On connections between orthogonal arrays and *D*-optimal designs for certain generalized linear models with group effects

John Stufken

Arizona State University

Design and Analysis of Experiments in Healthcare
July 6-10, 2015

Joint work with Xijue Tan

Outline

Yang and S, 2009, Annals of Statistics

They considered generalized linear and nonlinear models with two parameters, $\theta = (\theta_1, \theta_2)' \ldots$

…and one design variable, x, with values in an interval, possibly half-bounded or unbounded.

They used approximate designs: $\xi = \{(x_1, \omega_1), (x_2, \omega_2), \ldots\}$, where $\omega_i > 0$, $\sum_i \omega_i = 1$.

They focused on local optimality, and aimed for complete class results under the Loewner ordering.

Thus, with $M_\xi(\theta)$ as the information matrix for θ with design ξ, the aim was to find a small class of designs Ξ so that for any design ξ_0 there is a design $\xi_0^* \in \Xi$ with $M_{\xi_0^*}(\theta) \geq M_{\xi_0}(\theta)$.

Many common optimality criteria adhere to the Loewner ordering.

Such classes always exist (all designs; Carathéodory).

Yang and S, 2009, Annals of Statistics

They considered generalized linear and nonlinear models with two parameters, $\theta = (\theta_1, \theta_2)'$ ...

... and one design variable, x, with values in an interval, possibly half-bounded or unbounded.

They used approximate designs: $\xi = \{(x_1, \omega_1), (x_2, \omega_2), \ldots\}$, where $\omega_i > 0$, $\sum_i \omega_i = 1$.

They focused on local optimality, and aimed for complete class results under the Loewner ordering.

Thus, with $M_\xi(\theta)$ as the information matrix for θ with design ξ, the aim was to find a small class of designs Ξ so that for any design ξ_0 there is a design $\xi_0^* \in \Xi$ with $M_{\xi_0^*}(\theta) \geq M_{\xi_0}(\theta)$.

Many common optimality criteria adhere to the Loewner ordering.

Such classes always exist (all designs; Carathéodory).

Yang and S, 2009, Annals of Statistics

They considered generalized linear and nonlinear models with two parameters, $\theta = (\theta_1, \theta_2)'$ ...

... and one design variable, x, with values in an interval, possibly half-bounded or unbounded.

They used approximate designs: $\xi = \{(x_1, \omega_1), (x_2, \omega_2), \ldots\}$, where $\omega_i > 0$, $\sum_i \omega_i = 1$.

They focused on local optimality, and aimed for complete class results under the Loewner ordering.

Thus, with $M_\xi(\theta)$ as the information matrix for θ with design ξ, the aim was to find a small class of designs Ξ so that for any design ξ_0 there is a design $\xi_0^* \in \Xi$ with $M_{\xi_0^*}(\theta) \geq M_{\xi_0}(\theta)$.

Many common optimality criteria adhere to the Loewner ordering.

Such classes always exist (all designs; Carathéodory).

Yang and S, 2009, Annals of Statistics

They considered generalized linear and nonlinear models with two parameters, $\theta = (\theta_1, \theta_2)'$ ...

... and one design variable, x, with values in an interval, possibly half-bounded or unbounded.

They used approximate designs: $\xi = \{(x_1, \omega_1), (x_2, \omega_2), \ldots\}$, where $\omega_i > 0$, $\sum_i \omega_i = 1$.

They focused on local optimality, and aimed for complete class results under the Loewner ordering.

Thus, with $M_\xi(\theta)$ as the information matrix for θ with design ξ, the aim was to find a small class of designs Ξ so that for any design ξ_0 there is a design $\xi_0^* \in \Xi$ with $M_{\xi_0^*}(\theta) \geq M_{\xi_0}(\theta)$.

Many common optimality criteria adhere to the Loewner ordering.

Such classes always exist (all designs; Carathéodory).

Yang and S, 2009, Annals of Statistics

They considered generalized linear and nonlinear models with two parameters, $\theta = (\theta_1, \theta_2)'$ ...

...and one design variable, x, with values in an interval, possibly half-bounded or unbounded.

They used approximate designs: $\xi = \{(x_1, \omega_1), (x_2, \omega_2), \ldots\}$, where $\omega_i > 0$, $\sum_i \omega_i = 1$.

They focused on local optimality, and aimed for complete class results under the Loewner ordering.

Thus, with $M_\xi(\theta)$ as the information matrix for θ with design ξ, the aim was to find a small class of designs Ξ so that for any design ξ_0 there is a design $\xi_0^* \in \Xi$ with $M_{\xi_0^*}(\theta) \geq M_{\xi_0}(\theta)$.

Many common optimality criteria adhere to the Loewner ordering.

Such classes always exist (all designs; Carathéodory).

Yang and S, 2009, Annals of Statistics

They considered generalized linear and nonlinear models with two parameters, $\theta = (\theta_1, \theta_2)'$ ...

...and one design variable, x, with values in an interval, possibly half-bounded or unbounded.

They used approximate designs: $\xi = \{(x_1, \omega_1), (x_2, \omega_2), \ldots\}$, where $\omega_i > 0$, $\sum_i \omega_i = 1$.

They focused on local optimality, and aimed for complete class results under the Loewner ordering.

Thus, with $M_\xi(\theta)$ as the information matrix for θ with design ξ, the aim was to find a small class of designs Ξ so that for any design ξ_0 there is a design $\xi_0^* \in \Xi$ with $M_{\xi_0^*}(\theta) \geq M_{\xi_0}(\theta)$.

Many common optimality criteria adhere to the Loewner ordering.

Such classes always exist (all designs; Carathéodory).

Yang and S, 2009, Annals of Statistics

They considered generalized linear and nonlinear models with two parameters, $\theta = (\theta_1, \theta_2)'$ ...

...and one design variable, x, with values in an interval, possibly half-bounded or unbounded.

They used approximate designs: $\xi = \{(x_1, \omega_1), (x_2, \omega_2), \ldots\}$, where $\omega_i > 0$, $\sum_i \omega_i = 1$.

They focused on local optimality, and aimed for complete class results under the Loewner ordering.

Thus, with $M_\xi(\theta)$ as the information matrix for θ with design ξ, the aim was to find a small class of designs Ξ so that for any design ξ_0 there is a design $\xi_0^* \in \Xi$ with $M_{\xi_0^*}(\theta) \geq M_{\xi_0}(\theta)$.

Many common optimality criteria adhere to the Loewner ordering.

Such classes always exist (all designs; Carathéodory).

Yang and S, 2009, Annals of Statistics

Among others, they showed the following:

Define $c = \theta_1 + \theta_2 x$, and let $c \in [L, U] \ldots$

... then for the logistic or probit link with linear predictor c, a complete class (in terms of c) is formed by designs with at most two support points, where

(i) one of them is U if $U \leq 0$;

(ii) one of them is L if $L \geq 0$;

(iii) the two points are symmetric if $-L = U$;

(iv) the two points are symmetric or L is one of the points if $-L < U$;

(v) the two points are symmetric or U is one of the points if $-L > U$.

Thus, the two points are symmetric, or one of them is the endpoint that is closest to 0.

Yang and S, 2009, Annals of Statistics

Among others, they showed the following:

Define $c = \theta_1 + \theta_2 x$, and let $c \in [L, U] \ldots$

... then for the logistic or probit link with linear predictor c, a complete class (in terms of c) is formed by designs with at most two support points, where

(i) one of them is U if $U \leq 0$;

(ii) one of them is L if $L \geq 0$;

(iii) the two points are symmetric if $-L = U$;

(iv) the two points are symmetric or L is one of the points if $-L < U$;

(v) the two points are symmetric or U is one of the points if $-L > U$.

Thus, the two points are symmetric, or one of them is the endpoint that is closest to 0.

Yang and S, 2009, Annals of Statistics

Among others, they showed the following:

Define $c = \theta_1 + \theta_2 x$, and let $c \in [L, U] \ldots$

... then for the logistic or probit link with linear predictor c, a complete class (in terms of c) is formed by designs with at most two support points, where

(i) one of them is U if $U \leq 0$;

(ii) one of them is L if $L \geq 0$;

(iii) the two points are symmetric if $-L = U$;

(iv) the two points are symmetric or L is one of the points if $-L < U$;

(v) the two points are symmetric or U is one of the points if $-L > U$.

Thus, the two points are symmetric, or one of them is the endpoint that is closest to 0.

S and Yang, 2012, Statistica Sinica

What happens if we add classificatory variables (factors)?

With two factors at two levels each, a design consists of pairs $(x_{i_1 i_2 j}, \omega_{i_1 i_2 j})$, $i_1 = 1, 2$, $i_2 = 1, 2$, $j = 1, 2, \ldots$, where $\omega_{i_1 i_2 j} > 0$ and $\sum_{i_1} \sum_{i_2} \sum_j \omega_{i_1 i_2 j} = 1$.

Could add constraints for the cells formed by the factors, such as $\sum_j \omega_{11j} = .30$, and so on.

We studied the problem without such constraints.

What is a reasonable family of models for problems of this nature?

What are parametric functions that might be of interest?

S and Yang, 2012, Statistica Sinica

What happens if we add classificatory variables (factors)?

With two factors at two levels each, a design consists of pairs $(x_{i_1 i_2 j}, \omega_{i_1 i_2 j})$, $i_1 = 1, 2$, $i_2 = 1, 2$, $j = 1, 2, \ldots$, where $\omega_{i_1 i_2 j} > 0$ and $\sum_{i_1} \sum_{i_2} \sum_j \omega_{i_1 i_2 j} = 1$.

Could add constraints for the cells formed by the factors, such as $\sum_j \omega_{11j} = .30$, and so on.

We studied the problem without such constraints.

What is a reasonable family of models for problems of this nature?

What are parametric functions that might be of interest?

S and Yang, 2012, Statistica Sinica

What happens if we add classificatory variables (factors)?

With two factors at two levels each, a design consists of pairs $(x_{i_1 i_2 j}, \omega_{i_1 i_2 j})$, $i_1 = 1, 2$, $i_2 = 1, 2$, $j = 1, 2, \ldots$, where $\omega_{i_1 i_2 j} > 0$ and $\sum_{i_1} \sum_{i_2} \sum_j \omega_{i_1 i_2 j} = 1$.

Could add constraints for the cells formed by the factors, such as $\sum_j \omega_{11j} = .30$, and so on.

We studied the problem without such constraints.

What is a reasonable family of models for problems of this nature?

What are parametric functions that might be of interest?

S and Yang, 2012, Statistica Sinica

What happens if we add classificatory variables (factors)?

With two factors at two levels each, a design consists of pairs $(x_{i_1 i_2 j}, \omega_{i_1 i_2 j})$, $i_1 = 1, 2$, $i_2 = 1, 2$, $j = 1, 2, \ldots$, where $\omega_{i_1 i_2 j} > 0$ and $\sum_{i_1} \sum_{i_2} \sum_j \omega_{i_1 i_2 j} = 1$.

Could add constraints for the cells formed by the factors, such as $\sum_j \omega_{11j} = .30$, and so on.

We studied the problem without such constraints.

What is a reasonable family of models for problems of this nature?

What are parametric functions that might be of interest?

S and Yang, 2012, Statistica Sinica

What happens if we add classificatory variables (factors)?

With two factors at two levels each, a design consists of pairs $(x_{i_1 i_2 j}, \omega_{i_1 i_2 j})$, $i_1 = 1, 2$, $i_2 = 1, 2$, $j = 1, 2, \ldots$, where $\omega_{i_1 i_2 j} > 0$ and $\sum_{i_1} \sum_{i_2} \sum_j \omega_{i_1 i_2 j} = 1$.

Could add constraints for the cells formed by the factors, such as $\sum_j \omega_{11j} = .30$, and so on.

We studied the problem without such constraints.

What is a reasonable family of models for problems of this nature?

What are parametric functions that might be of interest?

S and Yang, 2012, Statistica Sinica

What happens if we add classificatory variables (factors)?

With two factors at two levels each, a design consists of pairs $(x_{i_1 i_2 j}, \omega_{i_1 i_2 j})$, $i_1 = 1, 2$, $i_2 = 1, 2$, $j = 1, 2, \ldots$, where $\omega_{i_1 i_2 j} > 0$ and $\sum_{i_1} \sum_{i_2} \sum_j \omega_{i_1 i_2 j} = 1$.

Could add constraints for the cells formed by the factors, such as $\sum_j \omega_{11j} = .30$, and so on.

We studied the problem without such constraints.

What is a reasonable family of models for problems of this nature?

What are parametric functions that might be of interest?

S and Yang, 2012, Statistica Sinica

For F factors and s_k levels for the kth factor, there are $s = s_1 \times s_2 \times \ldots \times s_F$ cells.

For an exact design, let Y_{ij} be the jth binary response in the ith cell, and consider

$$Pr(Y_{ij} = 1) = P(\alpha_0 + \alpha_i + \beta x_{ij}),$$

where P is a cumulative distribution function.

Some observations and notation:

* It is an overparametrized model.
* Can use a slope parameter that depends on the cells, β_i.
* Can model the α_i's, e.g. using a main-effects model or main-effects and lower order interactions.
* Will also write $i_1 i_2 \ldots i_F$ instead of i.

S and Yang, 2012, Statistica Sinica

For F factors and s_k levels for the kth factor, there are $s = s_1 \times s_2 \times \ldots \times s_F$ cells.

For an exact design, let Y_{ij} be the jth binary response in the ith cell, and consider

$$Pr(Y_{ij} = 1) = P(\alpha_0 + \alpha_i + \beta x_{ij}),$$

where P is a cumulative distribution function.

Some observations and notation:

* It is an overparametrized model.
* Can use a slope parameter that depends on the cells, β_i.
* Can model the α_i's, e.g. using a main-effects model or main-effects and lower order interactions.
* Will also write $i_1 i_2 \ldots i_F$ instead of i.

S and Yang, 2012, Statistica Sinica

For F factors and s_k levels for the kth factor, there are $s = s_1 \times s_2 \times \ldots \times s_F$ cells.

For an exact design, let Y_{ij} be the jth binary response in the ith cell, and consider

$$Pr(Y_{ij} = 1) = P(\alpha_0 + \alpha_i + \beta x_{ij}),$$

where P is a cumulative distribution function.

Some observations and notation:

* It is an overparametrized model.
* Can use a slope parameter that depends on the cells, β_i.
* Can model the α_i's, e.g. using a main-effects model or main-effects and lower order interactions.
* Will also write $i_1 i_2 \ldots i_F$ instead of i.

S and Yang, 2012, Statistica Sinica

For F factors and s_k levels for the kth factor, there are $s = s_1 \times s_2 \times \ldots \times s_F$ cells.

For an exact design, let Y_{ij} be the jth binary response in the ith cell, and consider

$$Pr(Y_{ij} = 1) = P(\alpha_0 + \alpha_i + \beta x_{ij}),$$

where P is a cumulative distribution function.

Some observations and notation:

* It is an overparametrized model.
* Can use a slope parameter that depends on the cells, β_i.
* Can model the α_i's, e.g. using a main-effects model or main-effects and lower order interactions.
* Will also write $i_1 i_2 \ldots i_F$ instead of i.

S and Yang, 2012, Statistica Sinica

For F factors and s_k levels for the kth factor, there are $s = s_1 \times s_2 \times \ldots \times s_F$ cells.

For an exact design, let Y_{ij} be the jth binary response in the ith cell, and consider

$$Pr(Y_{ij} = 1) = P(\alpha_0 + \alpha_i + \beta x_{ij}),$$

where P is a cumulative distribution function.

Some observations and notation:

∗ It is an overparametrized model.

∗ Can use a slope parameter that depends on the cells, β_i.

∗ Can model the α_i's, e.g. using a main-effects model or main-effects and lower order interactions.

∗ Will also write $i_1 i_2 \ldots i_F$ instead of i.

S and Yang, 2012, Statistica Sinica

For F factors and s_k levels for the kth factor, there are $s = s_1 \times s_2 \times \ldots \times s_F$ cells.

For an exact design, let Y_{ij} be the jth binary response in the ith cell, and consider

$$Pr(Y_{ij} = 1) = P(\alpha_0 + \alpha_i + \beta x_{ij}),$$

where P is a cumulative distribution function.

Some observations and notation:

* It is an overparametrized model.
* Can use a slope parameter that depends on the cells, β_i.
* Can model the α_i's, e.g. using a main-effects model or main-effects and lower order interactions.
* Will also write $i_1 i_2 \ldots i_F$ instead of i.

S and Yang, 2012, Statistica Sinica

Let $\theta = (\alpha_0, \alpha_1, \ldots, \alpha_s, \beta')'$, where β is either a scalar or the vector $(\beta_1, \ldots, \beta_s)'$

Define $c_{ij} = \alpha_0 + \alpha_i + \beta x_{ij}$ or $c_{ij} = \alpha_0 + \alpha_i + \beta_i x_{ij}$, depending on which model is used

For cell i, let $[L_i, U_i]$ be the design space for c_{ij}

Then, for the logistic or probit link, a complete class is formed by all designs with at most two support points per cell, and with additional specifications for each cell as in the model without factors (i.e., in each cell the two points are symmetric or one is equal to one of the endpoints for that cell).

The proof uses a simple additivity property of the information matrix

The result remains true if the model is "simplified" by modeling the α_i's (e.g. using only main-effects)

S and Yang, 2012, Statistica Sinica

Let $\theta = (\alpha_0, \alpha_1, \ldots, \alpha_s, \beta')'$, where β is either a scalar or the vector $(\beta_1, \ldots, \beta_s)'$

Define $c_{ij} = \alpha_0 + \alpha_i + \beta x_{ij}$ or $c_{ij} = \alpha_0 + \alpha_i + \beta_i x_{ij}$, depending on which model is used

For cell i, let $[L_i, U_i]$ be the design space for c_{ij}

Then, for the logistic or probit link, a complete class is formed by all designs with at most two support points per cell, and with additional specifications for each cell as in the model without factors (i.e., in each cell the two points are symmetric or one is equal to one of the endpoints for that cell).

The proof uses a simple additivity property of the information matrix

The result remains true if the model is “simplified” by modeling the α_i's (e.g. using only main-effects)

S and Yang, 2012, Statistica Sinica

Let $\theta = (\alpha_0, \alpha_1, \ldots, \alpha_s, \beta')'$, where β is either a scalar or the vector $(\beta_1, \ldots, \beta_s)'$

Define $c_{ij} = \alpha_0 + \alpha_i + \beta x_{ij}$ or $c_{ij} = \alpha_0 + \alpha_i + \beta_i x_{ij}$, depending on which model is used

For cell i, let $[L_i, U_i]$ be the design space for c_{ij}

Then, for the logistic or probit link, a complete class is formed by all designs with at most two support points per cell, and with additional specifications for each cell as in the model without factors (i.e., in each cell the two points are symmetric or one is equal to one of the endpoints for that cell).

The proof uses a simple additivity property of the information matrix

The result remains true if the model is "simplified" by modeling the α_i's (e.g. using only main-effects)

S and Yang, 2012, Statistica Sinica

Let $\theta = (\alpha_0, \alpha_1, \ldots, \alpha_s, \beta')'$, where β is either a scalar or the vector $(\beta_1, \ldots, \beta_s)'$

Define $c_{ij} = \alpha_0 + \alpha_i + \beta x_{ij}$ or $c_{ij} = \alpha_0 + \alpha_i + \beta_i x_{ij}$, depending on which model is used

For cell i, let $[L_i, U_i]$ be the design space for c_{ij}

Then, for the logistic or probit link, a complete class is formed by all designs with at most two support points per cell, and with additional specifications for each cell as in the model without factors (i.e., in each cell the two points are symmetric or one is equal to one of the endpoints for that cell).

The proof uses a simple additivity property of the information matrix

The result remains true if the model is “simplified” by modeling the α_i's (e.g. using only main-effects)

S and Yang, 2012, Statistica Sinica

Let $\theta = (\alpha_0, \alpha_1, \ldots, \alpha_s, \beta')'$, where β is either a scalar or the vector $(\beta_1, \ldots, \beta_s)'$

Define $c_{ij} = \alpha_0 + \alpha_i + \beta x_{ij}$ or $c_{ij} = \alpha_0 + \alpha_i + \beta_i x_{ij}$, depending on which model is used

For cell i, let $[L_i, U_i]$ be the design space for c_{ij}

Then, for the logistic or probit link, a complete class is formed by all designs with at most two support points per cell, and with additional specifications for each cell as in the model without factors (i.e., in each cell the two points are symmetric or one is equal to one of the endpoints for that cell).

The proof uses a simple additivity property of the information matrix

The result remains true if the model is "simplified" by modeling the α_i's (e.g. using only main-effects)

S and Yang, 2012, Statistica Sinica

Let $\theta = (\alpha_0, \alpha_1, \ldots, \alpha_s, \beta')'$, where β is either a scalar or the vector $(\beta_1, \ldots, \beta_s)'$

Define $c_{ij} = \alpha_0 + \alpha_i + \beta x_{ij}$ or $c_{ij} = \alpha_0 + \alpha_i + \beta_i x_{ij}$, depending on which model is used

For cell i, let $[L_i, U_i]$ be the design space for c_{ij}

Then, for the logistic or probit link, a complete class is formed by all designs with at most two support points per cell, and with additional specifications for each cell as in the model without factors (i.e., in each cell the two points are symmetric or one is equal to one of the endpoints for that cell).

The proof uses a simple additivity property of the information matrix

The result remains true if the model is "simplified" by modeling the α_i's (e.g. using only main-effects)

S and Yang, 2012, Statistica Sinica

With different slope parameters (β_i), will need at least two support points per cell if estimation of β_i's is of interest

If α_i's are not simplified, even with a common slope β, at least one observation per cell is needed if all $\alpha_0 + \alpha_i$'s are of interest

Will focus in the remainder on the common slope model with simplifying assumptions for the α_i's

S and Yang, 2012, Statistica Sinica

With different slope parameters (β_i), will need at least two support points per cell if estimation of β_i's is of interest

If α_i's are not simplified, even with a common slope β, at least one observation per cell is needed if all $\alpha_0 + \alpha_i$'s are of interest

Will focus in the remainder on the common slope model with simplifying assumptions for the α_i's

S and Yang, 2012, Statistica Sinica

With different slope parameters (β_i), will need at least two support points per cell if estimation of β_i's is of interest

If α_i's are not simplified, even with a common slope β, at least one observation per cell is needed if all $\alpha_0 + \alpha_i$'s are of interest

Will focus in the remainder on the common slope model with simplifying assumptions for the α_i's

S and Yang, 2012, Statistica Sinica

For a main-effects model, write $\alpha_{i_1\ldots i_F} = \alpha_1^{i_1} + \ldots + \alpha_F^{i_F}$, let

$$\eta_1 = (\alpha_0+\alpha_1^1+\ldots+\alpha_F^1, \alpha_1^2-\alpha_1^1, \ldots, \alpha_1^{s_1}-\alpha_1^1, \ldots, \alpha_F^2-\alpha_F^1, \ldots, \alpha_F^{s_F}-\alpha_F^1, \beta)',$$

and let η_2 be obtained from η_1 by deleting its first entry.

Without restrictions on design space, D-optimal designs for η_1 and η_2:

$$\xi^* = \{(c_{i1} = c^*, \omega_{i1} = \tfrac{1}{2s}), (c_{i2} = -c^*, \omega_{i2} = \tfrac{1}{2s})\},$$

where c^* maximizes

$$c^2\Psi^{m+2}(c) \text{ and } c^2\Psi^{m+1}(c),$$

respectively, $\Psi(c) = [P(c)']^2/[P(c)(1-P(c)]$, and $m = \sum_{k=1}^F (s_k - 1)$.

Results remain true with restrictions on the design space, provided that c^* and $-c^*$ are in the design space for each cell.

S and Yang, 2012, Statistica Sinica

For a main-effects model, write $\alpha_{i_1 \ldots i_F} = \alpha_1^{i_1} + \ldots + \alpha_F^{i_F}$, let

$$\eta_1 = (\alpha_0 + \alpha_1^1 + \ldots + \alpha_F^1, \alpha_1^2 - \alpha_1^1, \ldots, \alpha_1^{s_1} - \alpha_1^1, \ldots, \alpha_F^2 - \alpha_F^1, \ldots, \alpha_F^{s_F} - \alpha_F^1, \beta)',$$

and let η_2 be obtained from η_1 by deleting its first entry.

Without restrictions on design space, *D*-optimal designs for η_1 and η_2:

$$\xi^* = \{(c_{i1} = c^*, \omega_{i1} = \tfrac{1}{2s}), (c_{i2} = -c^*, \omega_{i2} = \tfrac{1}{2s})\},$$

where c^* maximizes

$$c^2 \Psi^{m+2}(c) \text{ and } c^2 \Psi^{m+1}(c),$$

respectively, $\Psi(c) = [P(c)']^2 / [P(c)(1 - P(c)]$, and $m = \sum_{k=1}^F (s_k - 1)$.

Results remain true with restrictions on the design space, provided that c^* and $-c^*$ are in the design space for each cell.

S and Yang, 2012, Statistica Sinica

For a main-effects model, write $\alpha_{i_1 \ldots i_F} = \alpha_1^{i_1} + \ldots + \alpha_F^{i_F}$, let

$$\eta_1 = (\alpha_0 + \alpha_1^1 + \ldots + \alpha_F^1, \alpha_1^2 - \alpha_1^1, \ldots, \alpha_1^{s_1} - \alpha_1^1, \ldots, \alpha_F^2 - \alpha_F^1, \ldots, \alpha_F^{s_F} - \alpha_F^1, \beta)',$$

and let η_2 be obtained from η_1 by deleting its first entry.

Without restrictions on design space, *D*-optimal designs for η_1 and η_2:

$$\xi^* = \{(c_{i1} = c^*, \omega_{i1} = \frac{1}{2s}), (c_{i2} = -c^*, \omega_{i2} = \frac{1}{2s})\},$$

where c^* maximizes

$$c^2 \Psi^{m+2}(c) \text{ and } c^2 \Psi^{m+1}(c),$$

respectively, $\Psi(c) = [P(c)']^2 / [P(c)(1 - P(c)]$, and $m = \sum_{k=1}^{F} (s_k - 1)$.

Results remain true with restrictions on the design space, provided that c^* and $-c^*$ are in the design space for each cell.

Outline

Tan and S, 2015a

Each cell has two support points; algorithms also gave designs with many support points, both for D-optimality and other criteria.

Modest goal: For a main-effects model, can we reduce the number of support points by using an orthogonal array (OA)?

An $OA(N, s_1 \times \ldots \times s_F, t)$ of strength t is an $N \times F$ array with s_k symbols in the kth column, so that for any t columns, all combinations that can appear as a row in the induced $N \times t$ subarray appear equally often.

If we can select 2 points in each cell corresponding to an $OA(N, s_1 \times \ldots \times s_F, t)$, then the number of support points is $2N$ rather than $2s$; so we want an array with $N < s$.

This idea works for a main-effects model and the logistic or probit links with an OA of strength 2.

Tan and S, 2015a

Each cell has two support points; algorithms also gave designs with many support points, both for *D*-optimality and other criteria.

Modest goal: For a main-effects model, can we reduce the number of support points by using an orthogonal array (OA)?

An $OA(N, s_1 \times \ldots \times s_F, t)$ of strength t is an $N \times F$ array with s_k symbols in the kth column, so that for any t columns, all combinations that can appear as a row in the induced $N \times t$ subarray appear equally often.

If we can select 2 points in each cell corresponding to an $OA(N, s_1 \times \ldots \times s_F, t)$, then the number of support points is $2N$ rather than $2s$; so we want an array with $N < s$.

This idea works for a main-effects model and the logistic or probit links with an OA of strength 2.

Tan and S, 2015a

Each cell has two support points; algorithms also gave designs with many support points, both for D-optimality and other criteria.

Modest goal: For a main-effects model, can we reduce the number of support points by using an orthogonal array (OA)?

An $OA(N, s_1 \times \ldots \times s_F, t)$ of strength t is an $N \times F$ array with s_k symbols in the kth column, so that for any t columns, all combinations that can appear as a row in the induced $N \times t$ subarray appear equally often.

If we can select 2 points in each cell corresponding to an $OA(N, s_1 \times \ldots \times s_F, t)$, then the number of support points is $2N$ rather than $2s$; so we want an array with $N < s$.

This idea works for a main-effects model and the logistic or probit links with an OA of strength 2.

Tan and S, 2015a

Each cell has two support points; algorithms also gave designs with many support points, both for *D*-optimality and other criteria.

Modest goal: For a main-effects model, can we reduce the number of support points by using an orthogonal array (OA)?

An $OA(N, s_1 \times \ldots \times s_F, t)$ of strength t is an $N \times F$ array with s_k symbols in the kth column, so that for any t columns, all combinations that can appear as a row in the induced $N \times t$ subarray appear equally often.

If we can select 2 points in each cell corresponding to an $OA(N, s_1 \times \ldots \times s_F, t)$, then the number of support points is $2N$ rather than $2s$; so we want an array with $N < s$.

This idea works for a main-effects model and the logistic or probit links with an OA of strength 2.

Tan and S, 2015a

Each cell has two support points; algorithms also gave designs with many support points, both for D-optimality and other criteria.

Modest goal: For a main-effects model, can we reduce the number of support points by using an orthogonal array (OA)?

An OA$(N, s_1 \times \ldots \times s_F, t)$ of strength t is an $N \times F$ array with s_k symbols in the kth column, so that for any t columns, all combinations that can appear as a row in the induced $N \times t$ subarray appear equally often.

If we can select 2 points in each cell corresponding to an OA$(N, s_1 \times \ldots \times s_F, t)$, then the number of support points is $2N$ rather than $2s$; so we want an array with $N < s$.

This idea works for a main-effects model and the logistic or probit links with an OA of strength 2.

Tan and S, 2015a

Statement of result: For the logistic or probit link, with a main-effects model and η_1 and η_2 as before, a design that places weight $\frac{1}{2N}$ on each of c^* and $-c^*$, which are defined as before, for the cells that correspond to an $OA(N, s_1 \times \ldots \times s_F, 2)$ is D-optimal.

Outline of proof: It suffices to check that the information matrices for these designs agree with those for the known D-optimal designs.

It turns out that for designs with at most two points in some or all classes, the information matrix for θ depends on 6 expressions: ...

Tan and S, 2015a

Statement of result: For the logistic or probit link, with a main-effects model and η_1 and η_2 as before, a design that places weight $\frac{1}{2N}$ on each of c^* and $-c^*$, which are defined as before, for the cells that correspond to an $OA(N, s_1 \times \ldots \times s_F, 2)$ is D-optimal.

Outline of proof: It suffices to check that the information matrices for these designs agree with those for the known D-optimal designs.

It turns out that for designs with at most two points in some or all classes, the information matrix for θ depends on 6 expressions: ...

Tan and S, 2015a

Statement of result: For the logistic or probit link, with a main-effects model and η_1 and η_2 as before, a design that places weight $\frac{1}{2N}$ on each of c^* and $-c^*$, which are defined as before, for the cells that correspond to an OA$(N, s_1 \times \ldots \times s_F, 2)$ is D-optimal.

Outline of proof: It suffices to check that the information matrices for these designs agree with those for the known D-optimal designs.

It turns out that for designs with at most two points in some or all classes, the information matrix for θ depends on 6 expressions: ...

Tan and S, 2015a

$\sum_i \sum_j \omega_{ij} \Psi(c_{ij})$ over all cells i and $j \leq 2$

$\sum_i \sum_j \omega_{ij} \Psi(c_{ij})$ over all cells i with factor k at level i_k and $j \leq 2$

$\sum_i \sum_j \omega_{ij} \Psi(c_{ij})$ over all cells i with factors $k \neq k'$ at levels i_k and $i_{k'}$ and $j \leq 2$

$\sum_i \sum_j \omega_{ij} c_{ij} \Psi(c_{ij})$ over all cells i and $j \leq 2$

$\sum_i \sum_j \omega_{ij} c_{ij} \Psi(c_{ij})$ over all cells i with factor k at level i_k and $j \leq 2$

$\sum_i \sum_j \omega_{ij} c_{ij}^2 \Psi(c_{ij})$ over all cells i and $j \leq 2$

Using that Ψ is an even function, it can be shown that the OA-based designs give the same values for these expressions as the D-optimal designs that use all cells.

Tan and S, 2015a

$\sum_i \sum_j \omega_{ij}\Psi(c_{ij})$ over all cells i and $j \leq 2$

$\sum_i \sum_j \omega_{ij}\Psi(c_{ij})$ over all cells i with factor k at level i_k and $j \leq 2$

$\sum_i \sum_j \omega_{ij}\Psi(c_{ij})$ over all cells i with factors $k \neq k'$ at levels i_k and $i_{k'}$ and $j \leq 2$

$\sum_i \sum_j \omega_{ij}c_{ij}\Psi(c_{ij})$ over all cells i and $j \leq 2$

$\sum_i \sum_j \omega_{ij}c_{ij}\Psi(c_{ij})$ over all cells i with factor k at level i_k and $j \leq 2$

$\sum_i \sum_j \omega_{ij}c_{ij}^2\Psi(c_{ij})$ over all cells i and $j \leq 2$

Using that Ψ is an even function, it can be shown that the OA-based designs give the same values for these expressions as the *D*-optimal designs that use all cells.

Tan and S, 2015a

Some benefits of the OA-based designs . . .

. . . simpler because of smaller support size ($2N$ points vs $2s$ points)

. . . correspond to more exact designs (weights $\frac{1}{2N}$ vs $\frac{1}{2s}$)

. . . can exist with more restrictive design space (c^* and $-c^*$ only need to be in the design space for the cells corresponding to the OA)

But we can often do better . . .

Tan and S, 2015a

Some benefits of the OA-based designs …

… simpler because of smaller support size ($2N$ points vs $2s$ points)

… correspond to more exact designs (weights $\frac{1}{2N}$ vs $\frac{1}{2s}$)

… can exist with more restrictive design space (c^* and $-c^*$ only need to be in the design space for the cells corresponding to the OA)

But we can often do better …

Tan and S, 2015a

Some benefits of the OA-based designs ...

...simpler because of smaller support size ($2N$ points vs $2s$ points)

...correspond to more exact designs (weights $\frac{1}{2N}$ vs $\frac{1}{2s}$)

...can exist with more restrictive design space (c^* and $-c^*$ only need to be in the design space for the cells corresponding to the OA)

But we can often do better ...

Tan and S, 2015a

Some benefits of the OA-based designs ...

...simpler because of smaller support size ($2N$ points vs $2s$ points)

...correspond to more exact designs (weights $\frac{1}{2N}$ vs $\frac{1}{2s}$)

...can exist with more restrictive design space (c^* and $-c^*$ only need to be in the design space for the cells corresponding to the OA)

But we can often do better ...

Tan and S, 2015a

Some benefits of the OA-based designs ...

... simpler because of smaller support size ($2N$ points vs $2s$ points)

... correspond to more exact designs (weights $\frac{1}{2N}$ vs $\frac{1}{2s}$)

... can exist with more restrictive design space (c^* and $-c^*$ only need to be in the design space for the cells corresponding to the OA)

But we can often do better ...

Tan and S, 2015a

Still assuming the main-effects model, if there is an $OA(N, s_1 \times \ldots \times s_F \times 2, 2)$, then there are designs that use only N support points, each with weight $\frac{1}{N}$, that have the same information matrix for θ as the D-optimal designs.

How to obtain such designs? Use c^* in the $N/2$ cells corresponding to the first F columns for which the last factor is at level 1, say; use $-c^*$ for the other $N/2$ cells.

Now only a single point needs to be in the design region for each of the N cells corresponding to the first F columns of the OA.

Tan and S, 2015a

Still assuming the main-effects model, if there is an $OA(N, s_1 \times \ldots \times s_F \times 2, 2)$, then there are designs that use only N support points, each with weight $\frac{1}{N}$, that have the same information matrix for θ as the D-optimal designs.

How to obtain such designs? Use c^* in the $N/2$ cells corresponding to the first F columns for which the last factor is at level 1, say; use $-c^*$ for the other $N/2$ cells.

Now only a single point needs to be in the design region for each of the N cells corresponding to the first F columns of the OA.

Tan and S, 2015a

Still assuming the main-effects model, if there is an $OA(N, s_1 \times \ldots \times s_F \times 2, 2)$, then there are designs that use only N support points, each with weight $\frac{1}{N}$, that have the same information matrix for θ as the D-optimal designs.

How to obtain such designs? Use c^* in the $N/2$ cells corresponding to the first F columns for which the last factor is at level 1, say; use $-c^*$ for the other $N/2$ cells.

Now only a single point needs to be in the design region for each of the N cells corresponding to the first F columns of the OA.

Tan and S, 2015a

Example: Let $F = s_1 = s_2 = 2$, and consider the following $OA(4, 2 \times 2 \times 2, 2)$:

1	1	1
1	2	2
2	1	2
2	2	1

The first two columns denote the cells, while the last column is used to select c^* or $-c^*$:

1	1	c^*
1	2	$-c^*$
2	1	$-c^*$
2	2	c^*

In this toy example, we still need to use all 4 cells (in general this number is reduced), but only a single support point in each cell.

Tan and S, 2015a

Example: Let $F = s_1 = s_2 = 2$, and consider the following $OA(4, 2 \times 2 \times 2, 2)$:

1	1	1
1	2	2
2	1	2
2	2	1

The first two columns denote the cells, while the last column is used to select c^* or $-c^*$:

1	1	c^*
1	2	$-c^*$
2	1	$-c^*$
2	2	c^*

In this toy example, we still need to use all 4 cells (in general this number is reduced), but only a single support point in each cell.

Tan and S, 2015a

Example: Let $F = s_1 = s_2 = 2$, and consider the following $OA(4, 2 \times 2 \times 2, 2)$:

$$\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{array}$$

The first two columns denote the cells, while the last column is used to select c^* or $-c^*$:

$$\begin{array}{ccc} 1 & 1 & c^* \\ 1 & 2 & -c^* \\ 2 & 1 & -c^* \\ 2 & 2 & c^* \end{array}$$

In this toy example, we still need to use all 4 cells (in general this number is reduced), but only a single support point in each cell.

Tan and S, 2015a

If it is not possible to add a 2-symbol column, but a column with an odd number of symbols can be added, then a similar result is available . . .

. . . if there is an OA($N, s_1 \times \ldots \times s_F \times (2u + 1), 2$), then there is a design that matches the information matrix and that has $(2u + 2)N/(2u + 1)$ support points; it has 2 support points in $N/(2u + 1)$ cells and 1 support point in the other $N - N/(2u + 1)$ cells

Do not take it to be uniform on its support: For cells with 2 points, use the weight $\frac{1}{2N}$, but for cells with 1 point use $\frac{1}{N}$ . . .

. . . thus the total weight for points in each of the N cells is $\frac{1}{N}$

Can show that a single point per cell can be used if this OA has additional structure (e.g. if it is of strength 3)

Tan and S, 2015a

If it is not possible to add a 2-symbol column, but a column with an odd number of symbols can be added, then a similar result is available ...

... if there is an OA$(N, s_1 \times \ldots \times s_F \times (2u+1), 2)$, then there is a design that matches the information matrix and that has $(2u+2)N/(2u+1)$ support points; it has 2 support points in $N/(2u+1)$ cells and 1 support point in the other $N - N/(2u+1)$ cells

Do not take it to be uniform on its support: For cells with 2 points, use the weight $\frac{1}{2N}$, but for cells with 1 point use $\frac{1}{N}$ ...

... thus the total weight for points in each of the N cells is $\frac{1}{N}$

Can show that a single point per cell can be used if this OA has additional structure (e.g. if it is of strength 3)

Tan and S, 2015a

If it is not possible to add a 2-symbol column, but a column with an odd number of symbols can be added, then a similar result is available ...

... if there is an OA$(N, s_1 \times \ldots \times s_F \times (2u+1), 2)$, then there is a design that matches the information matrix and that has $(2u+2)N/(2u+1)$ support points; it has 2 support points in $N/(2u+1)$ cells and 1 support point in the other $N - N/(2u+1)$ cells

Do not take it to be uniform on its support: For cells with 2 points, use the weight $\frac{1}{2N}$, but for cells with 1 point use $\frac{1}{N}$ ...

... thus the total weight for points in each of the N cells is $\frac{1}{N}$

Can show that a single point per cell can be used if this OA has additional structure (e.g. if it is of strength 3)

Tan and S, 2015a

If it is not possible to add a 2-symbol column, but a column with an odd number of symbols can be added, then a similar result is available ...

... if there is an OA($N, s_1 \times \ldots \times s_F \times (2u+1), 2$), then there is a design that matches the information matrix and that has $(2u+2)N/(2u+1)$ support points; it has 2 support points in $N/(2u+1)$ cells and 1 support point in the other $N - N/(2u+1)$ cells

Do not take it to be uniform on its support: For cells with 2 points, use the weight $\frac{1}{2N}$, but for cells with 1 point use $\frac{1}{N}$ ...

... thus the total weight for points in each of the N cells is $\frac{1}{N}$

Can show that a single point per cell can be used if this OA has additional structure (e.g. if it is of strength 3)

Tan and S, 2015a

If it is not possible to add a 2-symbol column, but a column with an odd number of symbols can be added, then a similar result is available . . .

. . . if there is an $OA(N, s_1 \times \ldots \times s_F \times (2u + 1), 2)$, then there is a design that matches the information matrix and that has $(2u + 2)N/(2u + 1)$ support points; it has 2 support points in $N/(2u + 1)$ cells and 1 support point in the other $N - N/(2u + 1)$ cells

Do not take it to be uniform on its support: For cells with 2 points, use the weight $\frac{1}{2N}$, but for cells with 1 point use $\frac{1}{N}$ . . .

. . . thus the total weight for points in each of the N cells is $\frac{1}{N}$

Can show that a single point per cell can be used if this OA has additional structure (e.g. if it is of strength 3)

Outline

Tan and S, 2015b

What if the α_i's in the linear predictor are modeled using main-effects and interactions?

Will keep it simple: add all two-factor interactions, or all two- and three-factor interactions, and so on; still using the logistic or probit link.

Consider finding a *D*-optimal design for $\eta = B\theta$, a "maximal" vector of estimable functions.

Unlike the main-effects model, S and Yang (2012) do not present a *D*-optimal design for this problem.

Information matrix becomes very complicated though, and we were not able to maximize the determinant of the information matrix for η directly.

But their complete class result still holds; so we can focus on designs with no more than 2 support points per cell.

Tan and S, 2015b

What if the α_i's in the linear predictor are modeled using main-effects and interactions?

Will keep it simple: add all two-factor interactions, or all two- and three-factor interactions, and so on; still using the logistic or probit link.

Consider finding a *D*-optimal design for $\eta = B\theta$, a "maximal" vector of estimable functions.

Unlike the main-effects model, S and Yang (2012) do not present a *D*-optimal design for this problem.

Information matrix becomes very complicated though, and we were not able to maximize the determinant of the information matrix for η directly.

But their complete class result still holds; so we can focus on designs with no more than 2 support points per cell.

Tan and S, 2015b

What if the α_i's in the linear predictor are modeled using main-effects and interactions?

Will keep it simple: add all two-factor interactions, or all two- and three-factor interactions, and so on; still using the logistic or probit link.

Consider finding a *D*-optimal design for $\eta = B\theta$, a "maximal" vector of estimable functions.

Unlike the main-effects model, S and Yang (2012) do not present a *D*-optimal design for this problem.

Information matrix becomes very complicated though, and we were not able to maximize the determinant of the information matrix for η directly.

But their complete class result still holds; so we can focus on designs with no more than 2 support points per cell.

Tan and S, 2015b

What if the α_i's in the linear predictor are modeled using main-effects and interactions?

Will keep it simple: add all two-factor interactions, or all two- and three-factor interactions, and so on; still using the logistic or probit link.

Consider finding a *D*-optimal design for $\eta = B\theta$, a "maximal" vector of estimable functions.

Unlike the main-effects model, S and Yang (2012) do not present a *D*-optimal design for this problem.

Information matrix becomes very complicated though, and we were not able to maximize the determinant of the information matrix for η directly.

But their complete class result still holds; so we can focus on designs with no more than 2 support points per cell.

Tan and S, 2015b

What if the α_i's in the linear predictor are modeled using main-effects and interactions?

Will keep it simple: add all two-factor interactions, or all two- and three-factor interactions, and so on; still using the logistic or probit link.

Consider finding a *D*-optimal design for $\eta = B\theta$, a "maximal" vector of estimable functions.

Unlike the main-effects model, S and Yang (2012) do not present a *D*-optimal design for this problem.

Information matrix becomes very complicated though, and we were not able to maximize the determinant of the information matrix for η directly.

But their complete class result still holds; so we can focus on designs with no more than 2 support points per cell.

Tan and S, 2015b

What if the α_i's in the linear predictor are modeled using main-effects and interactions?

Will keep it simple: add all two-factor interactions, or all two- and three-factor interactions, and so on; still using the logistic or probit link.

Consider finding a D-optimal design for $\eta = B\theta$, a "maximal" vector of estimable functions.

Unlike the main-effects model, S and Yang (2012) do not present a D-optimal design for this problem.

Information matrix becomes very complicated though, and we were not able to maximize the determinant of the information matrix for η directly.

But their complete class result still holds; so we can focus on designs with no more than 2 support points per cell.

Tan and S, 2015b

Guessed the form for a *D*-optimal design, using all cells.

Theoretically verified that the guess is correct using the generalized equivalence theorem.

Conclusion: *D*-optimal designs have the same form as for the main-effects model, but now with c^* as the maximizer of $c^2\Psi^r(c)$, where $r = \text{rank}(B)$.

Requires again that c^* and $-c^*$ are in the design region for every cell.

And as for the main-effects model, orthogonal arrays (of high enough strength) can be used to reduce the number of support points ...

Tan and S, 2015b

Guessed the form for a *D*-optimal design, using all cells.

Theoretically verified that the guess is correct using the generalized equivalence theorem.

Conclusion: *D*-optimal designs have the same form as for the main-effects model, but now with c^* as the maximizer of $c^2\Psi^r(c)$, where $r = \text{rank}(B)$.

Requires again that c^* and $-c^*$ are in the design region for every cell.

And as for the main-effects model, orthogonal arrays (of high enough strength) can be used to reduce the number of support points ...

Tan and S, 2015b

Guessed the form for a *D*-optimal design, using all cells.

Theoretically verified that the guess is correct using the generalized equivalence theorem.

Conclusion: *D*-optimal designs have the same form as for the main-effects model, but now with c^* as the maximizer of $c^2\Psi^r(c)$, where $r = \text{rank}(B)$.

Requires again that c^* and $-c^*$ are in the design region for every cell.

And as for the main-effects model, orthogonal arrays (of high enough strength) can be used to reduce the number of support points . . .

Tan and S, 2015b

Guessed the form for a D-optimal design, using all cells.

Theoretically verified that the guess is correct using the generalized equivalence theorem.

Conclusion: D-optimal designs have the same form as for the main-effects model, but now with c^* as the maximizer of $c^2\Psi^r(c)$, where $r = \text{rank}(B)$.

Requires again that c^* and $-c^*$ are in the design region for every cell.

And as for the main-effects model, orthogonal arrays (of high enough strength) can be used to reduce the number of support points . . .

Tan and S, 2015b

Guessed the form for a *D*-optimal design, using all cells.

Theoretically verified that the guess is correct using the generalized equivalence theorem.

Conclusion: *D*-optimal designs have the same form as for the main-effects model, but now with c^* as the maximizer of $c^2\Psi^r(c)$, where $r = \text{rank}(B)$.

Requires again that c^* and $-c^*$ are in the design region for every cell.

And as for the main-effects model, orthogonal arrays (of high enough strength) can be used to reduce the number of support points ...

Tan and S, 2015b

...with interactions of up to and including T factors, a uniform design that only uses support points in the cells corresponding to an OA$(N, s_1 \times \ldots \times s_F, 2T)$ is D-optimal for η.

If such an OA exists with an additional 2-symbol column, then a D-optimal design can be obtained that uses only a singly support point in each of the cells corresponding to the rows of the OA.

If only a $(2u + 1)$-symbol column can be added to the OA, then a D-optimal design can be obtained that uses a single support point in most cells corresponding to rows in the OA, but that uses two support points for $N/(2u + 1)$ cells.

Tan and S, 2015b

...with interactions of up to and including T factors, a uniform design that only uses support points in the cells corresponding to an $OA(N, s_1 \times \ldots \times s_F, 2T)$ is D-optimal for η.

If such an OA exists with an additional 2-symbol column, then a D-optimal design can be obtained that uses only a singly support point in each of the cells corresponding to the rows of the OA.

If only a $(2u+1)$-symbol column can be added to the OA, then a D-optimal design can be obtained that uses a single support point in most cells corresponding to rows in the OA, but that uses two support points for $N/(2u+1)$ cells.

Tan and S, 2015b

...with interactions of up to and including T factors, a uniform design that only uses support points in the cells corresponding to an OA$(N, s_1 \times \ldots \times s_F, 2T)$ is D-optimal for η.

If such an OA exists with an additional 2-symbol column, then a D-optimal design can be obtained that uses only a singly support point in each of the cells corresponding to the rows of the OA.

If only a $(2u + 1)$-symbol column can be added to the OA, then a D-optimal design can be obtained that uses a single support point in most cells corresponding to rows in the OA, but that uses two support points for $N/(2u + 1)$ cells.

Outline

Summary

OA-based designs can help to reduce the number of support points, correspond to larger families of exact designs, and reduce requirements on the design space for the existence of *D*-optimal designs.

Algorithms seem to have a hard time to find optimal designs, especially "small" designs, for these types of problems.

We focused on *D*-optimality . . .

. . . with no constraints on the cells.

Models tend to have a large number of parameters, which reduces the appeal of using local optimality . . .

. . . but other approaches also face challenges.

Summary

OA-based designs can help to reduce the number of support points, correspond to larger families of exact designs, and reduce requirements on the design space for the existence of *D*-optimal designs.

Algorithms seem to have a hard time to find optimal designs, especially “small” designs, for these types of problems.

We focused on *D*-optimality …

… with no constraints on the cells.

Models tend to have a large number of parameters, which reduces the appeal of using local optimality …

… but other approaches also face challenges.

Summary

OA-based designs can help to reduce the number of support points, correspond to larger families of exact designs, and reduce requirements on the design space for the existence of D-optimal designs.

Algorithms seem to have a hard time to find optimal designs, especially "small" designs, for these types of problems.

We focused on D-optimality ...

...with no constraints on the cells.

Models tend to have a large number of parameters, which reduces the appeal of using local optimality ...

...but other approaches also face challenges.

Summary

OA-based designs can help to reduce the number of support points, correspond to larger families of exact designs, and reduce requirements on the design space for the existence of D-optimal designs.

Algorithms seem to have a hard time to find optimal designs, especially "small" designs, for these types of problems.

We focused on D-optimality ...

... with no constraints on the cells.

Models tend to have a large number of parameters, which reduces the appeal of using local optimality ...

... but other approaches also face challenges.

Summary

OA-based designs can help to reduce the number of support points, correspond to larger families of exact designs, and reduce requirements on the design space for the existence of *D*-optimal designs.

Algorithms seem to have a hard time to find optimal designs, especially "small" designs, for these types of problems.

We focused on *D*-optimality . . .

. . . with no constraints on the cells.

Models tend to have a large number of parameters, which reduces the appeal of using local optimality . . .

. . . but other approaches also face challenges.

Summary

OA-based designs can help to reduce the number of support points, correspond to larger families of exact designs, and reduce requirements on the design space for the existence of *D*-optimal designs.

Algorithms seem to have a hard time to find optimal designs, especially "small" designs, for these types of problems.

We focused on *D*-optimality ...

...with no constraints on the cells.

Models tend to have a large number of parameters, which reduces the appeal of using local optimality ...

...but other approaches also face challenges.