How Likely Is Contagion in Financial Networks?

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Overview

We show how to estimate the probability of contagion and the increase in expected losses due to contagion with virtually no knowledge of the network topology.

This is important because in practice details of interbank network exposures are not known with much precision and they are constantly in flux.

Key parameters needed for the analysis

- bank size (assets)
- bank leverage
- proportion of liabilities to the nonfinancial sector
- proportion of assets from the nonfinancial sector
• If all banks have the same size, the probability of contagion is less than the probability of direct default for a wide class of shock distributions regardless of the network topology.

• Expected losses due to indirect (domino) effects are typically small compared to the expected losses from shocks to the firm’s own assets. The ratio of the two effects can be bounded without knowing the details of the network.

• Bankruptcy costs and mark-to-market write-downs in advance of default increase the probability of contagion and the expected size of the losses. These can be estimated using simple formulas that do not depend on the network topology.
This presentation represents the views of the authors and not the official views of the Office of Financial Research or the U.S. Department of the Treasury.
$\bar{p}_i = \text{node } i\text{'s total liabilities}$

$a_{ij} = \begin{cases} 
\frac{\bar{p}_{ij}}{\bar{p}_i}, & \bar{p}_i > 0; \\
0, & \text{otherwise.}
\end{cases}$

Relative liabilities

Payments due *from* node $k$

Payments due *to* node $j$

Outside assets $c_i$

Outside liabilities $b_i$
What We See

\[ \sum_j p_i a_{ij} \]

\[ \sum_k p_k a_{ki} \]

Outside assets \( c_i \)

Outside liabilities \( b_i \)

assumed > 0
Key parameters

Net worth

\[ w_i = \left( c_i + \sum_k \bar{p}_k a_{ki} \right) - \left( b_i + \sum_j \bar{p}_i a_{ij} \right) \]

Leverage of outside assets: outside assets/net worth

\[ \lambda_i = c_i / w_i \]

Financial connectivity: in-network liabilities/total liabilities

\[ \beta_i = (\bar{p}_i - b_i) / \bar{p}_i \]

Contagion index: \[ \beta_i w_i (\lambda_i - 1) \]
We compare the probability of indirect failure due to contagion with the probability of direct failure due to write-downs in a bank’s outside assets.

Contagion from bank \( i \) to a set of banks \( D \) is weak if the nodes in \( D \) are more likely to default through independent direct shocks to their outside assets than through an indirect shock from \( i \).
Theorem on weak contagion

Let \( d = |D| \)

\[ \tilde{\lambda}_D = \left[ \frac{\sum_{j \in D} \lambda_j^{-1}}{d} \right]^{-1} = \text{harmonic mean of the } \lambda_j 's \]

\[ \bar{w}_D = \frac{\sum_{j \in D} w_j}{d} = \text{arithmetic mean of } w_j 's \]

Theorem: Suppose the shocks are i.i.d. beta distributed.

Contagion from \( i \) to \( D \) is weak if

\[ \tilde{\lambda}_D \bar{w}_D \geq \beta_i w_i (\lambda_i - 1). \quad (1) \]

Contagion from \( i \) to \( D \) is impossible if

\[ \sum_{j \in D} w_j \geq \beta_i w_i (\lambda_i - 1). \quad (2) \]

Note that the right-hand side is the contagion index, which measures the impact of shocks to \( i \) on the rest of the financial system.
If all banks have the same amount of outside assets, contagion from any node to any set of nodes is weak.

Hence without some degree of heterogeneity contagion is weak irrespective of the topological structure of the interbank network.
How much do interbank connections amplify losses?

- At each node, replace the in-network assets and liabilities with fictitious out-of-network assets and liabilities, thus preserving the net worth of each node while amputating all interbank connections.

- Assume (conservatively) that the fictitious out-of-network assets are not vulnerable to shocks.

- Using the same shock distribution compare expected losses in the original network with expected losses in the ‘amputated’ network.

\[
\sum_k \overline{p}_k a_{ki} \\
\sum_j \overline{p}_i a_{ij}
\]

Outside assets \(c_i\)

Outside liabilities \(b_i\)
Loss amplification due to network

\[ X_i = \text{shock to i's outside assets} \]

\[ L^o = \sum X_i = \text{losses without network effects} \]

\[ L = \text{losses with network effects} \]

\[ \delta_i = P(X_i > w_i) = \text{probability that i defaults directly} \]

\[ \beta^+ = \max_i \beta_i = \text{maximum financial connectivity} \]

**Theorem.** Suppose the relative shocks \( X_i / c_i \) have a common log-concave distribution. Then

\[
\frac{E[L]}{E[L^o]} \leq 1 + \frac{\sum_i \delta_i c_i}{(1 - \beta^+) \sum c_i}
\]

\( 1/(1 - \beta^+) \) is the amplification factor due to financial connectivity
Other Factors Leading to Contagion

- *Mark-to-market losses from credit deterioration*: a loss of confidence in one bank’s ability to meet its obligations cascades through the network

- *Bankruptcy costs*: Failure produces losses above and beyond payment shortfall (delays, litigation, etc.)

- *Fire sales*
Bankruptcy costs

Every $1 of default in payments causes an additional $\gamma$ in bankruptcy costs
Credit quality deterioration

Original model

![Original model graph]

Modified

![Modified model graph]
Network effects with bankruptcy costs

\[
\frac{E[L]}{E[L^0]} \leq 1 + \frac{\sum_i \delta_i c_i}{(1 - (1 + \gamma)\beta^+ \sum_i c_i}
\]

European Banking Authority stress test data (2010).

\[\beta^+ = 0.43, \quad \gamma = 0.50\]

If \(\delta_i = 1\%\), the network adds 4.2\% to expected losses
• The contagion index measures the impact that the failure of a given bank has on the financial system.

• The probability that a given bank $i$ causes a group of sizable banks to fail due to contagion is less than the probability that the group defaults directly unless $i$ has a sufficiently high contagion index.

• If all banks are the same size as measured by outside assets, the probability of contagion is less than the probability of direct default for a wide class of shock distributions regardless of the network topology.
• Losses due to pure spillover effects in the network are typically small compared to the losses from direct default.

• Bankruptcy costs and mark-to-market write-downs in advance of default increase the probability of contagion and the expected size of the losses.

• Losses attributable to network effects can be bounded without knowing the details of the network.

• These losses are amplified by the degree of financial connectivity, which is a key indicator of systemic stability.
This paper:


Other network contagion models:

Thank You!