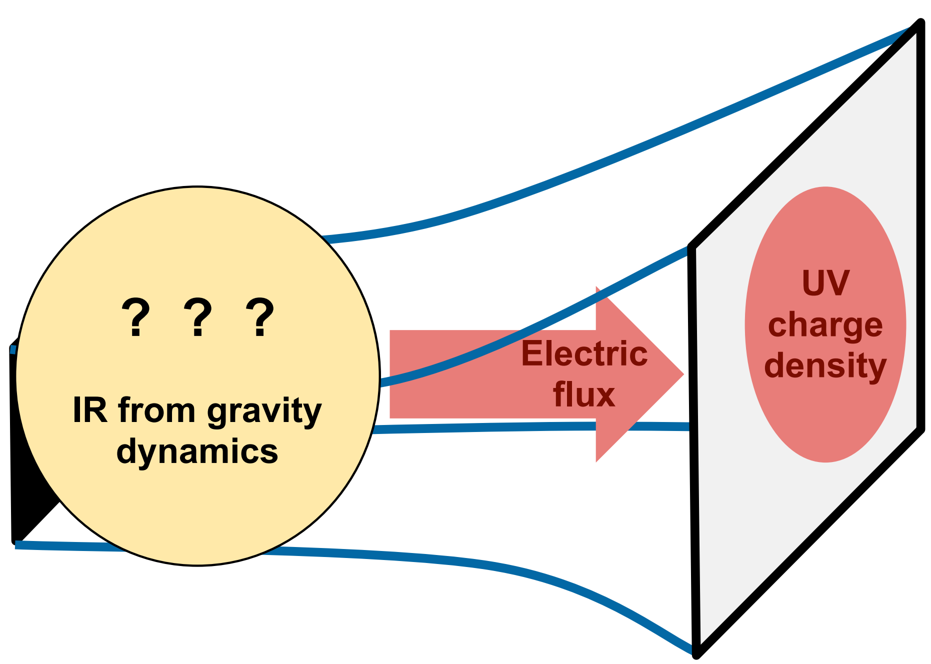
matrix boson,
"flavors" of fermions
in fundamental

B. Holographic inspirations for (bosonic) large N

Our unusual (in this context) N-ification is inspired in part by results from AdS/CFT.

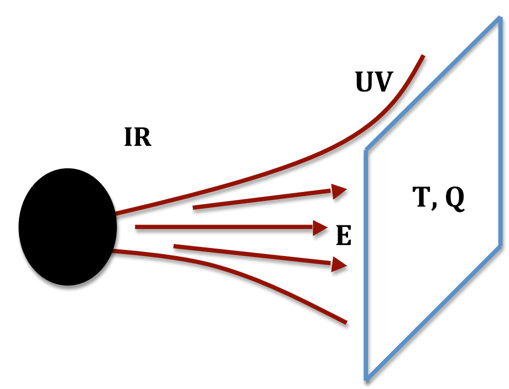
In general, in that setting, the question of finite charge density dynamics is mapped to a geometry question:

$$S = \int d^4x \sqrt{-g}\left(R - 2\Lambda + F^2 + \cdots\right)$$



The simplest theory to consider just has a Einstein gravity and a Maxwell field.

The Einstein-Maxwell theory supports charged AdS analogues of the Reissner-Nordstrom black brane:



$$ds^2 \equiv g_{MN} dx^M dx^N = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}\frac{dr^2}{f}$$

$$f = 1 + \frac{Q^2}{r^{2d-2}} - \frac{M}{r^d}, \qquad A_t = \mu\left(1 - \frac{r_0^{d-2}}{r^{d-2}}\right) .$$

The extremal limit manifests a near-horizon AdS2 geometry:

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + dx^2 + dy^2$$

From striking work of several groups, it has become clear that probe fermions in this geometry manifest non-Fermi liquid behavior.

S.S. Lee;
Liu, McGreevy, Vegh;
Cubrovic, Schalm, Zaanen; ...

This can be most simply understood by thinking in the dual quantum field theory directly.

Consider the field theory with action:

$$S = \quad S_{\text{strong}} + \sum_{J,J'} \int dt \ \left[c_J^\dagger (i\delta_{J,J'}\partial_t + \mu\delta_{J,J'} + t_{J,J'}) c_{J'} \right]$$
$$+ g \sum_J \int dt \left[c_J^\dagger \mathcal{O}_J^F + (\text{Hermitian conjugate}) \right].$$

A free fermion is coupled to a strongly coupled CFT by the lowest-dimension allowed operator, with coupling g.

We will make the assumption (justified in Einstein-Maxwell holography) that the structure of correlators in the CFT, is like that of a theory with emergent AdS2 geometry.

In perturbation theory in g, the leading effect on the fermion propagator comes from the "mixing" diagrams:

At large N, this geometric series is the exact answer.

Then, the c-fermion self-energy can be written in terms of the two-point function of $\mathcal{O}_{\mathrm{F}}$:

$$G_g(\mathbf{k},\omega) \sim \frac{1}{\omega - v|\mathbf{k}-\mathbf{k}_F(\mathbf{k})| - g^2\mathcal{G}(\mathbf{k},\omega)} \ .$$

Polchinski, Faulkner

$$\mathcal{G}(\omega) = \int dt\, e^{i\omega t} \langle \mathcal{O}_J^F(t) \mathcal{O}_J^{F\dagger}(0) \rangle \ .$$

In a "locally critical" theory, i.e. one with infinite dynamical exponent, one has:

$$\mathcal{G}(\omega) \;=\; c_\Delta \omega^{2\Delta-1}$$

This creates a non-Fermi liquid if

$$\Delta \leq 1.$$

This is evident because the simple pole (and hence the spectral weight of the quasiparticles) disappears from the fermion Greens function.

This gives a controlled derivation of NFL behavior in (very peculiar) large N theories. What about more conventional theories?

C. Our direct field theory RG

With our detour into geometry at an end, lets return to our field theory. What scaling do we do in our RG?

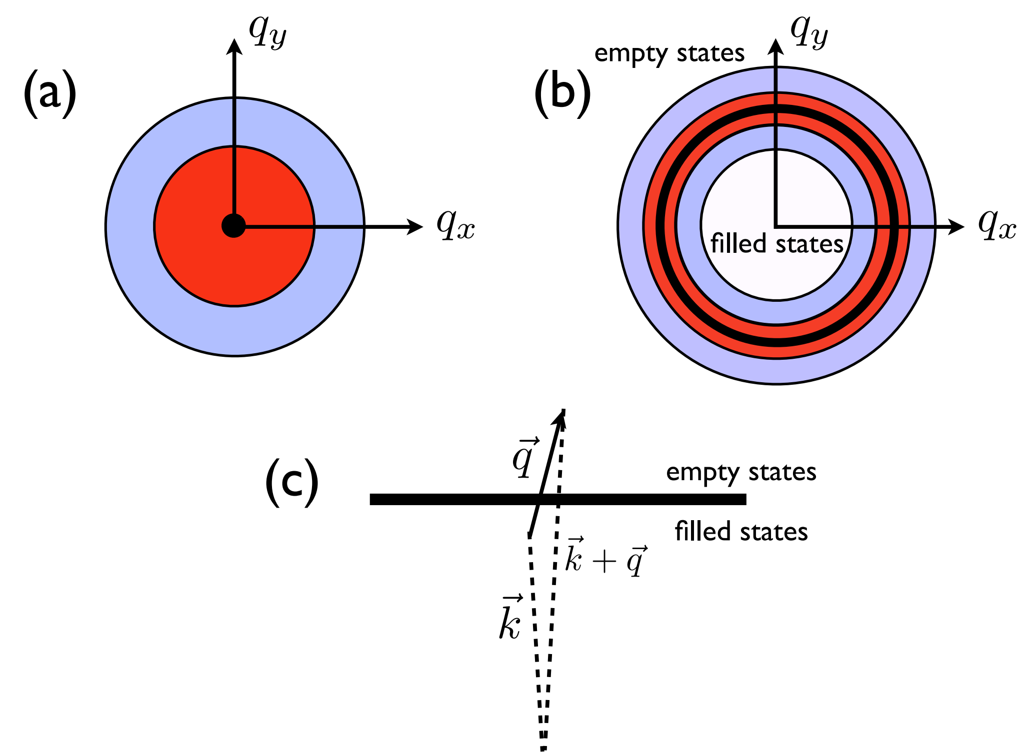


FIG. 2. Summary of tree-level scaling. High energy modes (blue) are integrated out at tree level and remaining low energy modes (red) are rescaled so as to preserve the boson and fermion kinetic terms. The boson modes (a) have the low energy locus at a point whereas the fermion modes (b) have their low energy locus on the Fermi surface. The most relevant Yukawa coupling (c) connects particle-hole states separated by small momenta near the Fermi surface; all other couplings are irrelevant under the scaling.

We expand fermion momenta about the closest point on the Fermi surface

$$\boldsymbol{k} = \hat{\Omega}(k_F + \ell).$$

and do Polchinski scaling:

$$k_0' = e^t k_0, \ \boldsymbol{k}_F' = \boldsymbol{k}_F, \ \ell' = e^t \ell$$

In contrast, we scale boson momenta isotropically:

$$k_0' = e^t k_0, \ \boldsymbol{k}' = e^t \boldsymbol{k}$$

This implies the following scaling of fields:

$$\psi' = e^{-3t/2}\psi, \;\; \phi' = e^{-\frac{(d+3)}{2}t}\phi$$

With the Polchinski scaling, we know the BCS four-Fermi interaction will remain marginal at tree level in all d. The bosonic quartic coupling scales as:

$$\lambda'_\phi = e^{(3-d)t}\lambda_\phi$$

So d=3 is the upper critical dimension.

Finally, expanding the momentum-dependent Yukawa coupling around the Fermi surface:

$$g(k,q) = g(\boldsymbol{k}_F, 0) + a_1\ell + a_2 q + \cdots$$

we see that:

$$\lim_{\boldsymbol{k}\to\boldsymbol{k}_F}\lim_{q\to 0} g'(\boldsymbol{k}',\boldsymbol{q}') = e^{\frac{3-d}{2}t} g(\boldsymbol{k},\boldsymbol{q})$$

This single term in the Yukawa coupling remains marginal in d=3; the further terms in the Taylor expansion are irrelevant.

We note that there are different (reasonable) possible choices for Fermi surface "patch" scaling. This scaling has been chosen for the following merits:

* It reverts to the obvious scalings as one decouples the boson and fermion, as perturbation theory should.

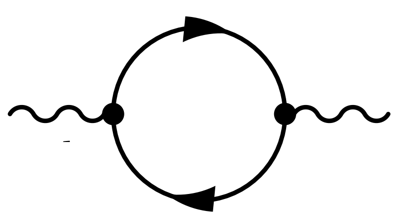
* BCS and forward scattering interactions of fermions remain obviously marginal at tree level.

One can think of the analysis to come as following the outcome of the battle between bosons and fermions:

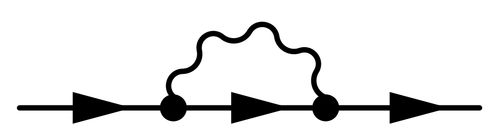
g=0: nearly free bosons, nearly free fermions decoupled from one another.

Non-zero g: non-trivial feedback between bosons and fermions.

1) bosons can decay
-> Landau damping.



2) fermions can decay
-> non-Fermi liquid behavior.



So who wins - the bosons or the fermions?

Mainstream philosophy: Hertz (1976)

This approach takes the viewpoint that damping of bosons due to fermions is the most significant effect.

Idea: integrating out all fermions results in free, overdamped bosons:

$$S_{eff} = \int_{k,\omega} \left[\omega^2 + k^2 + g^2 \frac{|\omega|}{\sqrt{k^2 + \omega^2}} \right] \phi^2$$

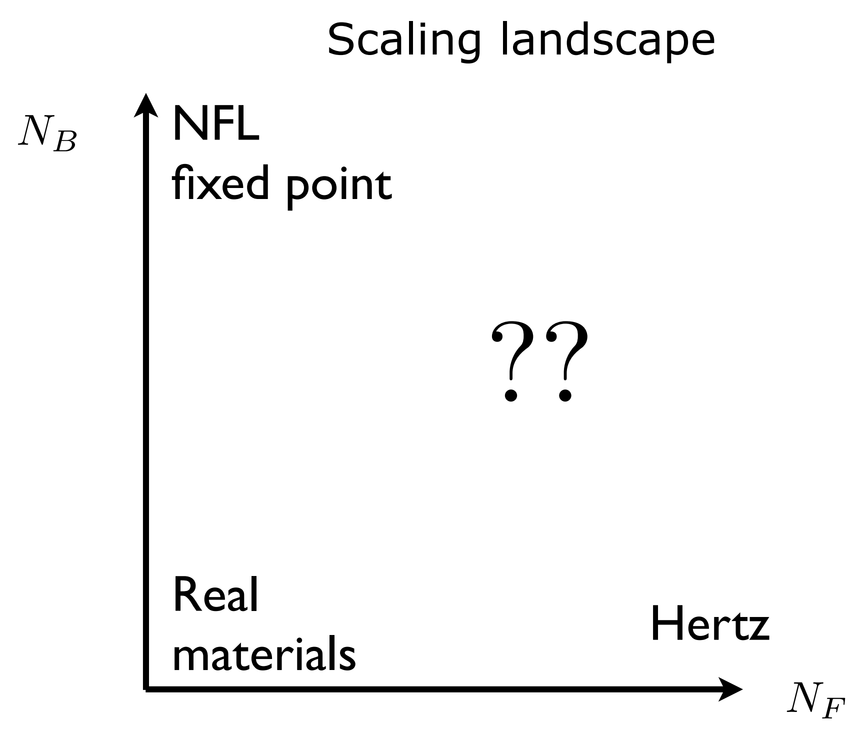
Higher order terms are ignored. They are irrelevant but singular - any approximate treatment of the theory could be dangerous!

Modern variants of Hertz's approach: feed the overdamped bosons back to the fermions to obtain a non-Fermi liquid. Look for self-consistency.

We are instead going to treat bosons and fermions on completely equal footing, and try to find simplifications in various large N or small epsilon limits.

We will find that the dominant behavior depends on the ratio of the dimensionless parameters in the problem.

This indicates that there should be a rich phase diagram in the large N theory:



III. RG in the epsilon expansion

Because we have couplings that are marginal in d=3, we can hope to find controlled fixed points in the epsilon expansion.

So, we do the RG. Our procedure will be to decimate in energy intervals, while integrating over all spatial momenta.

$$\Lambda e^{-t} < E < \Lambda$$

$$\text{external legs} : E < \Lambda e^{-t}$$

The diagrams that arise at one-loop order are:

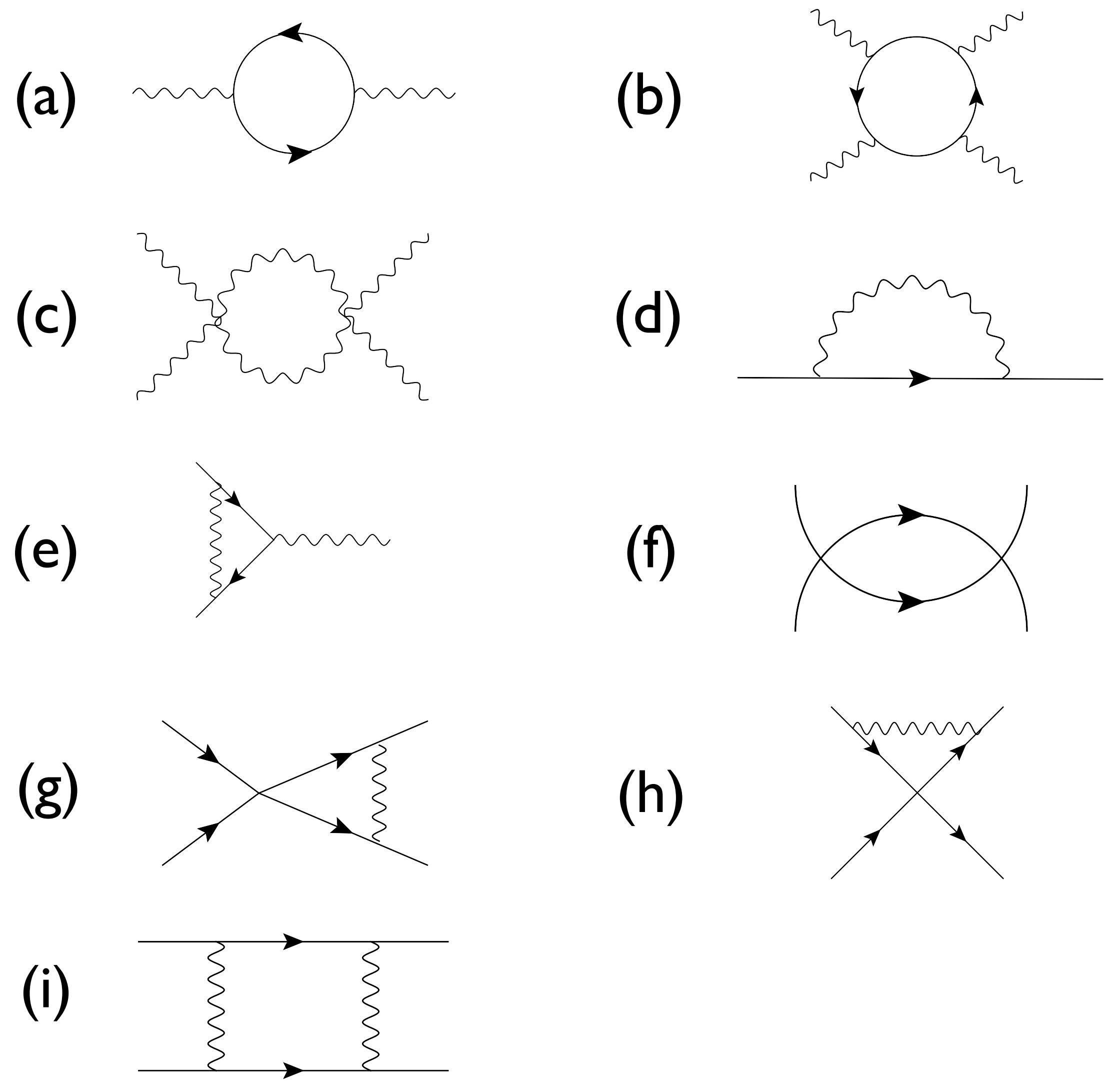
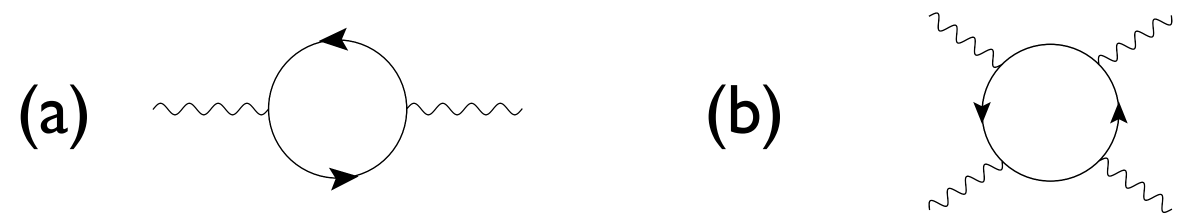


FIG. 2. One-loop diagrams. The boson self-energy (a), boson self-interactions (b,c), fermion self-energy (d), vertex correction (e) and particle-hole scattering (f). Diagrams (a) and (b) do not contribute to the renormalization group flow while (c) produces the ordinary Wilson-Fisher fixed point for bosons. Diagram (d) gives rise to fermion wave-function renormalization and (e) yields logarithmic Yukawa coupling constant renormalization. The usual marginal BCS interaction Fermi liquid theory (f) is altered by fermion wave-function renormalization as well as by diagram (g), both of which make the BCS interaction irrelevant.

An immediate and interesting result is that the diagrams (a) and (b) vanish:



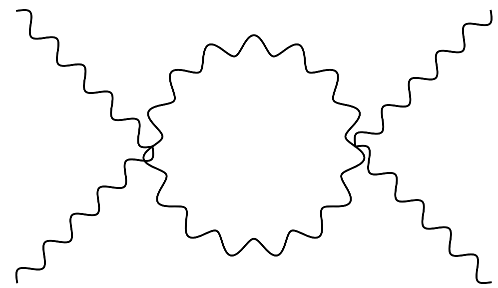
$$\Pi_{d\Lambda}(p) \propto \int_{d\Lambda} dq_0 \left[\mathrm{sgn}(q_0) - \mathrm{sgn}(q_0 + p_0)\right] = 0,$$

So the diagram which normally yields the dramatic effect of Hertz-Millis theory, does not contribute in the RG.

(Note that computing correlation functions would involve integrating down to zero energy, and can see singularities that are absent in the RG analysis.)

The diagram (c) yields the standard contribution arising in study of the Wilson-Fisher fixed point:

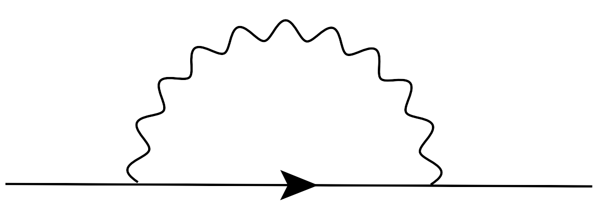
(c)



positive constant

$$\frac{d\lambda_\phi}{dt} = \epsilon\lambda_\phi - a_{\lambda_\phi}\lambda_\phi^2.$$

(d)



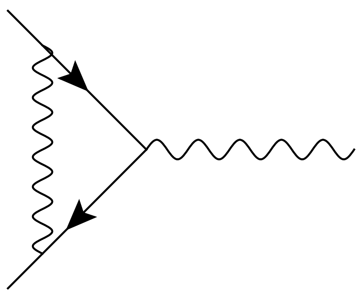
(d) gives a nontrivial fermion wavefunction renormalization:

$$\Sigma_{d\Lambda}(k) = \left(ik_0 g^2 a_g\right) d\log\Lambda,$$

another positive constant

The beta function for the Yukawa coupling arises from both the fermion wave-function renormalization, and the vertex correction. (The boson wave-function renormalization fortuitously vanishes at 1-loop).

The vertex correction is:

(e) 

$$\frac{\delta g_{d\Lambda}(k,0)}{g} = -C_3 \frac{\partial \Sigma_{d\Lambda}(k)}{\partial\,(ik_0)} = -g^2 C_3 a_g d \log \Lambda$$

The resulting flow equation is:

$$\frac{dg}{dt} = \frac{\epsilon}{2} g - (1 + C_3) g^3 a_g + \mathcal{O}(g^3 \epsilon)$$

with a fixed point: $g^2 = \frac{\epsilon}{2(1+C_3)a_g}$.

1 in basic theory
1/N at large N

Naive fixed point structure:

$$\lambda_\phi^* \sim \epsilon$$

$$g^* \sim \sqrt{\epsilon}$$

The Wilson-Fisher boson is unaffected at this order, but dresses the Fermi liquid into a non-Fermi liquid

The anomalous dimension of the fermion at this order is:

$$\gamma_\psi = \tfrac{\epsilon}{8} \ .$$

The fermion Green's function takes the form:

$$G(\omega, \ell) = \tfrac{1}{(i\omega - v_F \ell)^{1-2\gamma_\psi}} f(\omega/\ell) \ .$$

In particular, the lifetime is governed by:

$$\mathrm{Im}(\Sigma) \sim (g^*)^2 \omega^{1-\frac{\epsilon}{4}} \ .$$

Contrast with the standard approach, which gives:

$$\mathrm{Im}(\Sigma) \sim \omega^{\frac{d}{3}} = \omega^{1-\frac{\epsilon}{3}} \ .$$

ASIDE:

I did not discuss renormalisation of four-Fermi interactions here. A few subtleties arise:

* possible instabilities to superconductivity — BCS

* possible instabilities to chiral waves on the Fermi surface — DGR; Shuster, Son

Which occurs is model dependent and they happen at energies << the UV scale; we will ignore in this talk.

IV. Various large N limits

To push the analysis down to d=2, one needs to invoke large N. There are different ways to do this.

A. $N \to \infty, \;\; N_f$ fixed

In this limit, we consider the theory:

$$\mathcal{L}_{\psi} = \bar{\psi}^i \left[\partial_\tau + \mu - \epsilon(i\nabla)\right]\psi_i + \frac{\lambda_\psi}{N}\bar{\psi}^i\psi_i\bar{\psi}^j\psi_j$$

$$\mathcal{L}_{\phi} = \mathrm{tr}\left(m_\phi^2\phi^2 + (\partial_\tau\phi)^2 + c^2\left(\vec{\nabla}\phi\right)^2\right)$$

$$+\frac{\lambda_\phi^{(1)}}{8N}\mathrm{tr}(\phi^4) + \frac{\lambda_\phi^{(2)}}{8N^2}(\mathrm{tr}(\phi^2))^2$$

$$\mathcal{L}_{\psi,\phi} = \frac{g}{\sqrt{N}}\bar{\psi}^i\psi_j\phi_i^j$$

One can set:

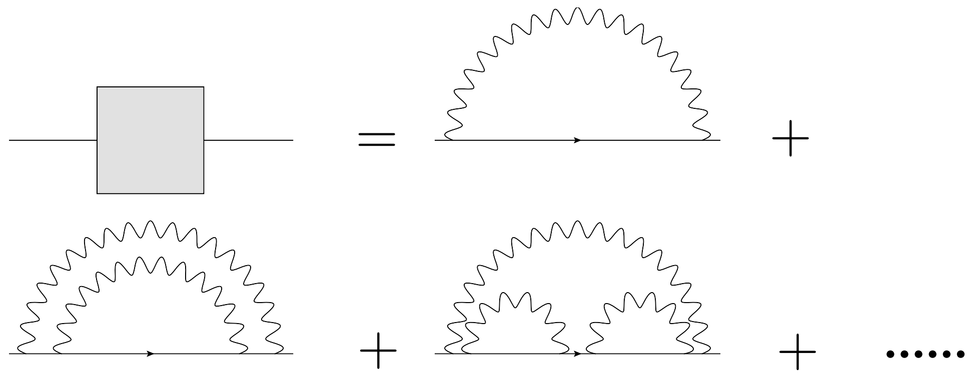
$$\lambda_\phi^{(1)} = 0$$

in a natural way (in the sense of `t Hooft). We do so.

In this theory, the only contribution to the four-Fermi beta function comes from the wavefunction renormalization of the fermion. One finds:

$$\frac{d}{dt}\lambda_\psi = -2a_g g^2 \lambda_\psi .$$

The result is a Wilson-Fisher boson which dresses a fermion into a non-Fermi liquid, stable against superconductivity.



$$G(k,\omega) = \frac{1}{(i\omega - v_F k_\perp)^{1/2}} f(\omega/k_\perp)$$

The bosons win at infinite N.

Very different from "z=3" Landau damping

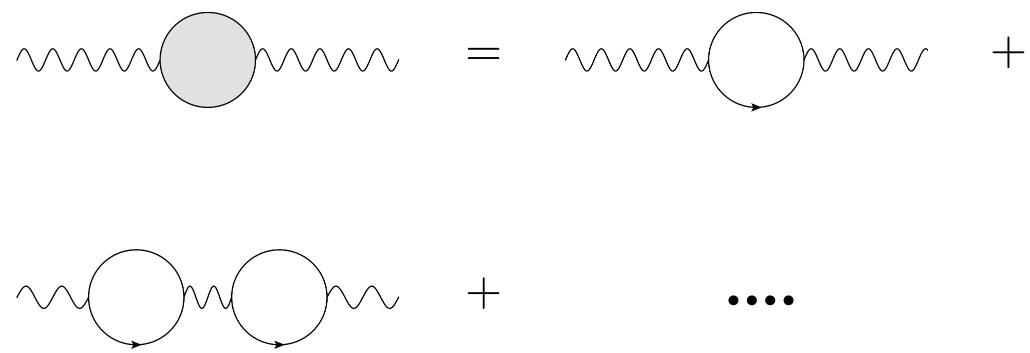
But first 1/N correction: $$\Pi(\omega) = \frac{g^2}{N}\,\omega\,\log(\omega)$$

Big effects at a low energy scale ~ $\frac{g^2}{N}$.

B. $N_f \to \infty, N$ fixed

In this limit, the resulting physics is very different.

The fermions win:



$N_F \to \infty$: Hertz's theory is exact. This limit is not scale invariant.

$$S_{eff} = \int_{k,\omega} \left[\omega^2 + k^2 + g^2 \frac{|\omega|}{\sqrt{k^2+\omega^2}} \right] \phi^2$$

First $1/N_F$ correction - leads to a different type of non-Fermi liquid:

$$c < v_F : \; \Sigma(\omega) = \frac{g^2}{N_F}\omega^{2/3} + \mathcal{O}(1/N_F^2)$$

Deep in the IR: feedback effects are highly non-trivial (Sung-Sik Lee).

Moral of the story: there are several distinct asymptotic limits with different scaling behaviors, dynamic crossovers in this problem.

It is interesting to ponder the Veneziano limit; we plan to return to this after sorting out the leading large N limits carefully.

Overall moral:

As you leave any extreme limit of small parameters, there are distinct complicated problems to face here. The result will likely be a zoo of possible behaviors...

$1/N_B$

$1/N_F$

$\epsilon = 3 - d$