Network Risk and Key Players: A Structural Analysis of Interbank Liquidity

Edward Denbee, Christian Julliard, Ye Li, Kathy Yuan

*London School of Economics, †CEPR ◇Bank of England ‡Columbia University

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The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.
Recent crisis stressed the need of understanding systemic risk generation and exposure in the banking industry.

Our paper: linear quadratic network framework; we can identify:
1. the amplification mechanism, or multiplier, of liquidity shocks;
2. the liquidity level key players (for bailout?);
3. the liquidity risk key players (to regulate?).

Also: we solve the CP problem and have implications for the efficiency of liquidity injections and Quantitative Easing.
Introduction

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On average, in 2009, £700bn of transactions were settled every day across the two UK systems, CREST and CHAPS: the UK nominal GDP settled every two days.

Daily Gross Settlement requires large intraday liquidity buffers. Almost all banks in CHAPS regularly have intraday liquidity exposures in excess of £1bn to individual counterparties. For larger banks these exposures are regularly greater than £3bn.

We study banks’ intraday liquidity holding decision in the network, and its implications for systemic liquidity risk.
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We study banks’ intraday liquidity holding decision in the network, and its implications for systemic liquidity risk.
Outline

1. Theoretical Framework
   - Network Specification
   - Bank Objective Function and Nash Equilibrium
   - Risk, and Level, Key Players

2. Empirical Analysis
   - Empirical Specification
   - Network and Data Description
   - Estimation Results

3. Conclusions

Appendix
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► Appendix
Network Specification

- A directed and weighted network of $n$ banks.

Network $g$ : characterized by $n$-square adjacency matrix $G$ with elements $g_{i,j}$, and $g_{i,i} = 0$.

$g_{i,j} \neq i$ : the fraction of borrowing by Bank $i$ from Bank $j$.

$\Rightarrow$ $G$ is a (right) stochastic matrix and is not symmetric.
Bank Objective Function

- **Bank i decision variables:**

  \( l_i := q_i + z_i \) : is the observable liquidity holding of bank \( i \), where:

  \[ q_i : \text{liquidity level of bank } i \text{ absent bilateral effects, given by} \]

  \[ q_i = q_i(x) := \alpha_i + \sum_{m=1}^{M} \beta_m x_i^m + \sum_{p=1}^{P} \beta_p x_i^p \]

  - \( \alpha_i \) : fixed effect
  - \( \sum_{m=1}^{M} \beta_m x_i^m \) : characteristics
  - \( \sum_{p=1}^{P} \beta_p x_i^p \) : common factors

  \( z_i \) : the network component of liquidity buffer stock.
Bank Objective Function cont’d

- A quadratic payoff function for buffer stock liquidity $z_i$

$$u_i(z_i|g) = \hat{\mu}_i \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right) - \frac{1}{2} \gamma \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)^2 + z_i \delta \sum_{j \neq i} g_{ij} z_j$$

Accessible Liquidity

"Collateralized" Liquidity

$$\hat{\mu}_i / \gamma = \bar{\mu}_i + \nu_i \sim i.i.d. (0, \sigma_i^2)$$

Note: $G$ predetermined at decision time (but can change over time).
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(Decentralized) Equilibrium Outcome

**Optimal** $z_i$:

$$z_i = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j} z_j + \nu_i$$

$$\phi := \delta / \gamma - \psi$$

If $\phi > 0$ complementarity (reciprocate/herding/leverage stacks/signaling e.g. Moore (2011)).

If $\phi < 0$ substitutability (free ride à la Bhattacharya and Gale (1987)).

Eqm : if $|\phi| < 1$

$$l_i^* = q_i(x) + z_i^* = q_i(x) + \{M(\phi, G)\} \mu$$

where $\mu := \gamma^{-1} [\mu_1, ..., \mu_n]'$, $\{\}$ is the row operator, and

$$M(\phi, G) := I + \phi G + \phi^2 G^2 + \phi^3 G^3 + ... = \sum_{k=0}^{\infty} \phi^k G^k.$$
## (Decentralized) Equilibrium Outcome

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Key Players

The total liquidity originating from the network externalities is

\[ 1'z^* = 1'M (\phi, G) \bar{\mu} + 1'M (\phi, G) \nu \]

level effect  

risk effect

where \( z^* \equiv [z_1^*, ..., z_n^*]', \( \bar{\mu} \equiv [\bar{\mu}_1, ..., \bar{\mu}_n]', \( \nu \equiv [\nu_1, ..., \nu_n]' \)

\( \Rightarrow \) tradeoff: both terms increasing in \( \phi \) (for \( \bar{\mu} > 0 \)).

Risk Key Player: (the one to worry about...)

\[
\max_i \frac{\partial 1'z^*}{\partial \nu_i} \sigma_i = \max_i 1' \{M (\phi, G)\}_{.,i} \sigma_i \rightarrow \text{outdegree centrality}
\]

Level Key Player: (the one you might want to bailout...)

\[
\max_i E [1'z^* - 1'z^*_i] = \max_i \{M (\phi, G)\}_{.,i} \bar{\mu}_i + 1' \{M (\phi, G)\}_{.,i} \bar{\mu}_i - m_{i,i} \bar{\mu}_i
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\( \rightarrow \) indegree centrality + shock analogous – correct double counting
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Theoretical Framework
Empirical Analysis
Conclusions

Empirical Specification
Network and Data Description
Estimation Results

Empirical Model

**SEM:** the theoretical framework is matched by a **Spatial Error Model**

\[
\begin{align*}
I_{i,t} &= \alpha_i + \sum_{m=1}^{M} \beta_{m}^{\text{bank}} x_{i,t}^m + \sum_{p=1}^{P} \beta_{p}^{\text{time}} x_{t}^p + z_{i,t} \\

z_{i,t} &= \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j,t} z_{j,t} + \nu_{i,t}, \quad \nu_{i,t} \sim iid \left(0, \sigma_i^2\right),
\end{align*}
\]

where \(g_{i,j,t}, x_{i,t}^m\) and \(x_{t}^p\) are predetermined at time \(t\).

**Note:**
1. Network as a shock propagation mechanism
   \(\Rightarrow (\text{average}) \quad \text{Network Multiplier}: \frac{1}{1 - \phi}\)
2. Total liquidity, \(L_t \equiv 1'[l_{1,t}, ..., l_{n,t}]\), is heteroskedastic:
   \[
   \text{Var}_{t-1}(L_t) = 1'M(\phi, G_t) \text{ diag } \left(\left\{\sigma_i^2\right\}_{i=1}^{n}\right) M(\phi, G_t)' 1.
   \]
3. Can perform Q-MLE (\(\phi\) overidentified if \(\text{rank}(M(\phi, G_t)) > 2\)
Empirical Model

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\[
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3. Can perform Q-MLE (\(\phi\) overidentified if \(\text{rank}(M(\phi, G_t)) > 2\))
SDM: For robustness, we also consider a direct network effect of banks observable characteristic, liquidity decisions, and possible match specific control variables, $x_{i,j,t}$ (Spatial Durbin Model)

$$l_{i,t} = \bar{\alpha}_i + \sum_{m=1}^{M} \beta^\text{bank}_m x^m_{i,t} + \sum_{p=1}^{P} \gamma^\text{time}_p x^p_t$$

$$+ \rho \sum_{j=1}^{n} g_{i,j,t} l_{j,t} + \sum_{j=1}^{n} g_{i,j,t} x_{i,j,t}\theta + v_{i,t}$$

Note: if $x_{i,j,t} := \text{vec}(x^m_{j\neq i,t})'$, $\rho = \phi$, $\theta = -\phi \text{vec}(\beta^\text{bank}_m)$,

$$\gamma^\text{time}_p = (1 - \phi)\beta^\text{time}_p \forall p \Rightarrow \text{back to SEM}$$

$\Rightarrow$ this more general spatial structure provides a specification test for our model.
Network Impulse-Response Functions

- The network impulse-response of total liquidity, \( L_t := \sum_{i=1}^{n} l_{i,t} \), to a one standard deviation shock to bank \( i \) is

\[
NIRF_i (\phi, G_t, \sigma_i) \equiv \frac{\partial L_t}{\partial \nu_{i,t}} \sigma_i = 1' \{ M (\phi, G_t) \} . i \sigma_i
\]

NIRFs:
1. are pinned down by the outdegree centrality and
   \[ \text{Risk Key Player} \equiv \arg\max_i NIRF_i (\phi, G_t, \sigma_i) \]
2. account for all direct and indirect links among banks since
3. are a natural decomposition of total liquidity variance

\[
Var_{t-1} (L_t) \equiv \text{vec} \left( \{NIRF_i (\phi, G_t, \sigma_i)\}_{i=1}^{n} \right)' \text{vec} \left( \{NIRF_i (\phi, G_t, \sigma_i)\}_{i=1}^{n} \right).
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  \]
Network and Other Data Description

**Sample:** from Feb 2006 to Sept 2010, daily data.

**Network Banks:** all CHAPS members (non CHAPS banks must channel their payments through these banks)

**Network Proxy:** \[ g_{i,j,t} = \text{the fraction of bank } i \text{'s loans borrowed from bank } j \] (computed as monthly averages in previous month)

**Dependent Variable:** liquidity available at the beginning of the day (account balance plus posting of collateral)

**Macro Controls** (aggregate risk proxies, lagged): LIBOR; Interbank Rate; Intraday Volatility of Liquidity Available; Turnover Rate in Payment System; Right Kurtosis of Aggregate Payment Time; time trend.

**Banks Characteristics** (lagged): Borrowing Rate; Right Kurtosis of Payment (Out) Time; Right Kurtosis of Payment (In) Time; Intraday Volatility of Liquidity Available; Total Intraday Payments; Liquidity Used; Repo liability to Total Asset Ratio; Cumulative Change in Retail Deposit to Total Asset Ratio; Total Lending and Borrowing in Interbank Market; Stock Return; CDS;
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Estimation Results

Two types of estimation:

1. Subsample estimations:
   - (good times) Pre Hedge Fund Crisis/ Northern Rock
   - (fin. crisis) Hedge Fund Crisis – Asset Purchase Program Announcement (Q.E.) Post Asset Purchase Program Announcement

2. Rolling estimations with 6-month window ⇒ allow $\phi$ to change at higher frequency.
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### SEM Estimation

<table>
<thead>
<tr>
<th>Network Effect: $\phi$</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.640*</td>
<td>0.166*</td>
<td>−0.151*</td>
</tr>
<tr>
<td></td>
<td>(52.44)</td>
<td>(7.06)</td>
<td>(−6.45)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>69.11%</td>
<td>89.71%</td>
<td>85.54%</td>
</tr>
<tr>
<td>(average) Network Multiplier</td>
<td>2.77*</td>
<td>1.12*</td>
<td>0.87*</td>
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</table>
Period 1: $NIRF^e(\phi, \bar{G}, 1)$ – Risk Key Players

Pre Northern Rock/Hedge Fund Crisis

Bank 1 Bank 2 Bank 3 Bank 4 Bank 5 Bank 6 Bank 7 Bank 8 Bank 9 Bank 10 Bank 11

Excess NIRF
+/- 2 s.e. C.I.
Excess network multiplier
+/- 2 s.e. C.I.
Period 1: Net Borrowing

Network Risk and Key Players
Period 1: Network Borrowing/Lending Flows
\( \hat{\phi} \): SEM Rolling Estimation (6-month window)
\( \hat{\phi} \) and \( \hat{\rho} \): SEM and SDM Rolling Estimation (6-month window)
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We provide:

- an implementable approach to assess interbank network risk:
  1. network shocks multiplier
  2. risk, and level, key players identification
  3. network impulse-response functions

Empirical Findings:

1. First estimation of network risk multiplier \( \Rightarrow \) a significant shock propagation mechanism for liquidity
2. The network multiplier and risk:
   - vary significantly over time, and can be very large.
   - implies complementarity (and high risk) before the crisis.
   - it’s basically zero post Bearn Stearns \( \Rightarrow \) rational reaction.
   - indicates free riding on the liquidity provided by the Quantitative Easing.
3. most of the systemic risk is generated by a small subset of key players (and not necessarily the obvious ones).
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4 Additional Data Info
- Second Largest Eigenvalue of $G_t$
- Average Clustering Coefficient
- Other Variables

5 Additional Estimation Result
- Full SEM Results
- Specification Test

6 Network Evolution
- NIRFs
- Net Borrowing and Flows
- Movie
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The Second Largest Eigenvalue of $G_t$
Cohesiveness

**Q:** How cohesive is this network?

**A:** Average Clustering Coefficient (Watts and Strogatz, 1998)

\[
ACC = \frac{1}{n} \sum_{i=1}^{n} CL_i(G),
\]

\[
CL_i(G) = \frac{\#\{jk \in G \mid k \neq j, j \in n_i(G), k \in n_i(G)\}}{\#\{jk \mid k \neq j, j \in n_i(G), k \in n_i(G)\}}
\]

where \( n \) is the number of members in the network and \( n_i(G) \) is the set of players between whom and player \( i \) there is an edge.

**Numerator:** \( \# \) of pairs of banks linked to \( i \) that are also linked to each other

**Denominator:** \( \# \) of pairs of banks linked to \( i \)
Average Clustering Coefficient of the Network

The graph shows the average clustering coefficient over time, with key events highlighted:

- 20070201: Subprime Default
- 20070809: Northern Rock/Hedge Fund Crisis
- 20080311: Bear Stearns
- 20080914: Lehman Brothers
- 20090919: Asset Purchase Programme Announced

The x-axis represents time, and the y-axis represents the clustering coefficient. The graph indicates a decline in clustering coefficient post-crisis events.
Aggregate Liquidity Available at the Beginning of a Day

20070201: Subprime Default
20070809: Northern Rock/Hedge Fund Crisis
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GBP

Time

Aggregate Liquidity Available (Weekly Average)
Interest Rate in Interbank Market

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![Graph showing interest rate changes with key events marked.](image-url)
Cross-Sectional Dispersion of Interbank Rate

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Times:
- 060525, 061016, 070308, 070801, 071220, 080516, 081007, 090227, 090723, 091211, 100510, 100929
Intraday Volatility of Aggregate Liquidity Available

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Turnover Rate in the Payment System

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Network Risk and Key Players

34/24
Right Kurtosis of Aggregate Payment Time

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060525 061016 070308 070811 080516 081007 090227 090723 091211 100510 100929

10 minutes
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### Macro Controls

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<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Liquidity (log)</td>
<td>−0.0020</td>
<td>0.3324*</td>
<td>0.5974*</td>
</tr>
<tr>
<td></td>
<td>(−0.04)</td>
<td>(4.59)</td>
<td>(14.73)</td>
</tr>
<tr>
<td>Right Kurtosis of Payments</td>
<td>−0.1654*</td>
<td>0.0265</td>
<td>0.0031</td>
</tr>
<tr>
<td></td>
<td>(−2.39)</td>
<td>(1.12)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>Volatility of Liquidity (log)</td>
<td>0.1750</td>
<td>0.1935*</td>
<td>0.0075</td>
</tr>
<tr>
<td></td>
<td>(1.37)</td>
<td>(7.15)</td>
<td>(0.52)</td>
</tr>
<tr>
<td>Turnover Rate</td>
<td>0.0097</td>
<td>0.0055*</td>
<td>0.0049*</td>
</tr>
<tr>
<td></td>
<td>(1.51)</td>
<td>(2.87)</td>
<td>(2.07)</td>
</tr>
<tr>
<td>LIBOR</td>
<td>0.6456*</td>
<td>0.3216*</td>
<td>−0.1633</td>
</tr>
<tr>
<td></td>
<td>(2.16)</td>
<td>(6.48)</td>
<td>(−1.12)</td>
</tr>
<tr>
<td>Interbank Rate Premium</td>
<td>1.9305*</td>
<td>−0.0505</td>
<td>0.9514*</td>
</tr>
<tr>
<td></td>
<td>(2.75)</td>
<td>(−0.61)</td>
<td>(2.86)</td>
</tr>
<tr>
<td>Constant</td>
<td>16.0761*</td>
<td>10.7165*</td>
<td>11.7844*</td>
</tr>
<tr>
<td></td>
<td>(5.14)</td>
<td>(5.66)</td>
<td>(9.70)</td>
</tr>
</tbody>
</table>
**Bank Characteristics**

<table>
<thead>
<tr>
<th></th>
<th>Estimate 1</th>
<th>Estimate 2</th>
<th>Estimate 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interbank Rate</strong></td>
<td>-0.5096</td>
<td>-0.2977*</td>
<td>0.1414</td>
</tr>
<tr>
<td></td>
<td>(-1.72)</td>
<td>(-6.02)</td>
<td>(1.0428)</td>
</tr>
<tr>
<td><strong>Intraday Payment Level (log)</strong></td>
<td>-0.1558*</td>
<td>-0.1595*</td>
<td>0.0478*</td>
</tr>
<tr>
<td></td>
<td>(-5.73)</td>
<td>(-8.87)</td>
<td>(2.51)</td>
</tr>
<tr>
<td><strong>Right Kurtosis of Payment In</strong></td>
<td>0.0359</td>
<td>-0.0045</td>
<td>-0.0395*</td>
</tr>
<tr>
<td></td>
<td>(1.90)</td>
<td>(-0.26)</td>
<td>(-3.39)</td>
</tr>
<tr>
<td><strong>Right Kurtosis of Payment Out</strong></td>
<td>0.1416*</td>
<td>0.1742*</td>
<td>0.0426*</td>
</tr>
<tr>
<td></td>
<td>(8.17)</td>
<td>(15.89)</td>
<td>(4.16)</td>
</tr>
<tr>
<td><strong>Vol of Liquidity Available (log)</strong></td>
<td>0.0558*</td>
<td>0.0524*</td>
<td>0.0417*</td>
</tr>
<tr>
<td></td>
<td>(39.72)</td>
<td>(50.23)</td>
<td>(36.73)</td>
</tr>
<tr>
<td><strong>Liquidity Used (log)</strong></td>
<td>0.0303*</td>
<td>-0.0023</td>
<td>0.0052</td>
</tr>
<tr>
<td></td>
<td>(3.00)</td>
<td>(-0.34)</td>
<td>(0.68)</td>
</tr>
<tr>
<td><strong>Top 4 Bank in Payment Activity</strong></td>
<td>1.3374*</td>
<td>1.6815*</td>
<td>2.3738*</td>
</tr>
<tr>
<td></td>
<td>(26.97)</td>
<td>(46.31)</td>
<td>(57.18)</td>
</tr>
<tr>
<td><strong>Repo Liability / Assets</strong></td>
<td>-0.7721</td>
<td>0.7401*</td>
<td>0.0575</td>
</tr>
<tr>
<td></td>
<td>(-0.92)</td>
<td>(14.46)</td>
<td>(0.64)</td>
</tr>
<tr>
<td><strong>Change in Deposit / Assets</strong></td>
<td>0.5050</td>
<td>-1.3275*</td>
<td>-1.2503*</td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
<td>(-6.65)</td>
<td>(-3.70)</td>
</tr>
<tr>
<td><strong>Total Lending and Borrowing (log)</strong></td>
<td>0.1209*</td>
<td>0.0249</td>
<td>-0.3231*</td>
</tr>
<tr>
<td></td>
<td>(3.56)</td>
<td>(0.99)</td>
<td>(-23.70)</td>
</tr>
<tr>
<td><strong>CDS (log)</strong></td>
<td>-0.0652</td>
<td>-0.0274*</td>
<td>0.0514*</td>
</tr>
<tr>
<td></td>
<td>(-1.49)</td>
<td>(-3.17)</td>
<td>(4.55)</td>
</tr>
<tr>
<td><strong>CDS Missing Dummy</strong></td>
<td>-2.1893*</td>
<td>-2.2618*</td>
<td>-0.8502*</td>
</tr>
<tr>
<td></td>
<td>(-11.38)</td>
<td>(-32.04)</td>
<td>(-8.37)</td>
</tr>
</tbody>
</table>
**Specification Test**

**SDM:** For robustness, we also consider a direct network effect of banks observable characteristic, liquidity decisions, and possible match specific control variables, $x_{i,j,t}$ (Spatial Durbin Model)

$$l_{i,t} = \bar{\alpha}_i + \sum_{m=1}^{M} \beta_{m}^{\text{bank}} x_{i,t}^m + \sum_{p=1}^{P} \gamma_{p}^{\text{time}} x_t^p$$

$$+ \rho \sum_{j=1}^{n} g_{i,j,t} l_{j,t} + \sum_{j=1}^{n} g_{i,j,t} x_{i,j,t} \theta + \nu_{i,t}$$

**Note:** if $x_{i,j,t} := \text{vec}(x_{j\neq i,t})'$, $\rho = \phi$, $\theta = -\phi \text{vec}(\beta_{m}^{\text{bank}})$, $\gamma_{p}^{\text{time}} = (1 - \phi) \beta_{p}^{\text{time}} \forall p$ ⇒ back to SEM

⇒ this more general spatial structure provides a specification test for our model.
\( \hat{\phi} \) and \( \hat{\rho} \): SEM and SDM Rolling Estimation (6-month window)
Outline

4 Additional Data Info
- Second Largest Eigenvalue of $G_t$
- Average Clustering Coefficient
- Other Variables

5 Additional Estimation Result
- Full SEM Results
- Specification Test

6 Network Evolution
- NIRFs
- Net Borrowing and Flows
- Movie

Appendix
**Period 2:** \( NIRF^e(\phi, \bar{G}, 1) \) – Risk Key Players

**Note:** network risk reduction despite increased borrowing & lending
Period 3: $NIRF^e(\phi, \bar{G}, 1) - Risk Key Players$

**Post Asset Purchase Programme Announcement**

- Excess NIRF
- +/- 2 s.e. C.I.
- Excess network multiplier
- +/- 2 s.e. C.I.
Period 1: Net Borrowing

![Net Borrowing Diagram]

- Bank 1
- Bank 2
- Bank 3
- Bank 4
- Bank 5
- Bank 6
- Bank 7
- Bank 8
- Bank 9
- Bank 10
- Bank 11

Net Borrowing: Net Borrowing
Bank Index: Bank Index

-1e+11
-5e+10
0e+00
5e+10
1e+11

Denbee, Julliard, Li and Yuan

Network Risk and Key Players

Data
Additional Estimation Result
Network Evolution
NIRFs
Net Borrowing and Flows
Movie
**Period 2: Net Borrowing**

The graph shows the net borrowing of various banks during Period 2. Each bank is represented by a vertical line at its corresponding index on the x-axis, with the net borrowing amount indicated by the distance from the x-axis.

- **Bank 1** to **Bank 11** are the banks listed on the x-axis.
- **Bank 4**, **Bank 6**, **Bank 7**, **Bank 8**, **Bank 9**, and **Bank 10** are highlighted with vertical lines indicating significant net borrowing amounts.

The net borrowing values range from $-1\times10^{11}$ to $1\times10^{11}$.
**Period 3: Net Borrowing**

![Graph showing net borrowing for Bank Index 1 to 11 with specific values for each bank.](image)
**Period 1: Network Borrowing/Lending Flows**

![Network Diagram]

- Bank 9
- Bank 10
- Bank 11
- Bank 1
- Bank 2
- Bank 3
- Bank 4
- Bank 5
- Bank 6
- Bank 7
- Bank 8
Period 2: Network Borrowing/Lending Flows
Period 3: Network Borrowing/Lending Flows
Daily Network Borrowing/Lending Flows