Control of interbank contagion under partial information

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Overview

Our objective is to propose a tractable framework for the **control** of systemic risk in stochastic financial systems and to use this framework to derive insights into the relation between connectivity and control.
We focus on

- **Mathematical modeling:** setting optimization problems of a lender of last resort who decides to make interventions in a core-periphery network prone to contagion risk;
- **Methodology:** Proving a state-space collapse theorem that allows for tractability. The main idea is that contagion can be (under certain assumptions) tracked using a lower dimension Markov chain.

Contents

I. General model description

I.1 The system before the shock

Stylized hierarchical financial network

Two classes of banks:

- c **core** banks ; each holding y_c in external projects
- p **peripheral** banks ; each holding y_p in external projects

$$\text{Total number of banks : } n = c + p.$$

and

- 1 external creditor (formally represented as node 0).

Financial network: intermediates credit lines from the external creditor to peripheral, through core, thus leading to new investment in additional projects by peripheral banks.

$\sharp$ credit lines from external creditor to core $= \sharp$ extended by core to peripheral.

Credit intermediation by core banks

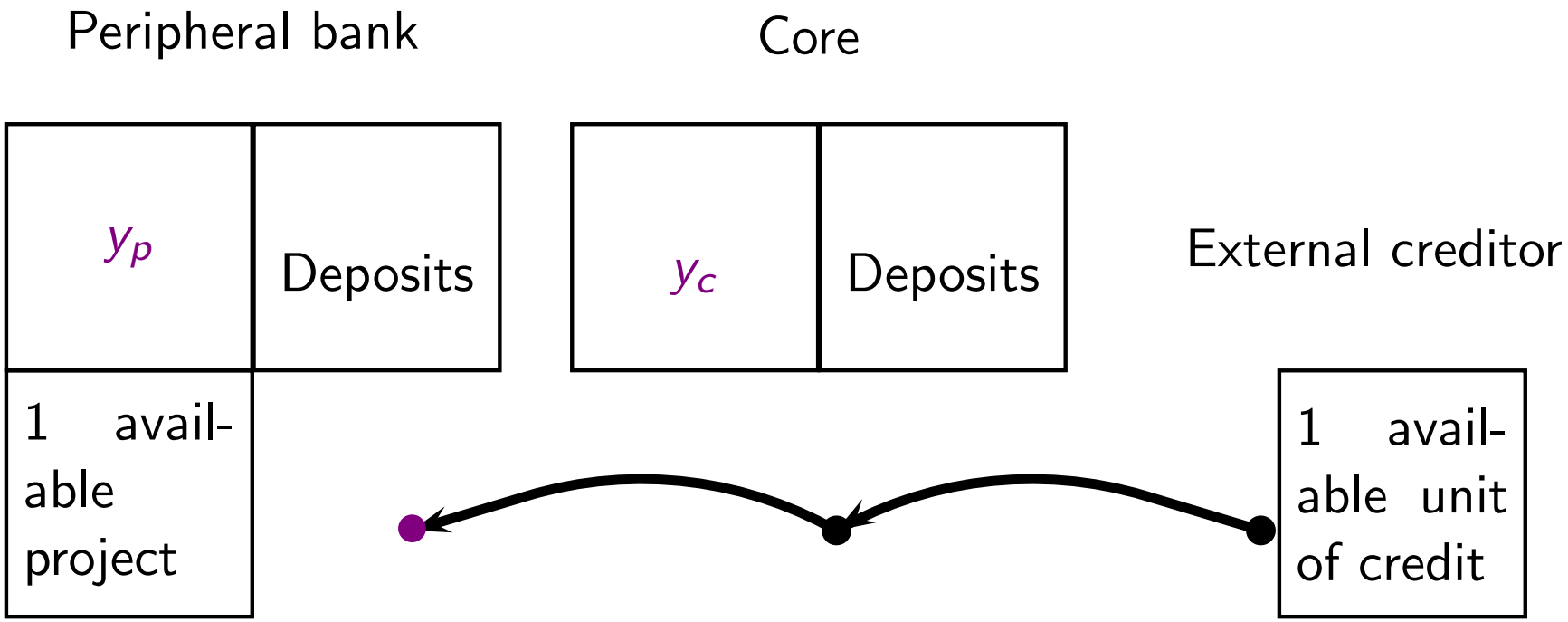


Figure: one unit of credit from external creditor is intermediated by a core bank to a peripheral bank, that invests it into one unit of additional project. (Particular case of one core and one peripheral)

Network model

- Core banks have connectivity λ, i.e. they have
  - λ borrowers (core or peripheral banks),
  - λ lenders (core banks or external creditor).
- Peripheral banks have at most connectivity 1 and can only borrow from core banks.

We represent the financial system as a directed unweighted multi-network $([n] \cup \{0\}, \mathcal{E})$, where

- $[n] := \{1, \ldots, n\}$ represents the set of banks ($n = c + p$)
- $\{0\}$ represents the external creditor
- $\mathcal{E}$ the set of links (i, j), where i is the lender and j is the borrower.

Links are unweighted, that is, there is a standardized value of a loan normalized to 1 (numéraire).

Value of the financial system

Total amount of external projects

$$\bar{J} := cy_c + py_p + N \times 1$$

- The two first terms are independent of the network since they are projects funded by each bank through their own deposits.
- N is the number of additional projects extended by peripheral banks using credit from core banks

$$N := \#\{(i,j) \in \mathcal{E}, i \in [c], j \in [p]\}$$

  and 1 corresponds to a unit of project.

As connectivity increases in the network, N may increase but contagion risk also increases. When a core fails, it withdraws its credit lines and liquidates its projects (y_c units). When a peripheral fails, it liquidates its projects (y_p units).

Direction of credit lines and contagion

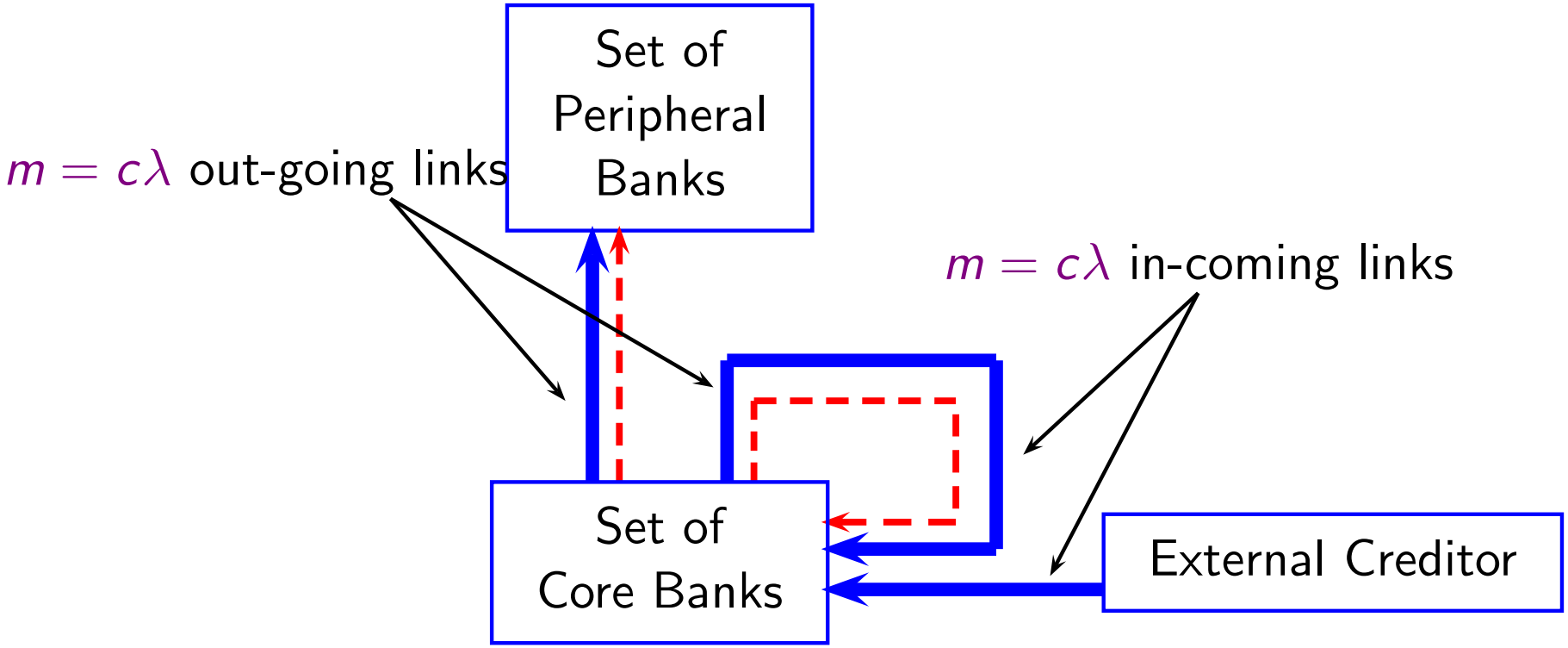


Figure: More connectivity of core banks amounts to more credit lines towards peripheral banks but also more cycles among core banks. Contagion happens in the direction of credit lines: when a bank fails it withdraws all credit lines from its borrowers.

Probability setup

We fix a probability space $(G_{n+1,m}, \mathbb{P})$, where $G_{n+1,m}$ denotes the set of networks with $n+1$ nodes and at most m links.

While the connectivity is fixed, the counterparties (lenders and borrowers) of the core banks are randomly chosen.

Note that it is sufficient to establish the counterparties of the core banks, since the peripheral banks and the creditor can only connect to core banks.

Under the probability measure $\mathbb{P}$, the law of $\mathcal{E}$ is given as follows.

- Each core bank is assigned λ in-coming half links and λ out-going half links,
  corresponding to the number of lenders and borrowers.
- Each peripheral bank is assigned one in-coming half-link.

In total we thus have
- $m := c\lambda$ out-going half links (offers of a credit line)
- $m + p$ in-coming half links (candidates to borrow one credit line).

We draw the borrowers of core banks:
We draw uniformly and independently, without replacement, m in-coming half-links out of the total $m + p$ in-coming half-links.
We match the m out-going half-links with the drawn m in-coming links.
When an out-going half-link is matched with an in-coming half-link, a link is established.

Only when the chosen candidate is a peripheral bank, a new unit of project is created. When it is a core bank, then this does not correspond to a new unit of project, but to an additional intermediary.

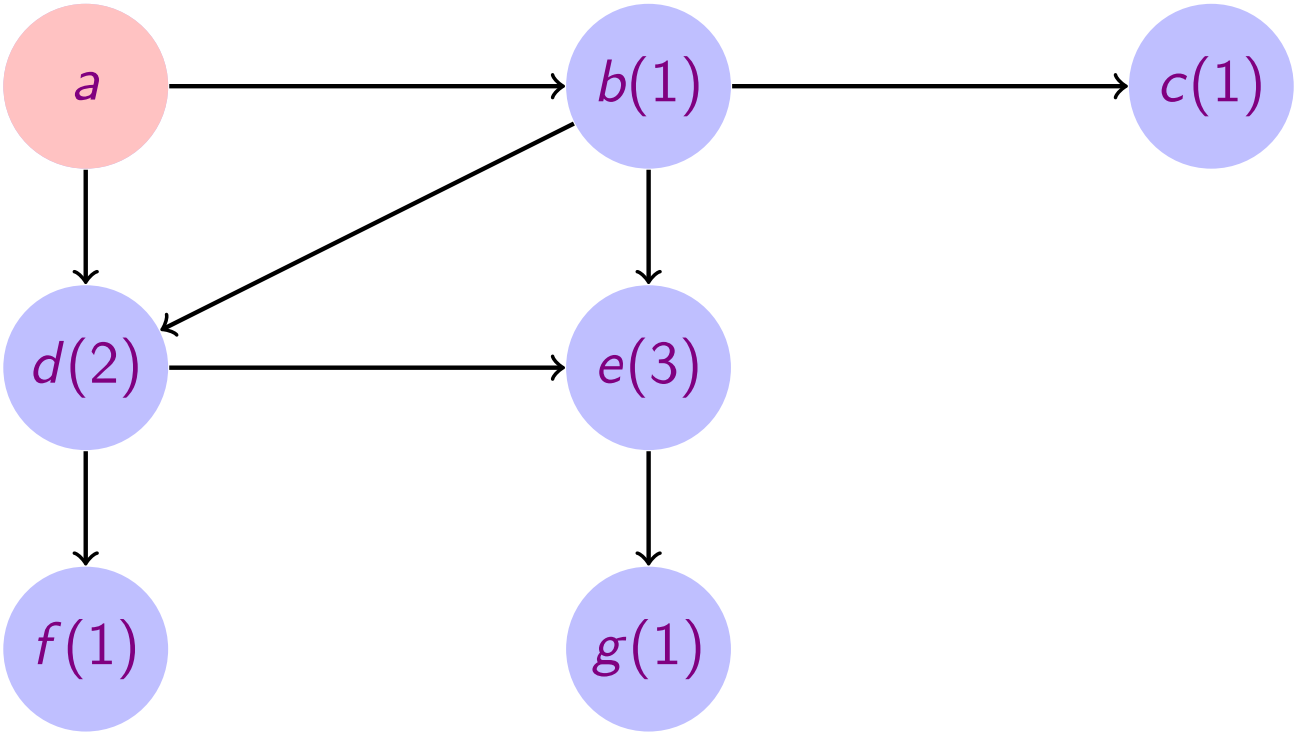
I.2 Contagion and Information

Distance to failure

Each bank is endowed with a random variable $\theta_0(i), i \in [n]$, called "*distance to failure*", which represents how many credit lines can be withdrawn from bank i, before bank i fails.
It takes values in $\{0, \ldots, \vartheta_{max}\}$, where distance 0 marks a failed bank.

We assume that when a bank fails, it withdraws all credit lines from its debtors and also liquidated entirely its projects.

Default cascades: a simple example



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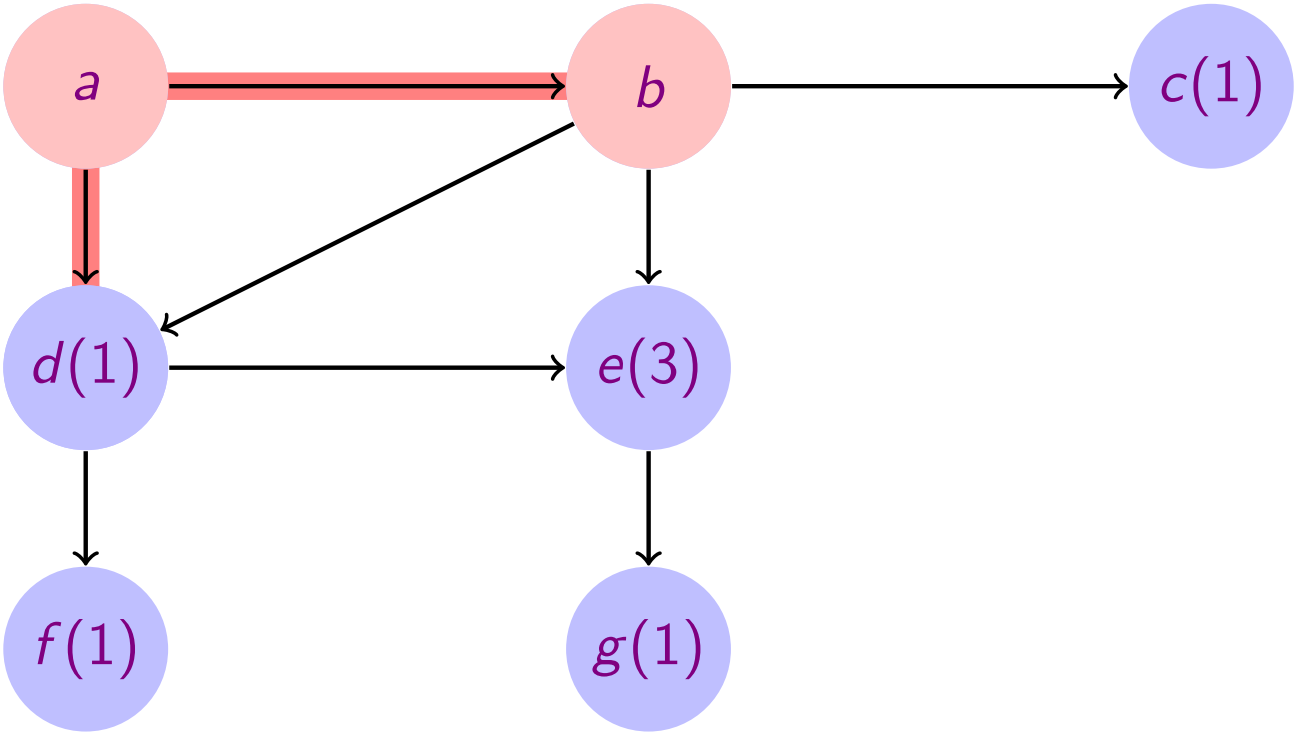

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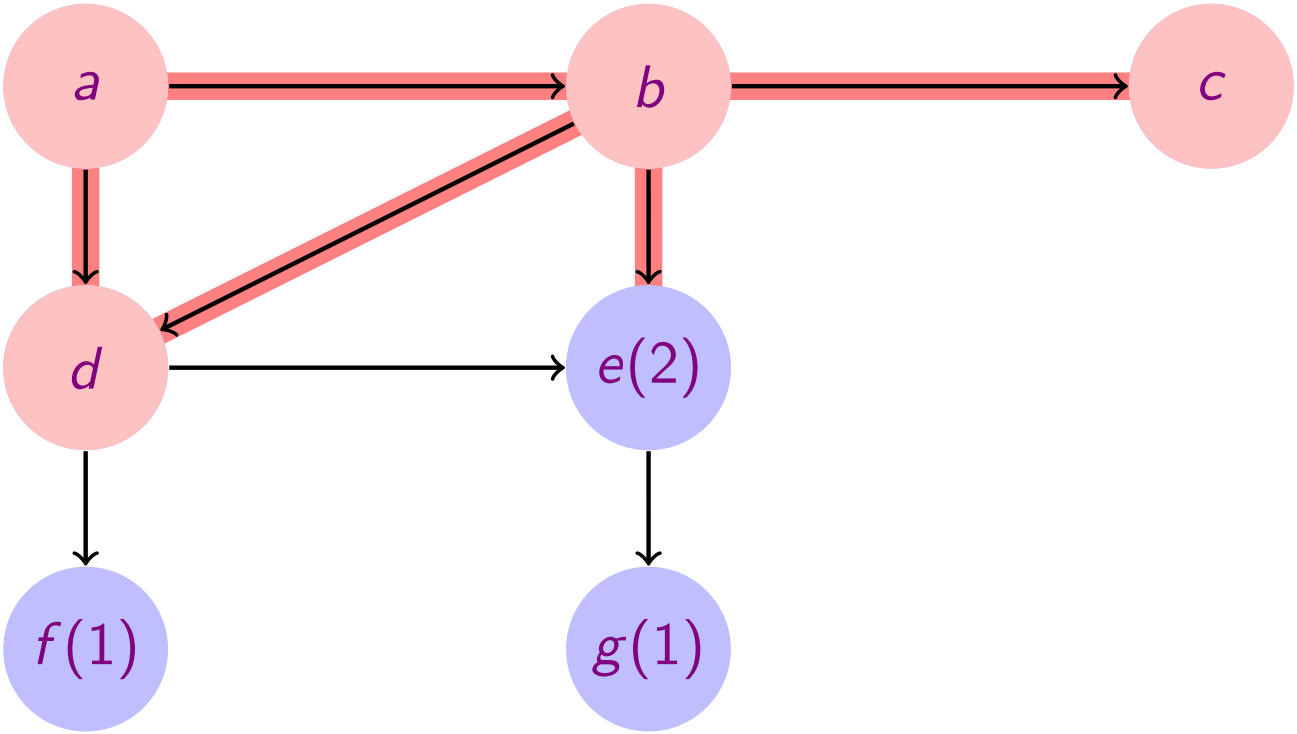

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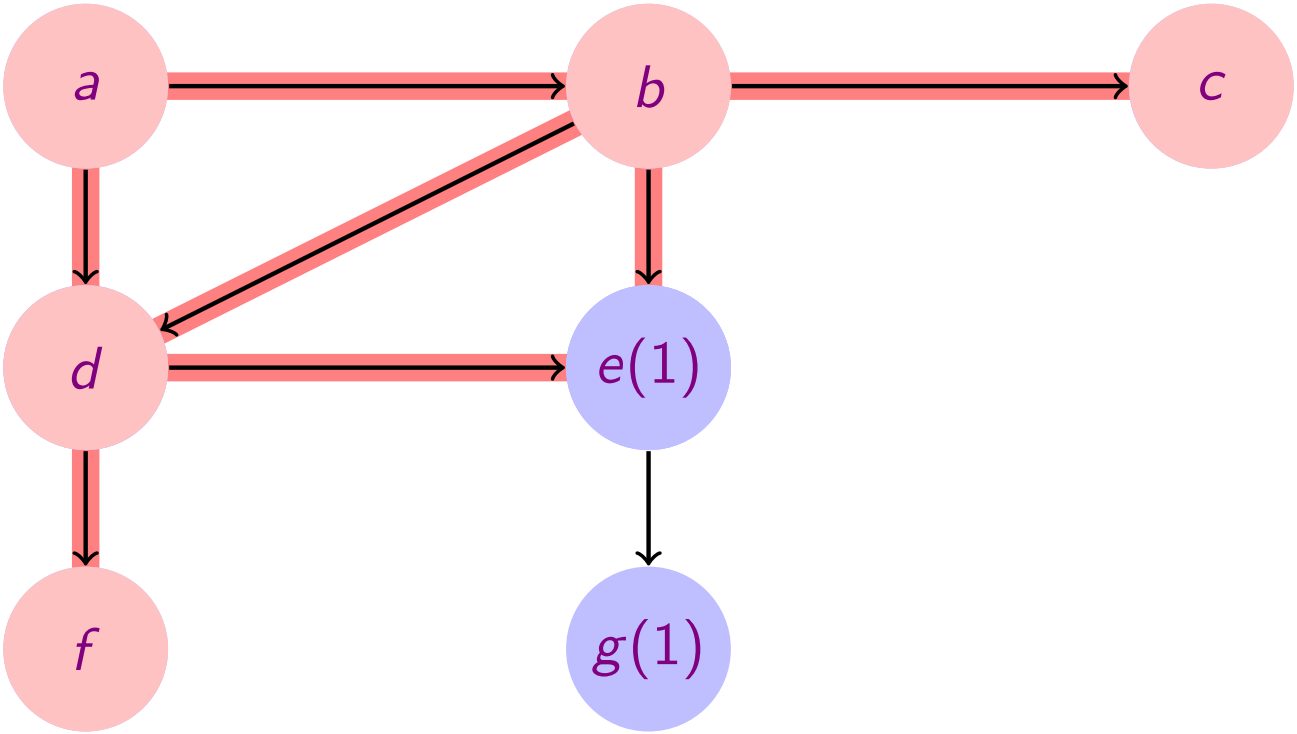
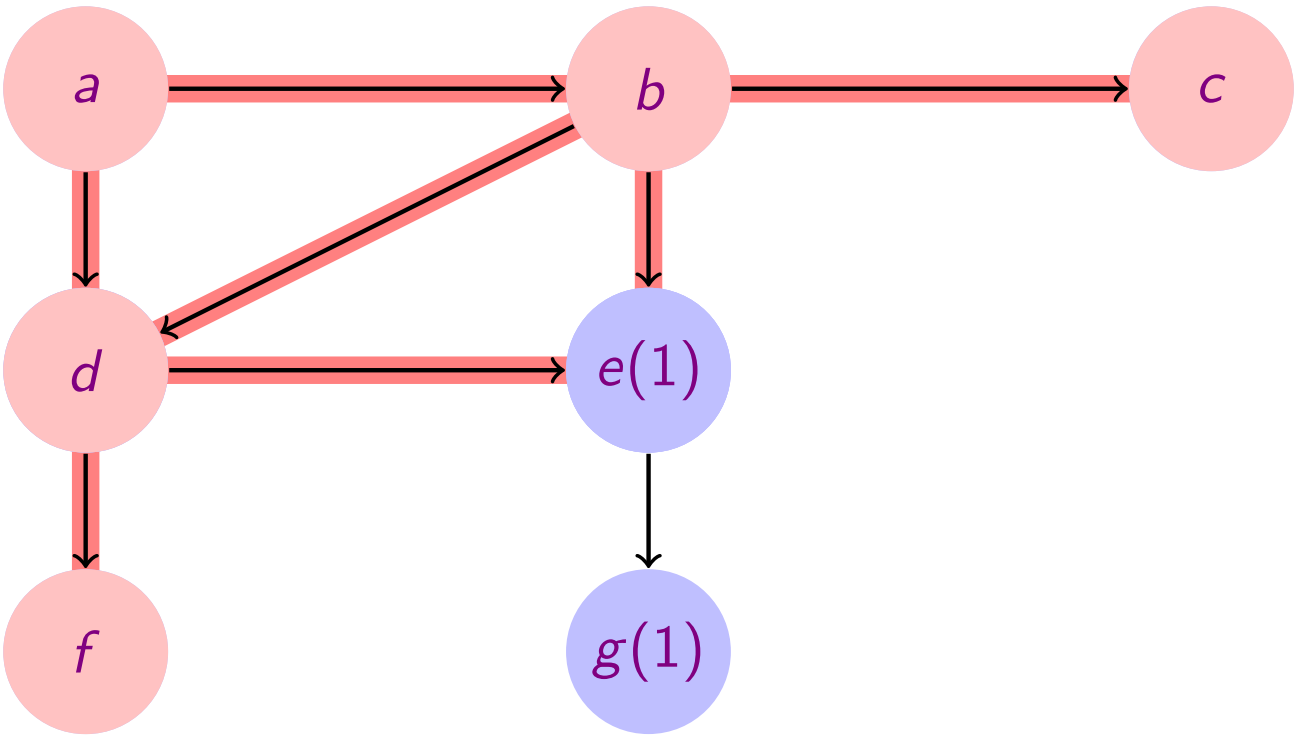


Default cascades: a simple example

The failure cluster in red
The revealed links are in red: For tractability, they will be revealed one by one



Link-revealing filtration

We endow the probability space $(G_{n+1.m}, \mathbb{P})$ with a filtration $(\mathcal{G}_k)_{k \leq m}$, called "**link-revealing**" filtration, that models the financial contagion and the flow of information:

II. Optimal intervention problem

We assume now that there exists a controller that intervenes a maximum number of times M (budget constraint) at steps $1 \leq k \leq m$.
When the controller intervenes on a bank, its distance to failure increases by 1.
The controller may intervene at each step only on one bank, which we understand is the end of the revealed link. For this type of control, we introduce the control set

$$\mathcal{U}_M := \{(u_k \in \{0,1\}, 1 \leq k \leq m), \sum_{1 \leq k \leq m} u_k \leq M\}.$$

For the optimization criterion we consider, this space is sufficient, i.e., it would not be optimal to intervene on multiple banks at the same time or on banks which are not the end of the revealed link.

2.1 Controlled cascade of information and contagion

We now define the controlled cascade of information and contagion, where the control process takes values in $\mathcal{U}_M$ and is adapted to the link-revealing filtration.

- **Initial condition.**
  - Let $\theta_0(i), i \in [n]$ given.
  - All links entering initially failed nodes are hidden.

- **Dynamics.**

At each step $k = 1, \ldots, m$, as long as there are hidden links entering failed banks, we choose uniformly one of them and reveal it.

- We refer to the end of the revealed link j_k, as the "new observation" at step k.
- The distance to failure of j_k decreases without intervention.
- If j_k fails (reaches distance 0), then we augment the set of hidden links.

We let T the stopping time of the cascade.

2.2 Optimal control under partial information

Optimization criterion

Value of the financial system at the end of the cascade process (defined similarly to the case without initial shock).

$$J_T = \#\{i \in [c], \theta_T(i) > 0\} y_c + \#\{i \in [p], \theta_T(i) > 0\} y_p + \mathbb{E}[N_T \mid \mathcal{G}_T],$$

where

$$N_T := \#\{(i,j) \in \mathcal{E}, i \in [c] j \in [p], \theta_T(j) > 0)\}.$$

The first two terms of J_T count in only the projects of non-failed banks, since the projects of failed banks are liquidated.
The last term in J_T represents the expectation of the number of loans renewed by core banks to the peripheral banks, i.e., the external projects that were funded by peripheral banks by borrowing from core banks.

Problem:

$$\Phi_0 := \max_{u \in \mathcal{U}^a} \mathbb{E}\big(J_T | \mathcal{G}_0\big)$$

where $\mathcal{U}^a$ denotes the set of $(\mathcal{G}_k)_{1 \leq k \leq m}$-adapted processes with values in $\mathcal{U}_M$.

Since the control space $\mathcal{U}^a$ is finite, there exists an optimal control. However, *a priori* this control depends on the whole history of the system, which would make the problem intractable.

2.2 State space collapse theorem

Aggregate state variables

Contagion and the optimal control can be determined using some aggregates of the state variables.

Intuitively, nodes that have the same connectivity and distance to failure are identical. Therefore, we only need to keep track of their number during the cascade, rather than their individual state. Using the state space collapse, the problem is now tractable and can be solved using dynamic programming.

III. Numerical analysis

III. 2. Numerical experiments

We solve our Problem by implementing the dynamic programming equation with the numerical values of the parameters given below unless otherwise stated.

Number of core banks	$c = 5$
Number of peripheral banks	$p = 40$
Number of projects of core banks	$y_c = 1$
Number of projects of peripheral banks	$y_p = 1$
Connectivity	$\lambda \in [1, 20]$
Maximum distance to failure	$\vartheta_{max} = 3$
Intervention budget	$M = 4$

Table: Parameters of the stylized model.

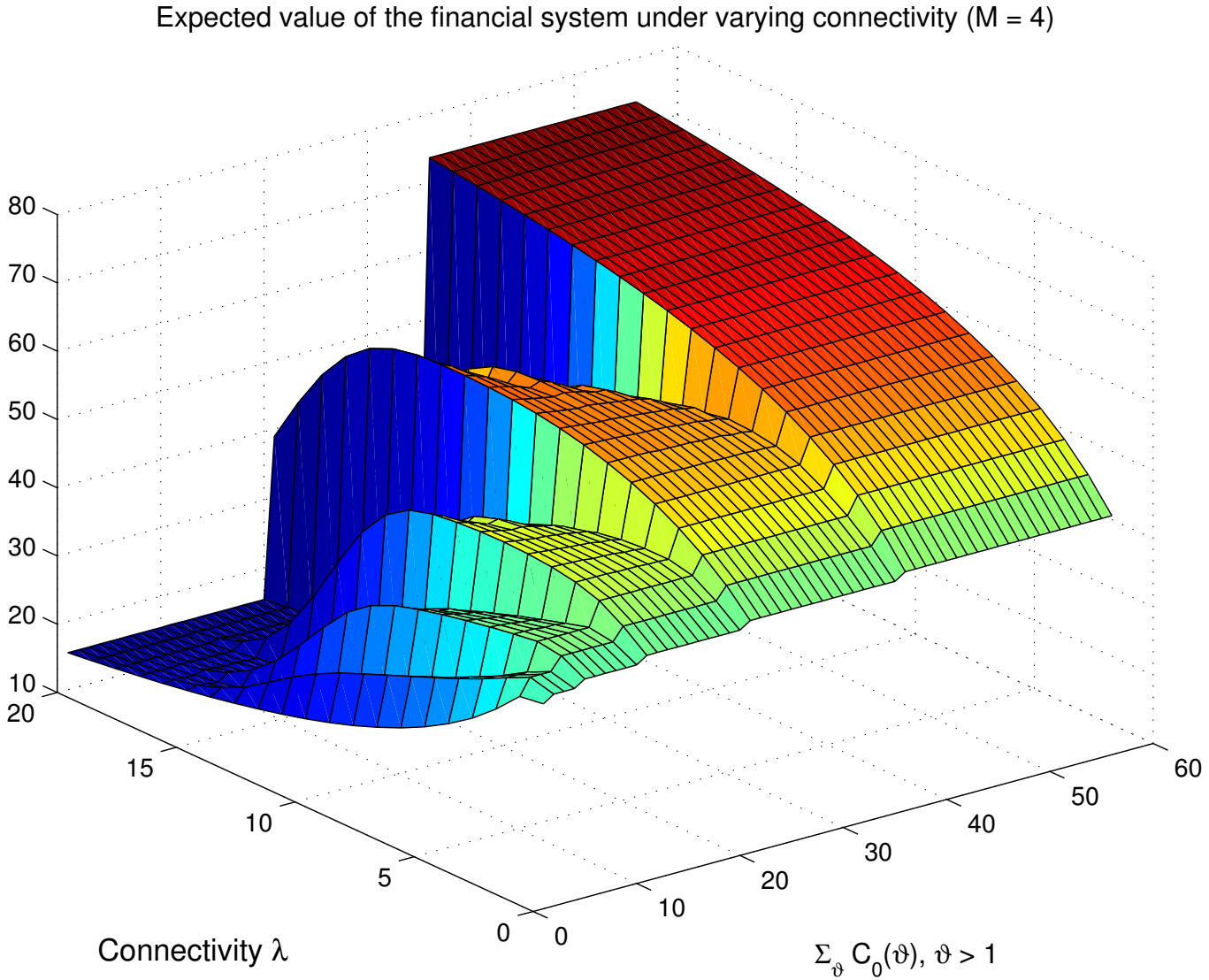


Figure: horizontal axis : initial state of core banks $(C_0(1), C_0(2), C_0(3))$ in increasing order of $\sum_{\vartheta=1}^{3} C_0(\vartheta)$ (number of non-failed core banks). $P_0 = p$. The slices correspond to different values of $\sum_{\vartheta=1}^{3} C_0(\vartheta) \in [0, c]$. For connectivity 0, the network is absent.

The relation between value and connectivity is non-monotonous.

- When there is no initial no failed core banks ($\sum_{\vartheta=1}^{3} C_0(\vartheta) = c$), then the value of the system always increases with connectivity.
- When there is at least one initially failed core bank ($\sum_{\vartheta=1}^{3} C_0(\vartheta) < c$), then the value first increases, then decreases with connectivity.

A priori, connectivity increases the value of the financial system. However, when there are initially core failed banks, contagion also increases and the value of the system is given by these two opposite effects.

Value of the financial system with different intervention budgets

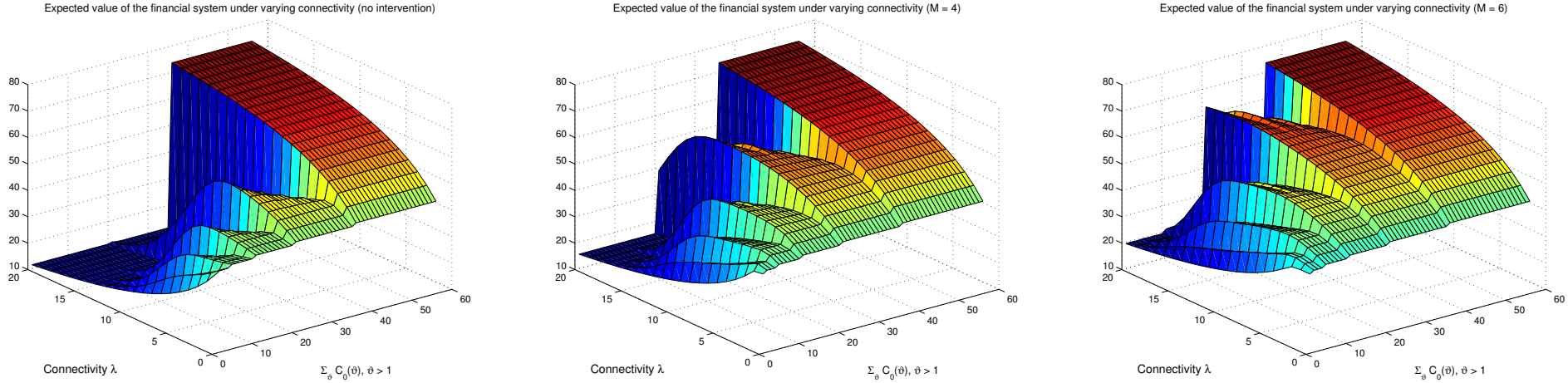


Figure: Value of the financial system (Φ_0) under varying connectivity and different intervention budgets: $M = 0$ (no intervention), $M = 4$, $M = 6$. There is an optimal level of connectivity for each initial state for a given budget, and this optimum is increasing in the budget.

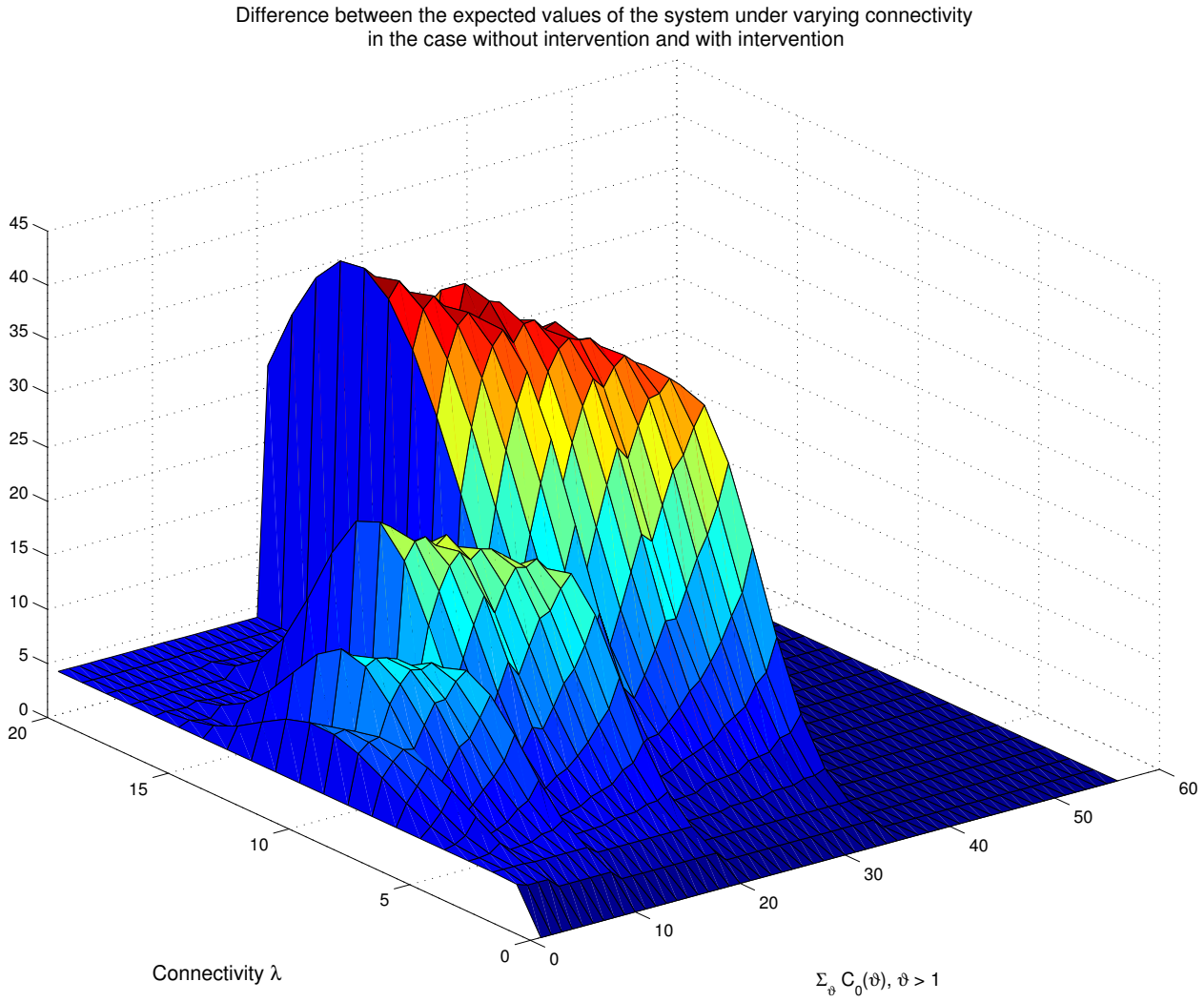


Figure: Difference between values without and with intervention ($M = 4$).
Optimal intervention ensures that the benefit from connectivity is not surpassed by the contagion cost, up to a certain connectivity. Note that the difference in value with intervention and without intervention surpasses the intervention budget. This conclusion depends on our assumption on the complete liquidations.

Policy of the controller

We compare now the number of interventions on core and peripheral banks for varying connectivity.

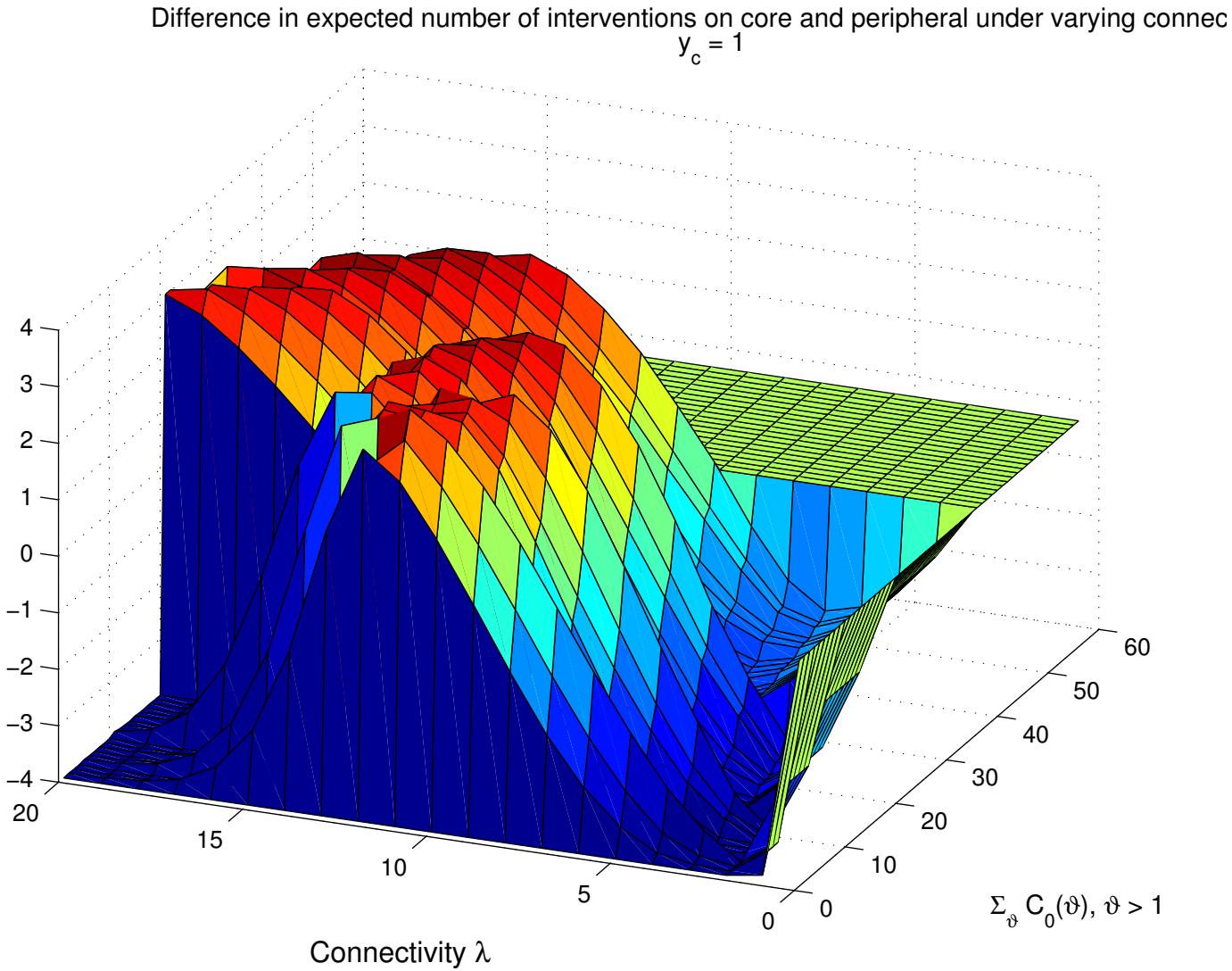


Figure: Difference in expected number of interventions on core and peripheral.
If no initial failed core banks, then no intervention as no contagion. When at least one initially failed core bank, we distinguish the following: For low connectivity, the number of interventions in peripheral banks increases with connectivity. For medium connectivity it is optimal to increase the number of interventions on core banks. For high connectivity , again optimal to intervene on peripheral banks.

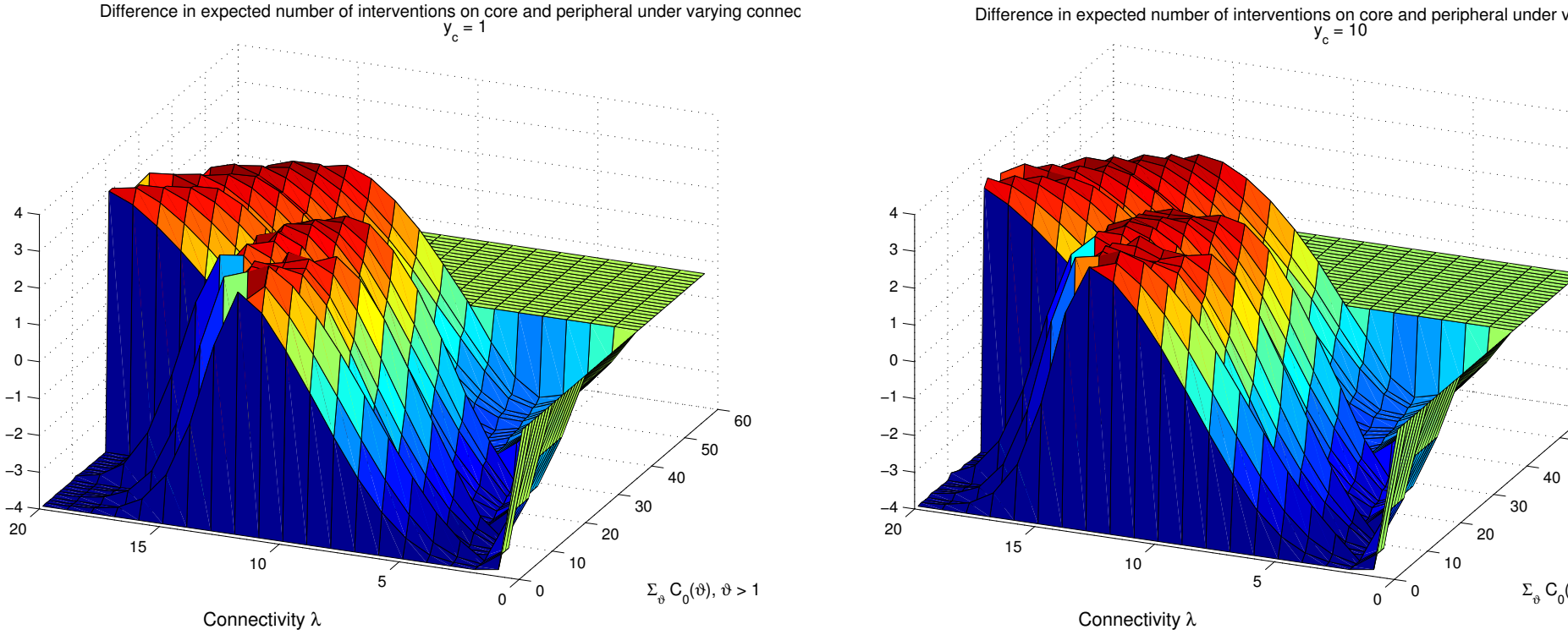


Figure: Difference between the expected number of interventions on core and peripheral banks, under two cases $y_c = y_p = 1$ (so difference among core and peripheral banks is connectivity alone) and $y_c = 10$.

Conclusions and Perspectives

Main insights:
Relation of the value of the financial system with connectivity and optimal intervention.
Many ways in which more realism can be added to the model:

- Relaxing the assumption of full liquidation and full withdrawal of credit lines in case of failures
- Design of optimal sharing rules of the benefit from connectivity and control among the controller and the financial system, for example in the form of interest rate.
- Allow for an exogenous rise in the distance to failure.