Environmental Superstatistics

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Content

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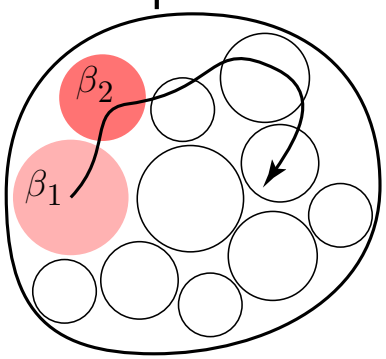
Simple example:

Consider Brownian particle moving through spatio-temporal inhomogeneous environment with temperature fluctuations on a large scale.



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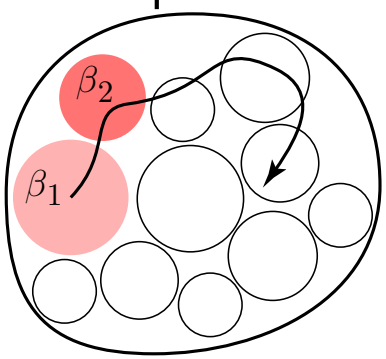

Mixture of two statistics:

- locally ordinary Brownian motion at inverse temperature $\boldsymbol{\beta_i}$
- superimposed to this is the stochastic process of inverse temperatures $\boldsymbol{\beta_i}$

Can construct dynamical realization in terms of Langevin equation

$$\dot{v} = -\gamma v + \sigma L(t)$$

with fluctuating parameters γ, σ (C.B., Phys. Rev. Lett., 2001).

Example: assume probability distribution of $\beta = \frac{\gamma}{2\sigma^2}$ in the various cells is χ^2-distribution of degree n

$$f(\beta) \sim \beta^{n/2-1} e^{-\frac{n\beta}{2\beta_0}}$$

(e.g. $\beta = \sum_{i=1}^{n} X_i^2$) and that β varies on a much larger time scale than local relaxation time.

Conditional prob. $p(v|\beta)) \sim e^{-\frac{1}{2}\beta v^2}$

Joint prob. $p(v, \beta) = f(\beta) p(v|\beta)$

Marginal prob. $p(v) = \int_0^\infty f(\beta) p(v|\beta) d\beta$

Integration yields

$$p(v) \sim \frac{1}{(1 + \frac{1}{2}\tilde{\beta}(q-1)v^2)^{1/(q-1)}}$$

(power-law generalized Boltzmann factors with $q = 1 + \frac{2}{n+1}$, $\tilde{\beta} = 2\beta_0/(3-q)$, and $E = \frac{1}{2}v^2$)

$\beta_0 = \int f(\beta)\beta d\beta =$ average of β

Very broad interpretation—β need not be inverse temperature

Can generalize the above example to general probability densities $f(\beta)$ and general Hamiltonians.

Superposition of two different statistics: that of β and that of ordinary stat. mech. Short name:

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Can further generalize to systems outside physics: Consider a relatively simple dynamics locally, depending on parameters $\boldsymbol{\beta}, \boldsymbol{\lambda}, \ldots$ (biology, finance, ...). Consider a complex environment (networks, nonequilibrium situations, ...) responsible for $\boldsymbol{\beta}, \boldsymbol{\lambda}$, ... fluctuations on a much larger scale.

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If there is sufficient time scale separation, solve the simple dynamics locally and superimpose the stochastic process of β, λ, ...

Take this as a simple model of the complex system under consideration. Often some partial analytic treatment is possible and essential features of the complex system can be understood in this way.

Effective Boltzmann factors $B(E)$ of this approach are given by

$$B(E) = \int_0^\infty f(\beta) e^{-\beta E} d\beta$$

$f(\beta)$: probability distribution of β.

Many results can be proved for general $f(\beta)$.

Some recent theoretical developments of the superstatistics concept:

- Can prove superstatistical generalization of fluctuation theorems (C.B. and E.G.D. Cohen, Physica A (2004))
- Can develop variational principle for large-energy asymptotics of general superstatistics (H. Touchette and C.B., Phys. Rev. E (2005))
- Can formally define generalized entropies for general superstatistics (Tsallis and Souza, Phys. Rev. E (2003))
- Can study various theoretical extensions of the superstatistics concept (Chavanis (2005), Vignat, Plastino (2005), Grigolini et al. (2005), Crooks (2006), Naudts (2007), Abe (2007))
- Study general symmetry group properties of superstatistics (Gell-Mann et al, PNAS 2012)
- Can consider superstatistical random matrix theory (Abul-Magd 2006-2012)
- Can apply superstatistical techniques to networks (Abe & Thurner 2005)
- Superstatistical path integrals (Jizba & Kleinert 2008-2012)

...and some more practical applications:

- Can apply superstatistical methods to analyse statistics of 3d hydrodynamic turbulence (C.B., PRL 2001, 2007, A. Reynolds, PRL 2003, C.B., Europhysics Lett. 2003, E. Bodenschatz 2004, C.B., E.G.D. Cohen and H.L. Swinney, Phys. Rev. E 2005, ...)
- Can apply it to atmospheric turbulence (wind velocity fluctuations), Rizzo & Rapisarda (2004), Kantz et al, Peinke et al., C.B. et al.)
- Can apply superstatistical methods to finance (Bouchard 2003, Ausloos 2003, Duarte Queiros 2005, Anteneodo 2008, ...2013)
- Can apply it to cosmic ray and high energy scattering statistics (C.B. 2004, 2009, Wilk 2013)
- Can apply it to hydroclimatic fluctuations (Porporato et al. 2006)
- Can apply it to train delay statistics (Briggs et al. 2007)
- Medical applications (Chen et al. 2008)
- Plenty of scope to apply it to atmospheric and climatic complex data sets

2 Typical classes of superstatistics

In experiments, one often observes 3 physically relevant universality classes (C.B., E.G.D. Cohen, H.L.Swinney, PRE 2005):

- (a) χ^2-superstatistics (= Tsallis statistics)
- (b) inverse χ^2-superstatistics
- (c) lognormal superstatistics

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Why? Consider, e.g., case (a). Assume there are many microscopic RV ξ_j, $j = 1, \ldots, J$, contributing to β in an additive way. For large J, sum $\frac{1}{\sqrt{J}} \sum_{j=1}^{J} \xi_j$ will approach a Gaussian random variable X_1 due to the Central Limit Theorem.

There can be n Gaussian random variables $X_1, \ldots, X_n$ due to various relevant degrees of freedom in the system.

β positive $\Rightarrow \beta = \sum_{i=1}^{n} X_i^2$ is χ^2-distributed with degree n,

$$f(\beta) = \frac{1}{\Gamma(\frac{n}{2})} \left(\frac{n}{2\beta_0} \right)^{n/2} \beta^{n/2-1} e^{-\frac{n\beta}{2\beta_0}} \tag{1}$$

(b) Same considerations can be applied if the 'temperature' β^{-1} rather than β itself is the sum of several squared Gaussian random variables arising out of many microscopic degrees of freedom ξ_j. Resulting $f(\beta)$ is the inverse χ^2-distribution:

$$f(\beta) = \frac{\beta_0}{\Gamma(\frac{n}{2})} \left(\frac{n\beta_0}{2}\right)^{n/2} \beta^{-n/2-2} e^{-\frac{n\beta_0}{2\beta}}. \tag{2}$$

It generates superstatistical distributions $p(E) \sim \int f(\beta) e^{-\beta E}$ that that decay as $e^{-\tilde{\beta}\sqrt{E}}$ for large E.

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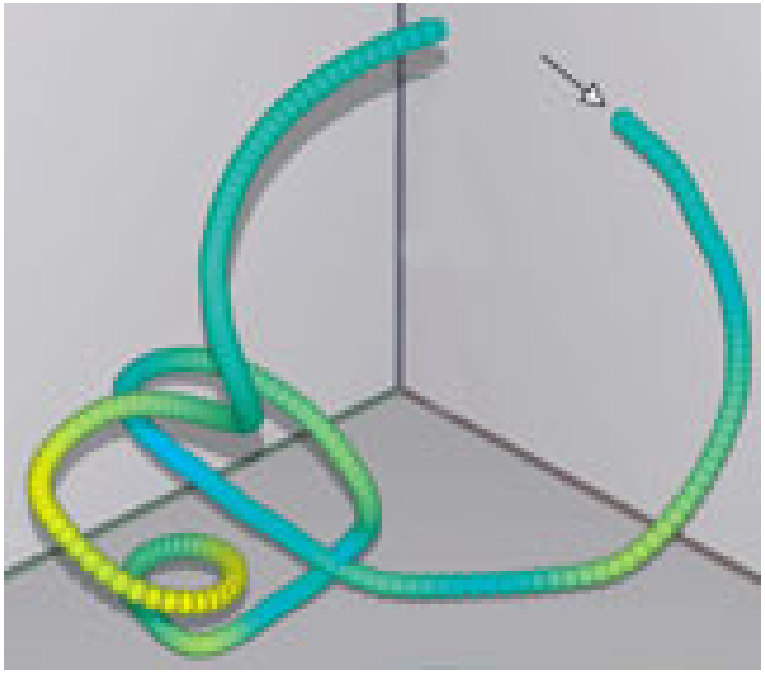
(c) β may be generated by multiplicative random processes. Local cascade random variable $X_1 = \prod_{j=1}^{J} \xi_j$, where J is the number of cascade steps and the ξ_j are positive microscopic random variables. By the Central Limit Theorem, for large J the RV $\frac{1}{\sqrt{J}} \log X_1 = \frac{1}{\sqrt{J}} \sum_{j=1}^{J} \log \xi_j$ becomes Gaussian for large J. Hence X_1 is log-normally distributed. In general there may be n such product contributions to β, i.e., $\beta = \prod_{i=1}^{n} X_i$. Then $\log \beta = \sum_{i=1}^{n} \log X_i$ is a sum of Gaussian random variables; hence it is Gaussian as well. Thus β is log-normally distributed, i.e.,

$$f(\beta) = \frac{1}{\sqrt{2\pi} s \beta} \exp\left\{\frac{-(\ln \frac{\beta}{\mu})^2}{2s^2}\right\}, \tag{3}$$

3 Application to Lagrangian fully developed turbulence

Bodenschatz et al., Nature (2001)

Measurements of acceleration $\vec{a}$ of single tracer particle in turbulent flow.



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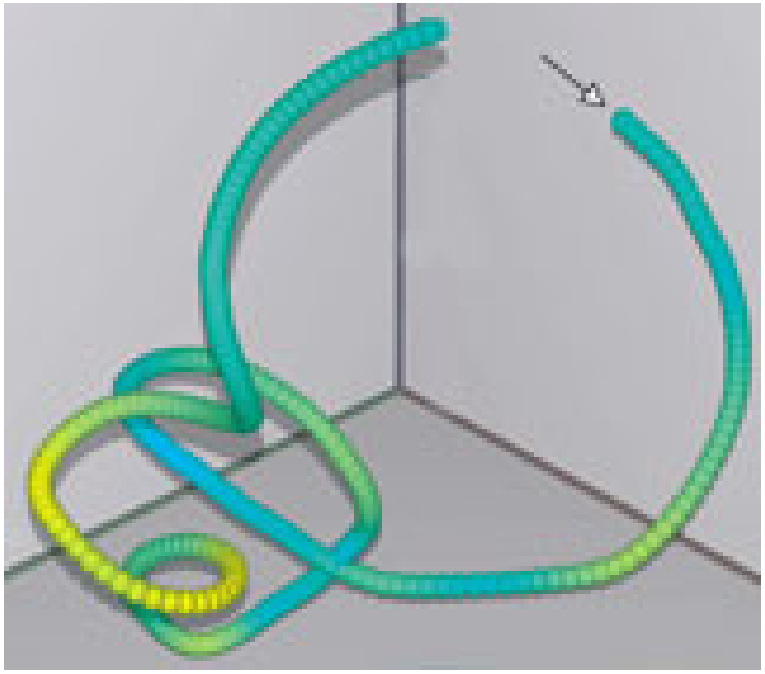
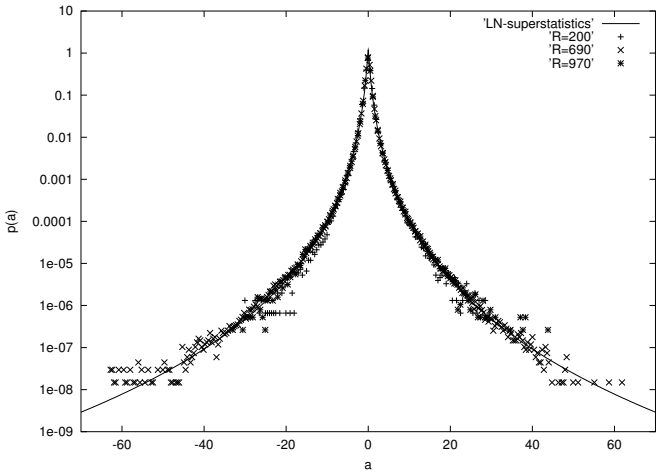


colour code:

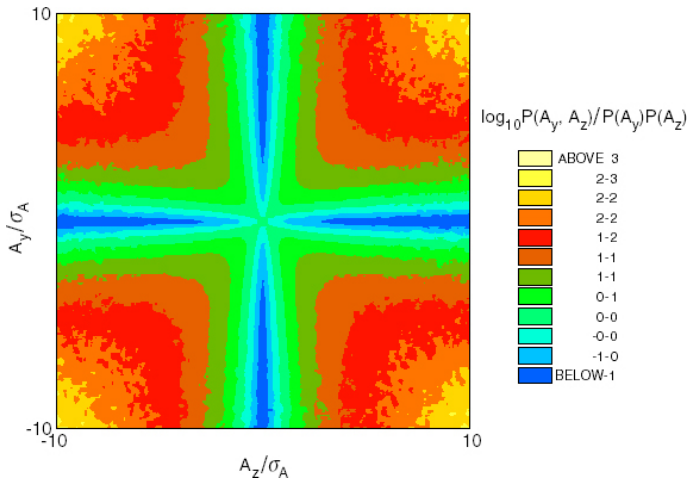
blue.....green...yellow

$|\vec{a}| = 0$.....16000 m/s^2

Experimental data one wants to understand:



histogram of accelerations (strongly non-Gaussian)



correlations of acceleration components (cross-like structure)

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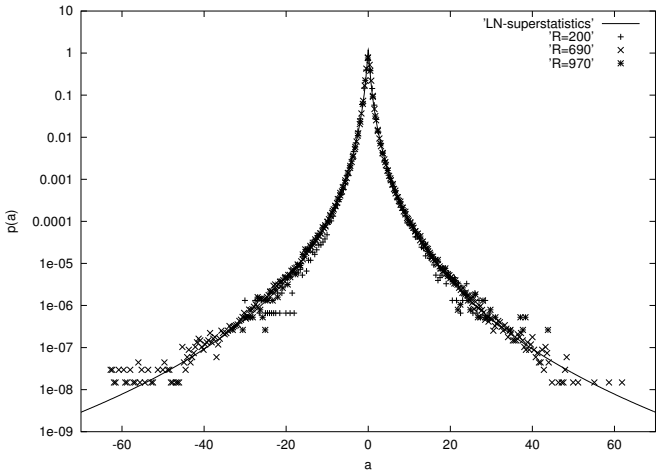

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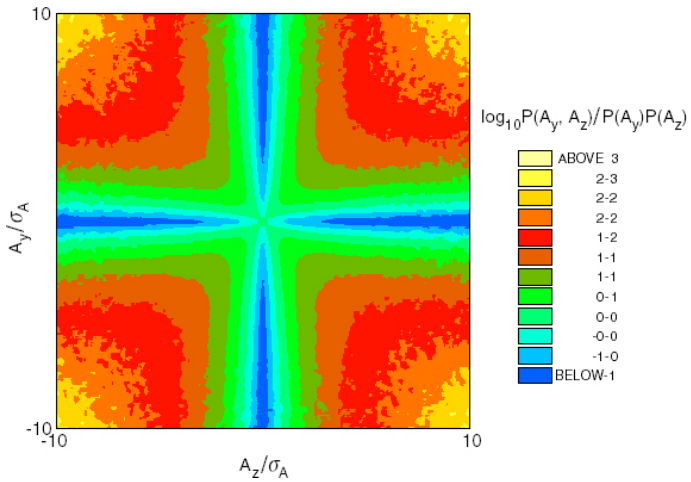

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2 possible theoretical approaches:

—give up! (system too complex!)

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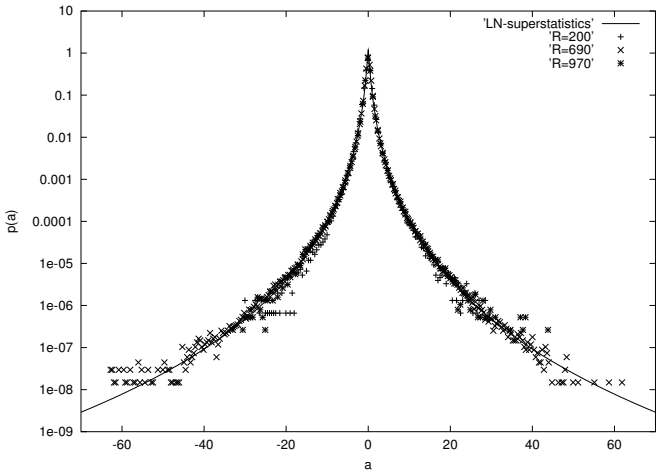

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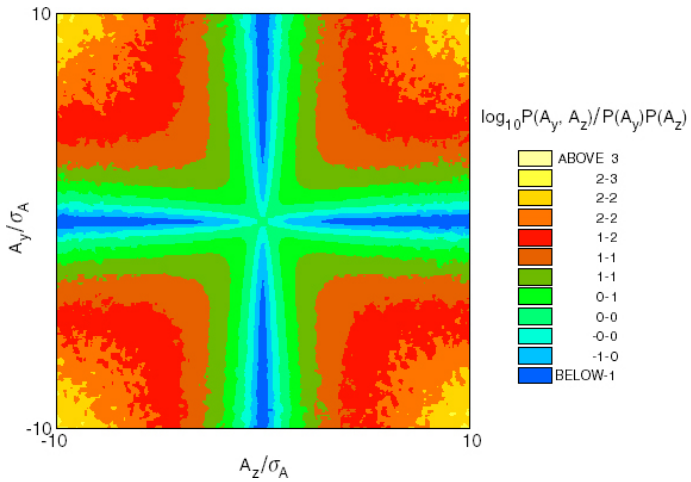

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2 possible theoretical approaches:

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—construct simple superstatistical solvable model

Superstatistical Lagrangian model for 3-dim velocity difference of tracer particle $\vec{u}(t) := \vec{v}(t+\tau) - \vec{v}(t)$. (C.B., PRL 98, 064502 (2007))

(note that $\vec{a} = \vec{u}/\tau$ for small τ)

$$\dot{\vec{u}} = -\gamma\vec{u} + B\vec{n} \times \vec{u} + \sigma\vec{L}(t). \tag{4}$$

γ and B: constants

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Noise strength σ and the unit vector $\vec{n}$ evolve stochastically on a large time scale T_σ and $T_{\vec{n}}$, respectively. ($\Longrightarrow$ superstatistics) $T_\sigma\gamma \sim T_L/\tau_\eta \sim R_\lambda >> 1$

Time scale $T_{\vec{n}} >> \tau_\eta$ describes the average life time of a region of given vorticity surrounding the test particle.

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Define $\beta := 2\gamma/\sigma^2$

$\beta^{-1} \sim \nu^{1/2}\langle\epsilon\rangle^{-1/2}\epsilon$, where ν is the kinematic viscosity and $\langle\epsilon\rangle$ the average energy dissipation.

Probability density of the stochastic process $\beta(t)$ assumed to be a lognormal distribution

$$f(\beta) = \frac{1}{\beta s\sqrt{2\pi}} \exp\left\{\frac{-(\log\frac{\beta}{m})^2}{2s^2}\right\}. \tag{5}$$

For very small τ the acceleration of the particle is given by $a_x = u_x/\tau$ and one gets the 1-point distribution

$$p(a_x) = \frac{\tau}{2\pi s} \int_0^\infty d\beta \, \beta^{-1/2} \exp\left\{ \frac{-(\log \frac{\beta}{m})^2}{2s^2} \right\} e^{-\frac{1}{2}\beta\tau^2 a_x^2}. \tag{6}$$

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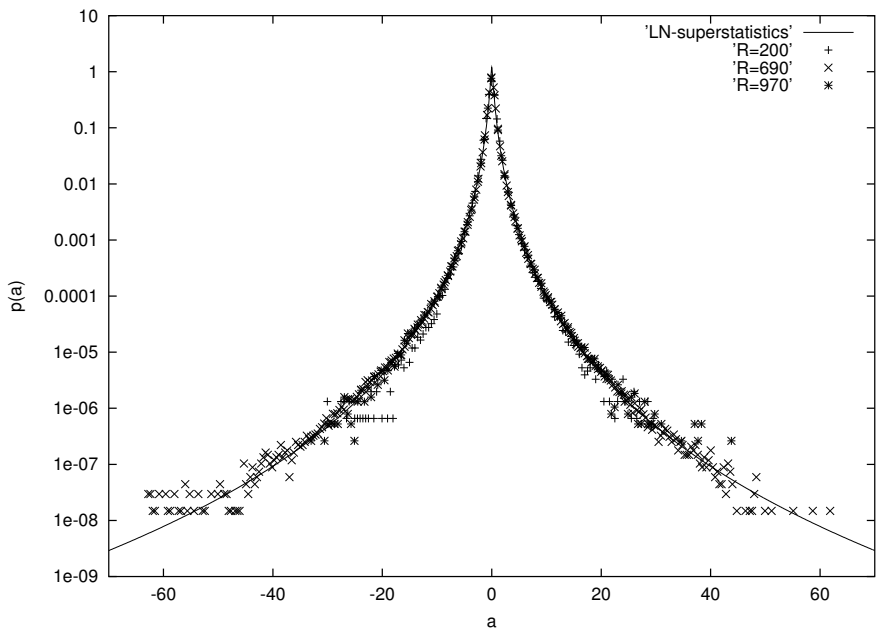
Experimentally measured probability distribution of acceleration:



strongly non-Gaussian

3-dim superstatistics induces correlations between components:
Study $R := p(a_x, a_y)/(p(a_x)p(a_y))$.
For independent acceleration components this ratio would always be given by $R = 1$.
However, our 3-dim superstatistical model yields prediction

$$R = \frac{\int_0^\infty \beta f(\beta) e^{-\frac{1}{2}\beta\tau^2(a_x^2+a_y^2)} d\beta}{\int_0^\infty \beta^{1/2} f(\beta) e^{-\frac{1}{2}\beta\tau^2 a_x^2} d\beta \int_0^\infty \beta^{1/2} f(\beta) e^{-\frac{1}{2}\beta\tau^2 a_y^2} d\beta} \tag{7}$$

General formula.

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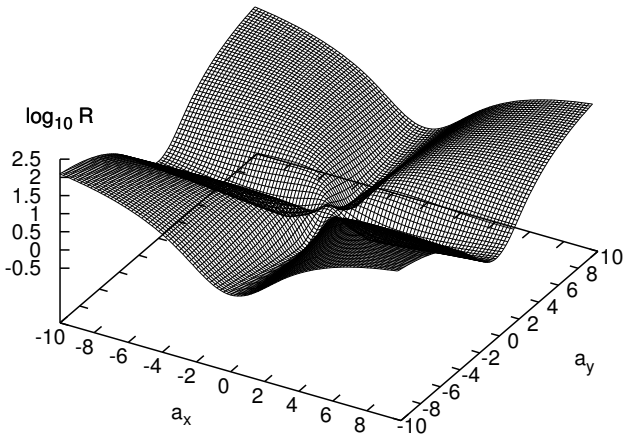
General formula.
Note that $R = 1$ for $f(\beta) = \delta(\beta - \beta_0)$

$\boldsymbol{R := p(a_x, a_y)/(p(a_x)p(a_y))}$ as predicted by lognormal superstatistics:



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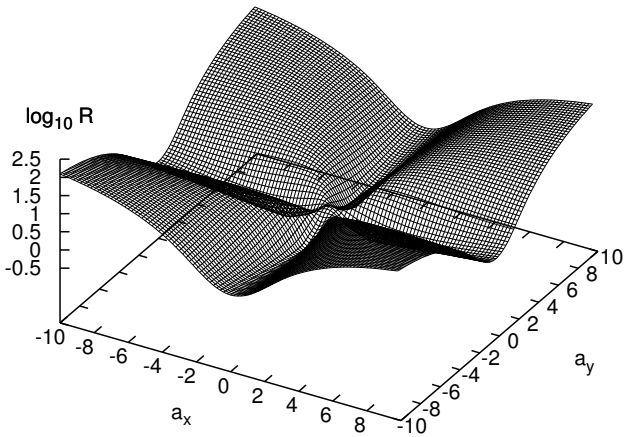
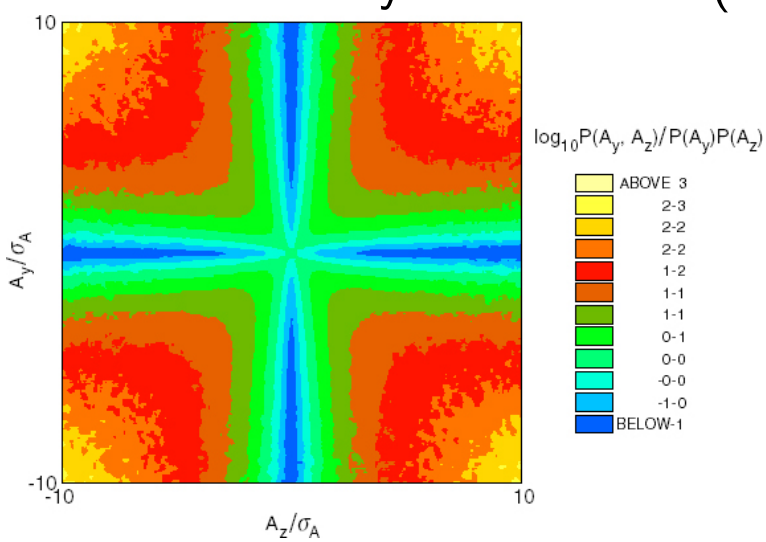


$\boldsymbol{R}$ as measured by Bodenschatz (New Journal Physics 2005)



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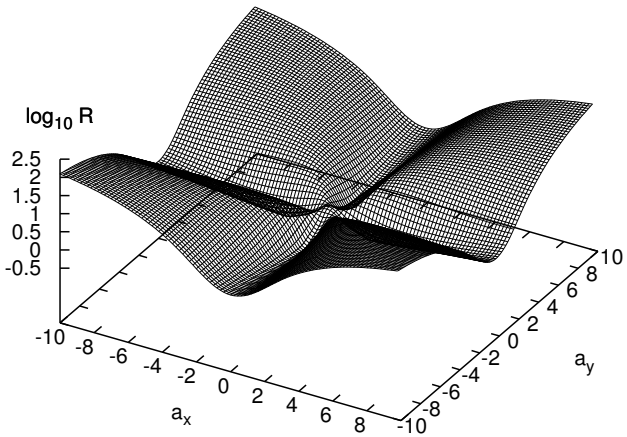

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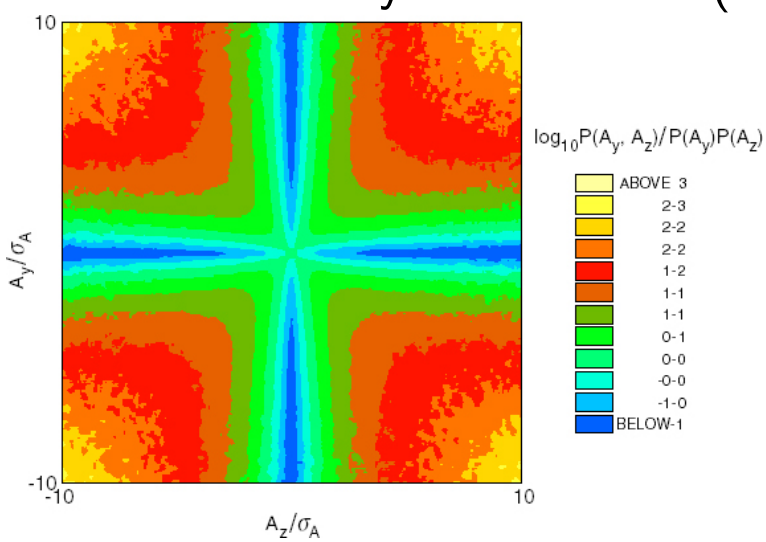
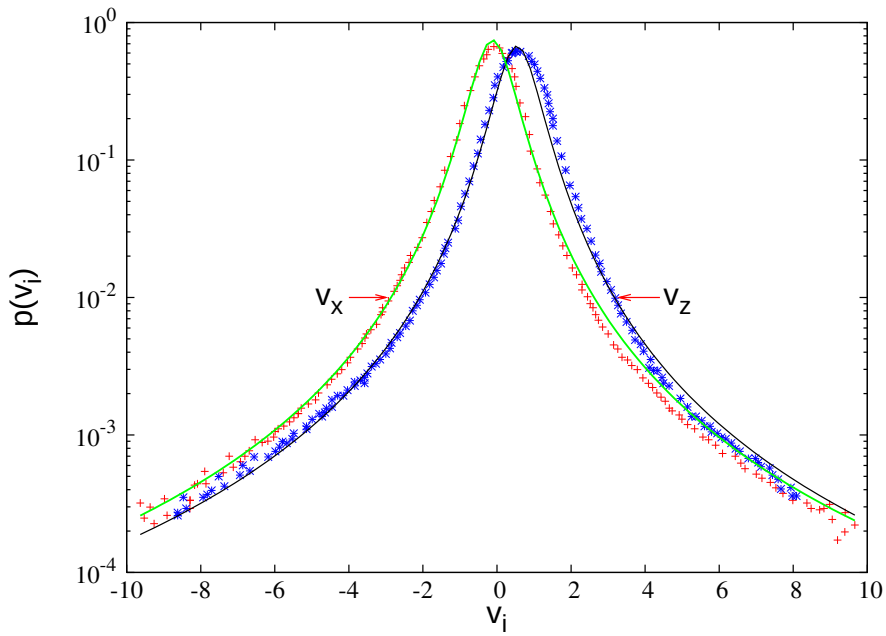
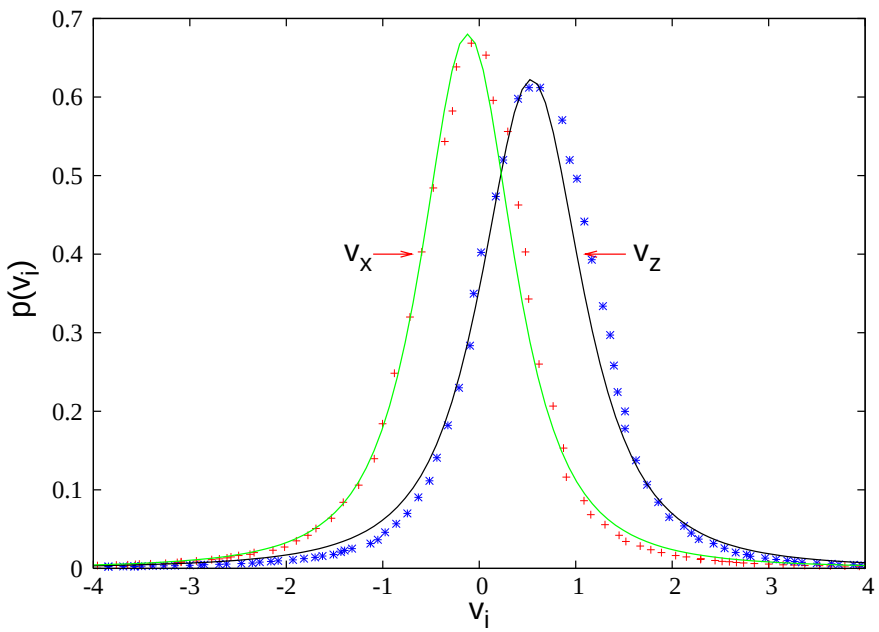


Big mystery: Why do highly nonlinear complex systems effectively generate such a simple superstatistical process?

4 Brief encounter with quantum turbulence



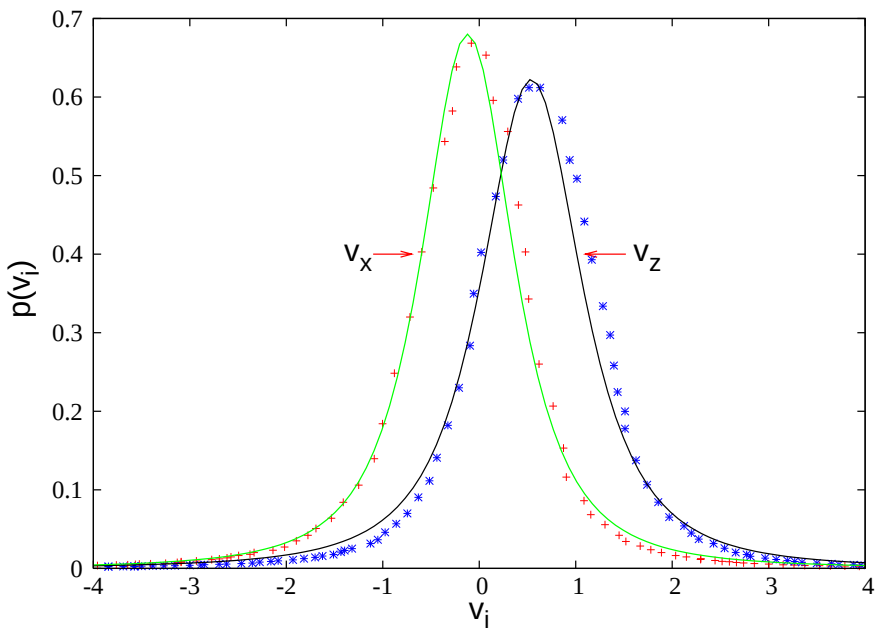
Paoletti et al (PRL 2008) observed power law tails, for velocity distribution of particle embedded in turbulent quantum liquid. Very complex problem—circulation is quantized, complex pattern of 1-dim topological vortices



Superstatistical model of quantum turbulence yields the prediction $p(v) = \frac{\sqrt{\beta_0}}{(2+\beta_0(v-c)^2)^{\frac{3}{2}}}$

Fluctuation parameter can be interpreted as vertical distance to nearest vortex filament.

C.B. and S. Miah, PRE 87, 031002(R) (2013)



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Apparently, quantum turbulence well described by chi-square superstatistics.

5 From (environmental) time series to superstatistics: Wind gusts

Treating atmospheric and climate dynamics as a kind of black box and having only a long sequence of measured data points $u(t)$ at hand, how can we check if a superstatistical model is a good approach? And how to extract the relevant superstatistical parameters out of the given time series?

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General Method (Details in C.Beck, E.G.D. Cohen, H. Swinney, PRE 72, 056133 (2005); E. Van der Straeten, C. Beck, PRE 80, 036108 (2009)):

Divide long time series $u(t)$ (average 0) into slices of length Δ. Calculate average curtosis of pieces of length Δ

$$\kappa(\Delta) = \frac{1}{N} \sum_{l=1}^{N} \kappa_{\Delta,l} \tag{8}$$

where the local kurtosis is given by

$$\kappa_{\Delta,l} = \frac{\langle u^4 \rangle_{\Delta,l}}{\langle u^2 \rangle_{\Delta,l}^2} \tag{9}$$

Long superstatistical time scale T is extracted by the condition

$$\kappa_T = 3 \tag{10}$$

Local deviations from Gaussianity on time scale T measured by

$$\theta_{T,l} := \langle u^4 \rangle_{T,l} - 3\langle u^2 \rangle_{T,l}^2 \tag{11}$$

Let

$$\epsilon := \frac{1}{3} \frac{\sum_{j=1}^{N} \theta_{T,j}}{\sum_{j=1}^{N} \langle u^2 \rangle_{T,j}^2} \tag{12}$$

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- Time series generated by complex system under consideration is superstatistical iff $|\epsilon| << 1$.
- Given T, choose $\Delta = T$ and make histogram of $\beta_{\Delta,l} = 1/\langle u^2 \rangle_{\Delta,l}$ to get $f(\beta)$.
- Original distribution given by $p(u) = \int_0^\infty d\beta f(\beta) \sqrt{\frac{\beta}{2\pi}} e^{-\frac{1}{2}\beta u^2}$

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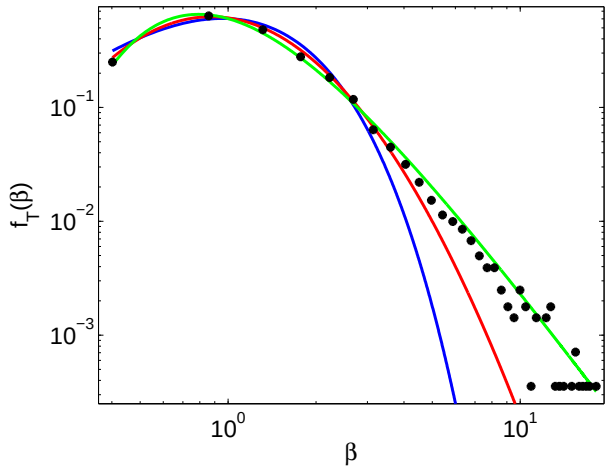
$$\theta_{T,l} := \langle u^4 \rangle_{T,l} - 3\langle u^2 \rangle_{T,l}^2 \tag{11}$$

Let

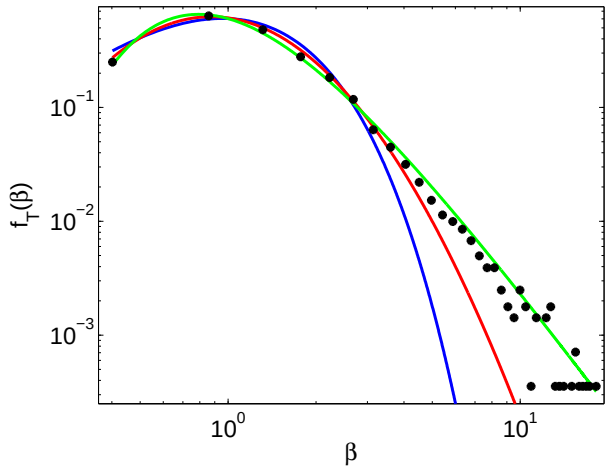
$$\epsilon := \frac{1}{3} \frac{\sum_{j=1}^{N} \theta_{T,j}}{\sum_{j=1}^{N} \langle u^2 \rangle_{T,j}^2} \tag{12}$$

- Time series generated by complex system under consideration is superstatistical iff $|\epsilon| << 1$.
- Given T, choose $\Delta = T$ and make histogram of $\beta_{\Delta,l} = 1/\langle u^2 \rangle_{\Delta,l}$ to get $f(\beta)$.
- Original distribution given by $p(u) = \int_0^\infty d\beta f(\beta) \sqrt{\frac{\beta}{2\pi}} e^{-\frac{1}{2}\beta u^2}$

Much more details, and application to surrogate data, in
E. Van der Straeten, C. Beck, PRE 80, 036108 (2009)



Extracted $f(\beta)$ from time series of wind velocity differences $v(t) - v(t+\delta)$
red: log-normal, blue: chi-square, green: inverse chi-square (Lammefjord site)



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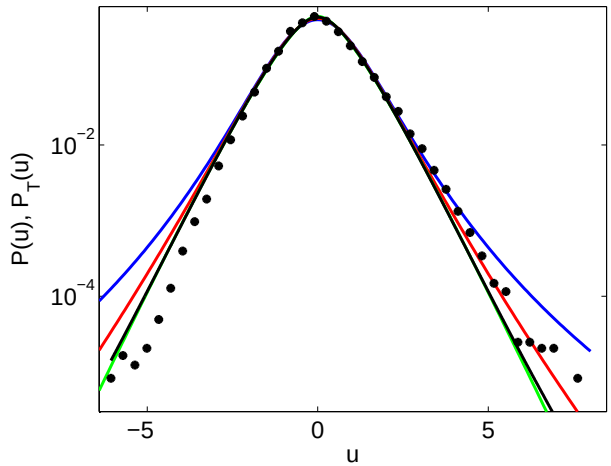
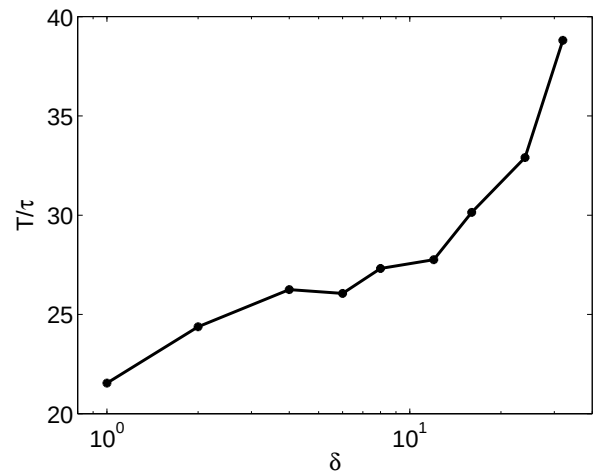
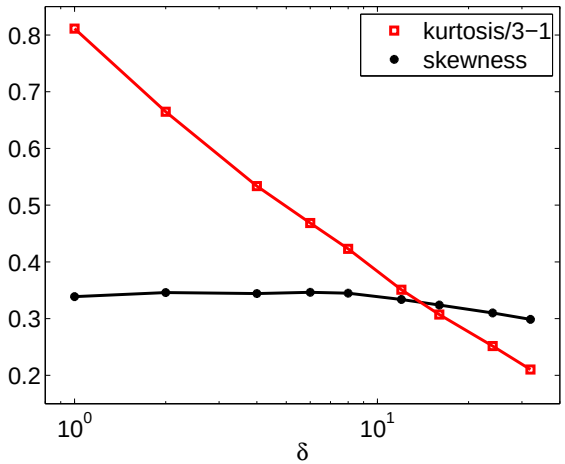
Apparently, atmospheric turbulence obeys inverse chi-square superstatistics



The original histogram



Superstatistical time scale separation: Ratio of long to short time scale



Kurtosis and skewness of the data

6 Sealevel fluctuations

Very recently, as part of the EPSRC FLOOD MEMORY grant (joint project of Newcastle, Queen Mary University of London, Edinburgh, Aberdeen, Swansea, Univ. West England, Cranfield, Nottingham, Southampton, Univ West England, NOC) we looked at sealevel fluctuations.

This is joint work with Pau Rabassa.

There is a deterministic (quasiperiodic) part of the time series (astronomical tide) plus fluctuations.

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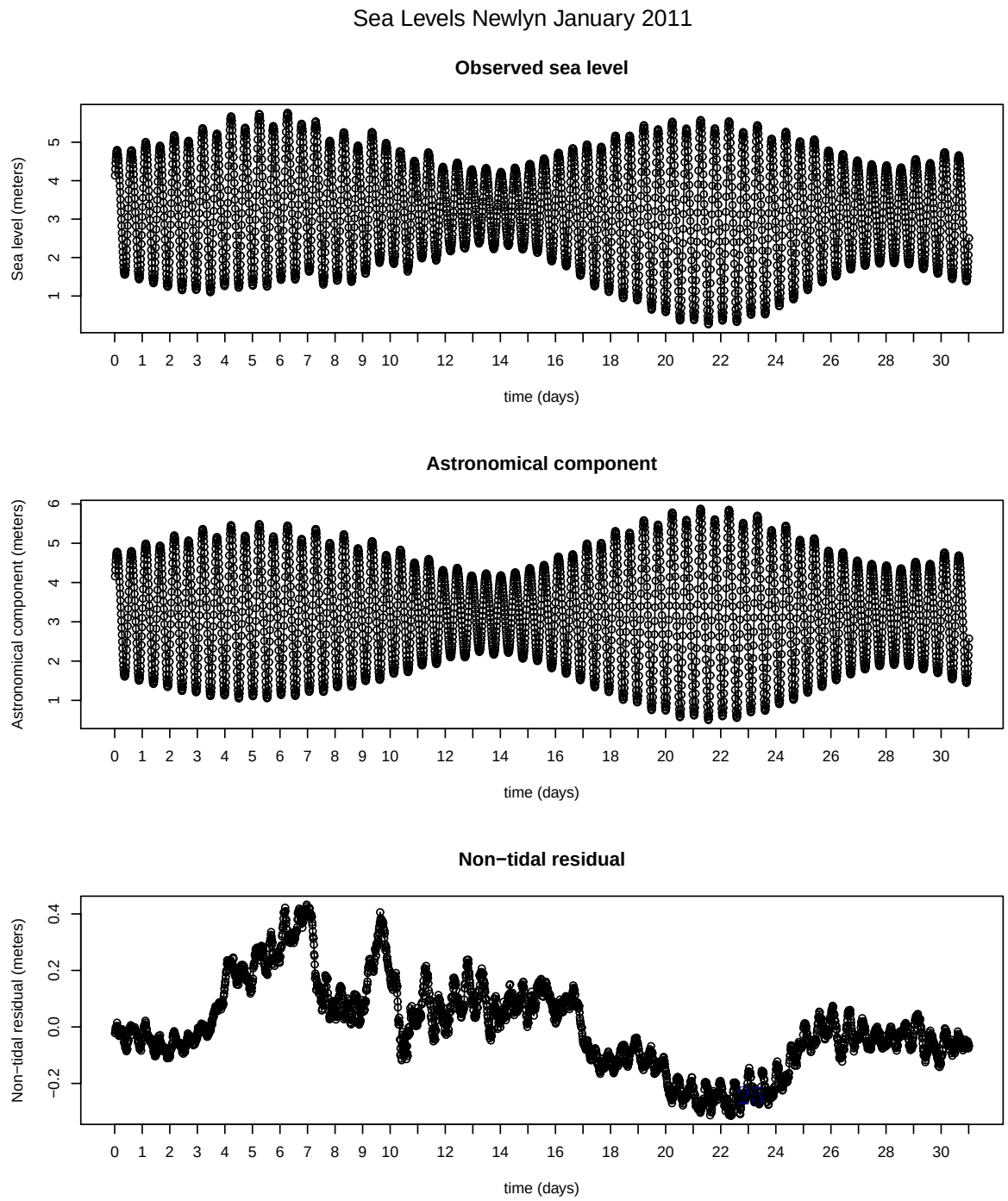
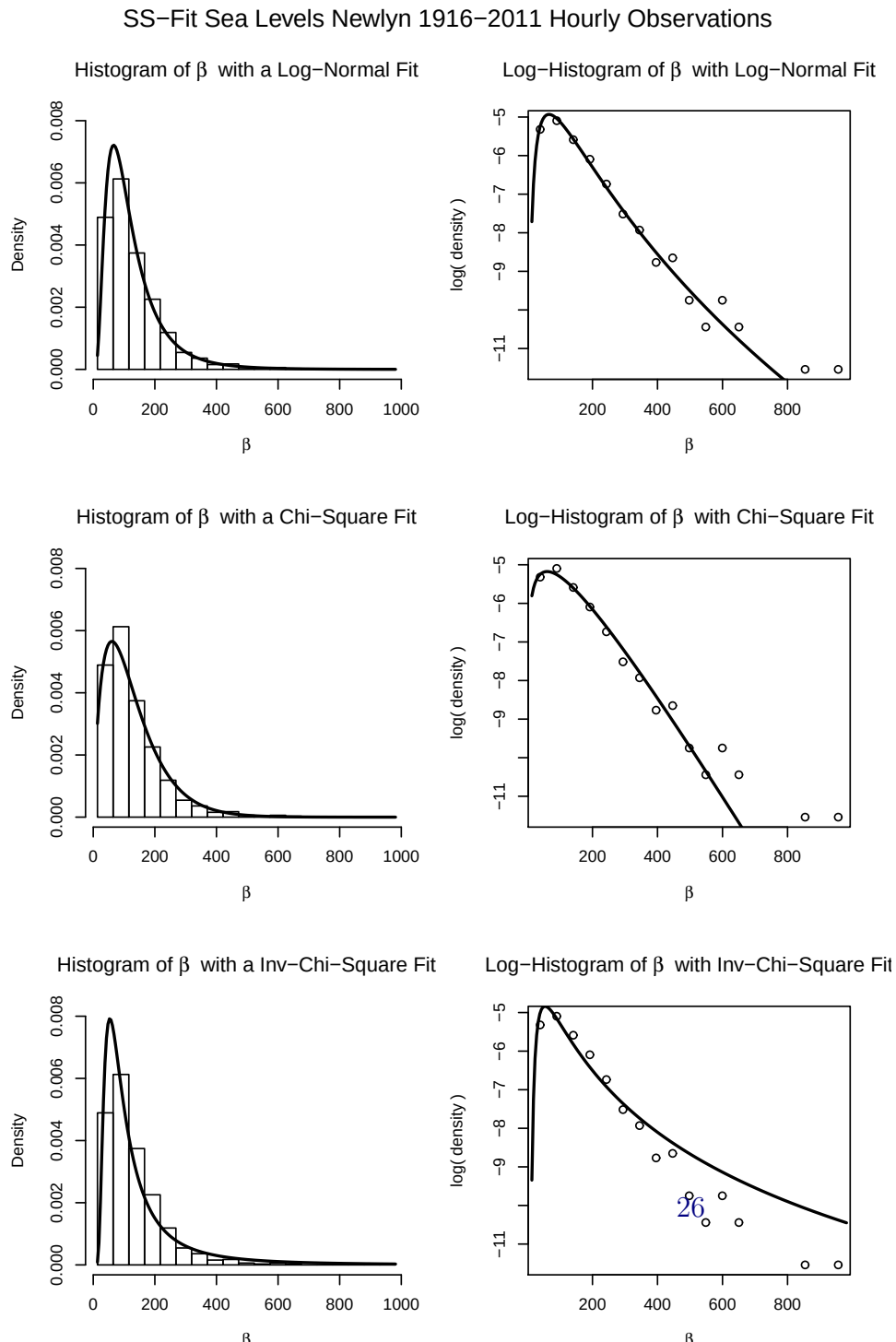
Note that extreme event statistics (the GEV distribution) is very sensitive to the precise type of superstatistics: E.g. chi-square superstatistics generates power law distributions, lognormal does not.


Sea Levels Newlyn January 2011
Observed sea level
Sea level (meters)
time (days)
Astronomical component
Astronomical component (meters)
time (days)
Non−tidal residual
Non−tidal residual (meters)
time (days)


SS−Fit Sea Levels Newlyn 1916−2011 Hourly Observations
Histogram of β with a Log−Normal Fit
Log−Histogram of β with Log−Normal Fit
Histogram of β with a Chi−Square Fit
Log−Histogram of β with Chi−Square Fit
Histogram of β with a Inv−Chi−Square Fit
Log−Histogram of β with Inv−Chi−Square Fit
Density
log(density)
β

Another interesting thing to look at: **Surface temperature statistics**

Interesting to just look at distributions of inverse temperature $f(\beta)$ as observed at various locations on Planet Earth—- superstatistics with these $f(\beta)$ relevant for thermodynamic devices that are supposed to work in open air.

Did that for Darwin, Santa Fe, Dubai, Sydney, London, Vancouver, Hong Kong, Ottawa, Eureka, ...

C. Yalcin, C.B., Physica A 392, 5431 (2013)

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The original superstatistics concept was for sharply peaked distributions $f(\beta)$ with a single maximum (C.B., E.G.D. Cohen, 2003). But in an environmental context this concept needs to broadened: Typically observed distributions at various locations on Planet Earth are double-peaked (due to seasonal variations), not single-peaked.

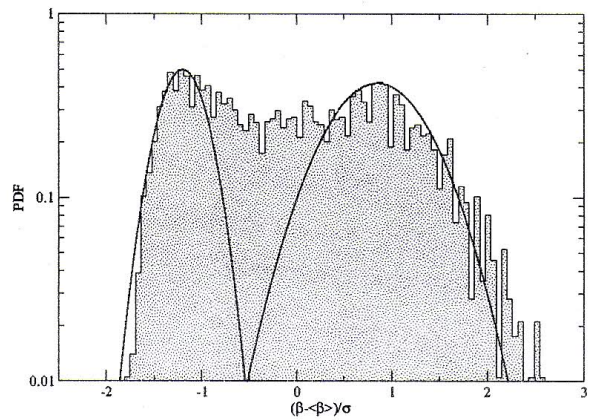

FIG. 23: Distribution of the daily measured inverse mean temperature in Santa Fe for 1998-2011

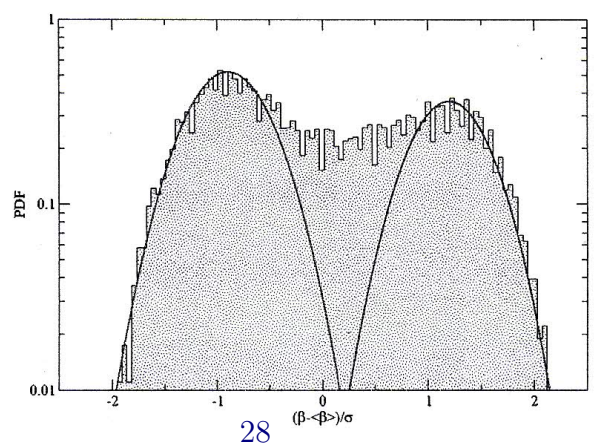

FIG. 24: Distribution of the daily measured inverse mean temperature in Dubai for 1974-2011

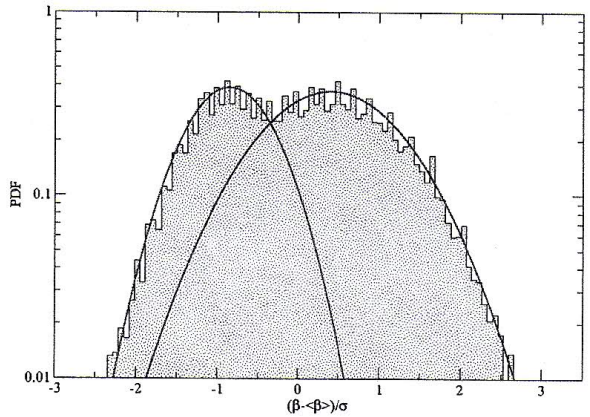

FIG. 26: Distribution of the daily measured inverse mean temperature in Central England (Lancashire, London and Bristol) for 1910-2011

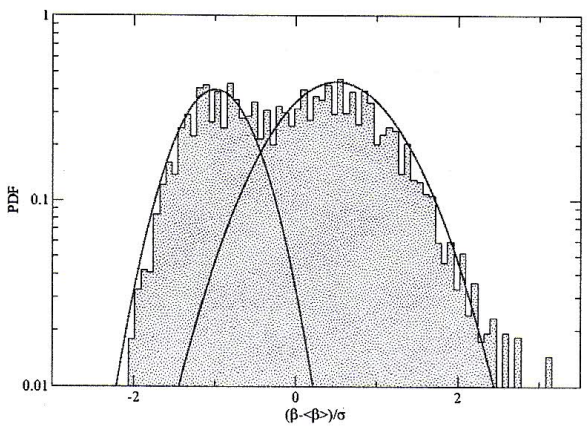

FIG. 27: Distribution of the daily measured inverse mean temperature in Vancouver for 1937-2011

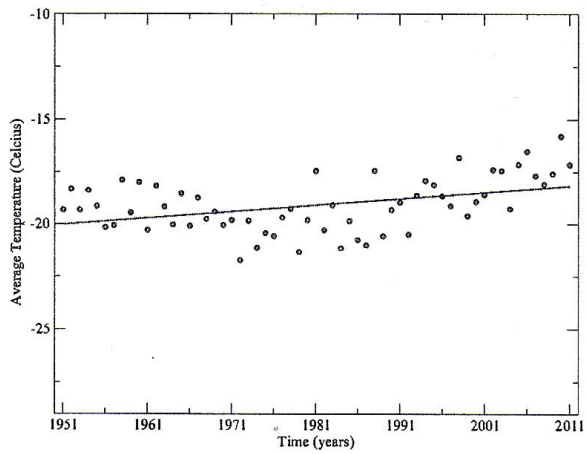

FIG. 32: Average of daily measured mean temperature of every single year in Eureka 1951-2011

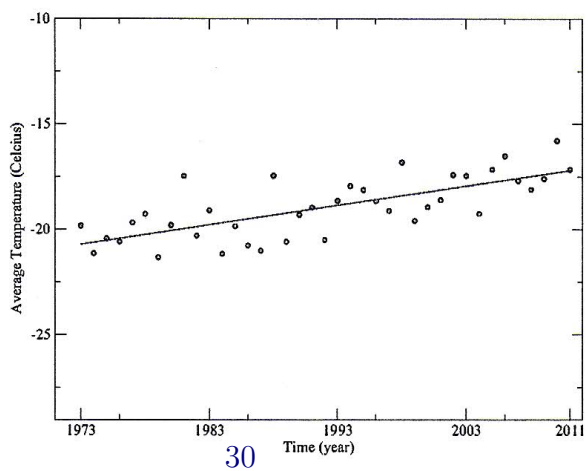

FIG. 33: Average of daily measured mean temperature of every single year in Eureka 1973-2011

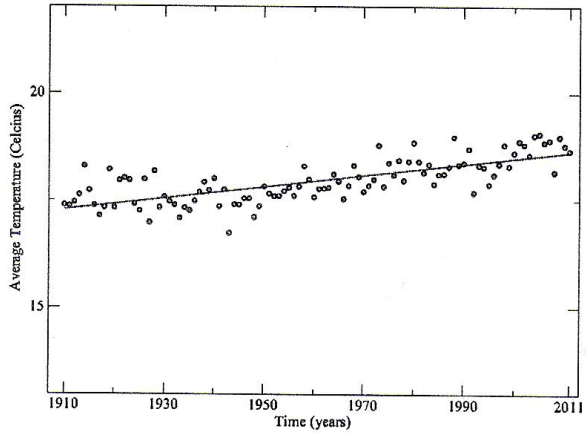

FIG. 35: Average of daily measured mean temperature of every single year in Sydney 1910-2011

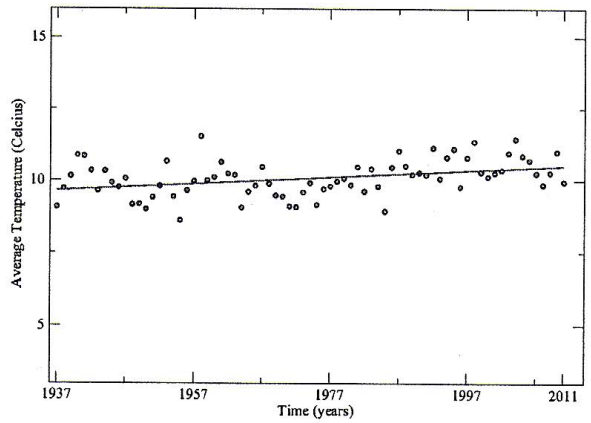

FIG. 36: Average of daily measured mean temperature of every single year in Vancouver 1937-2011

Summary:

- Superstatistical techniques can generally be applied to a variety of complex systems if there is sufficient time scale separation.
- There is theoretical evidence for three major universality classes: χ^2-superstatistics, inverse χ^2-superstatistics, and lognormal superstatistics. These arise as universal limit statistics for many different systems.
- A superstatistical model of Lagrangian fully developed 3d turbulence (C.B., PRL 2007) is in excellent agreement with the experimental data for probability densities, correlations between components, decay of correlations, Lagrangian scaling exponents, ... (lognormal superstatistics).
- Quantum turbulence described by χ^2 superstatistics (C.B., S. Miah, PRE 87, 031002(R) (2013))
- Atmospheric turbulence (wind velocity fluctuations) well-described by inverse χ^2 superstatistics
- Preliminary investigations show that sealevel fluctuations are well described by a superstatistical model that is close to lognormal superstatistics.