$t \gg 1$

1601.05957, PLD, Majumdar,Schehr
1703.03310, Sasarov, Meerson, Prolhac } $\Phi_-(z)$ via Painleve


Alexandre Krajenbrink

1802.08618, AK,PLD 1808.07710 AK,PLD, Prolhac via Cumulants

1803.05887, AK,PLD,Corwin, Ghosal, Tsai via CG

1809.03410, Tsai via SAO, rigorous

1811.00509, AK,PLD 4 methods (CG,SAO,Painleve,cumulants)

$$L = t \sum_i \phi(u + t^{-\frac{2}{3}} a_i)$$

1802.03273, 1810.07129, Corwin, Ghosal $|H|^{5/2}$ rigorous

$t \ll 1$

1603.03302, PLD, Majumdar,Schehr, Rosso droplet

1705.04654, 1804.08800 AK,PLD Brownian, 1/2 space

1808.07710 AK,PLD, Prolhac expansion up to t^3 + large t guess

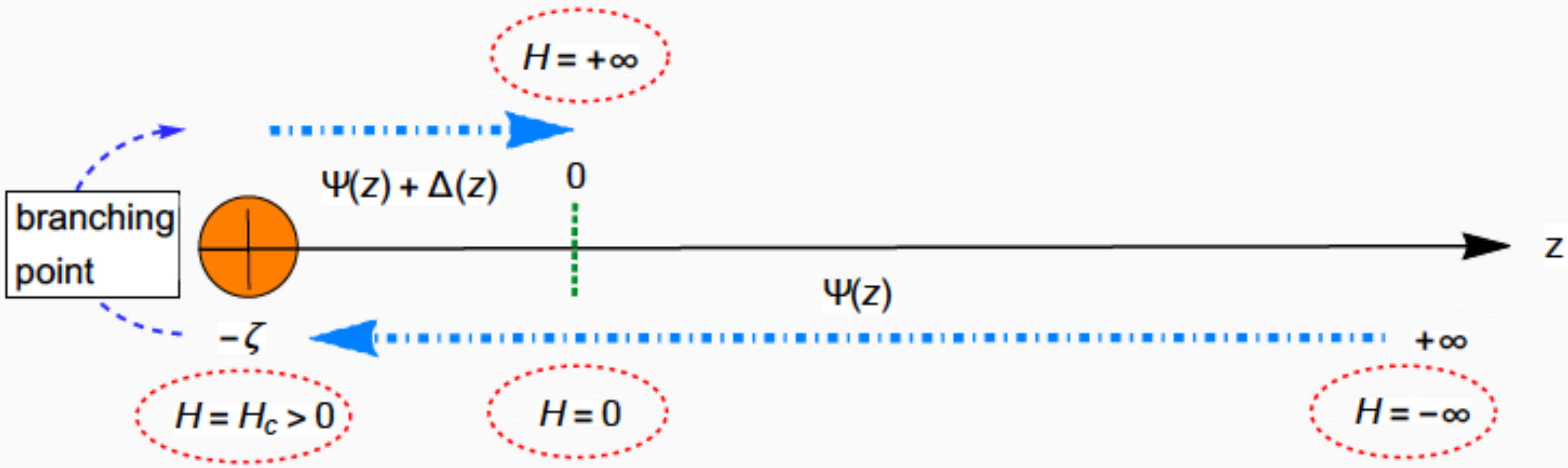
Optimal fluct th. WNT

Korshunov et al. (2007)

Meerson et al. 1512.04910+…

Numerics 1802.02106 Hartmann et al.

Analytic continuation



The branching point is the one of $\Psi(z)$

Exact short-time height distribution in 1D KPZ equation and edge fermions at high temperature

PLD, S. Majumdar, A. Rosso, G. Schehr, Phys. Rev. Lett. 117 070403 (2016).

The rate function of the droplet IC is given by

- For $H \leq H_c = \log \zeta(\frac{3}{2})$

$$\Phi(H) = -\frac{1}{\sqrt{4\pi}} \min_{z \in [-1,+\infty[} [z e^H + \mathrm{Li}_{5/2}(-z)]$$

- for $H \geq H_c$

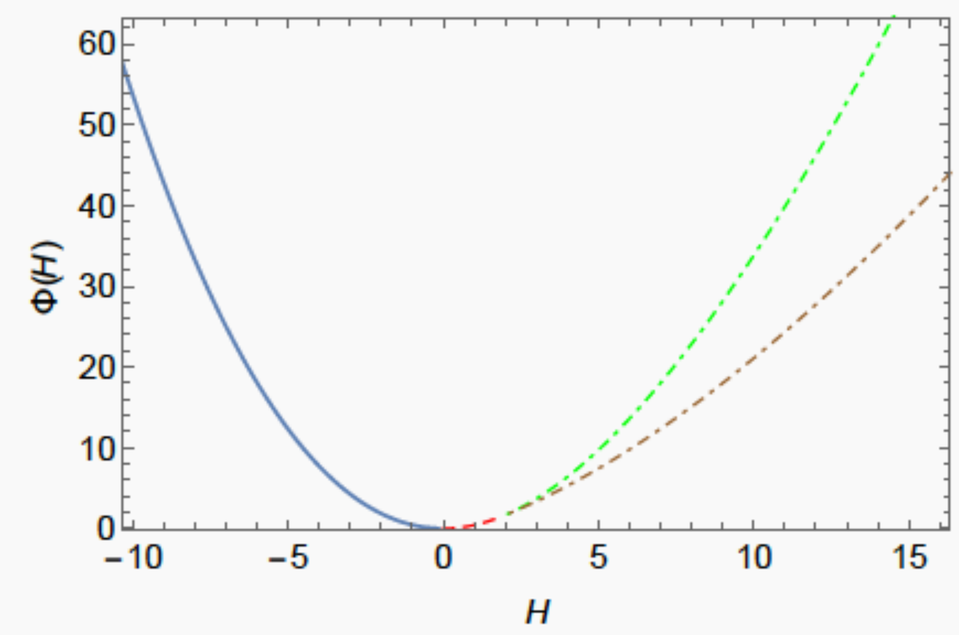
$$\Phi(H) = -\frac{1}{\sqrt{4\pi}} \min_{z \in [-1,0[} [z e^H + \mathrm{Li}_{5/2}(-z) - \frac{8\sqrt{\pi}}{3}[-\log(-z)]^{3/2}]$$

$\Phi(H)$ is analytic, the left tail is $\Phi(H) \simeq_{H\to-\infty} \frac{4}{15\pi}|H|^{5/2}$ and the right tail is $\Phi(H) \simeq_{H\to+\infty} \frac{4}{3} H^{3/2}$.

Exact short-time height distribution for the Brownian IC

A. Krajenbrink, PLD, Phys. Rev. E 96, 020102 (2017)

$$P(H,t) \sim \exp\left(-\frac{\Phi(H)}{\sqrt{t}}\right)$$



Singularity and dynamical phase transition

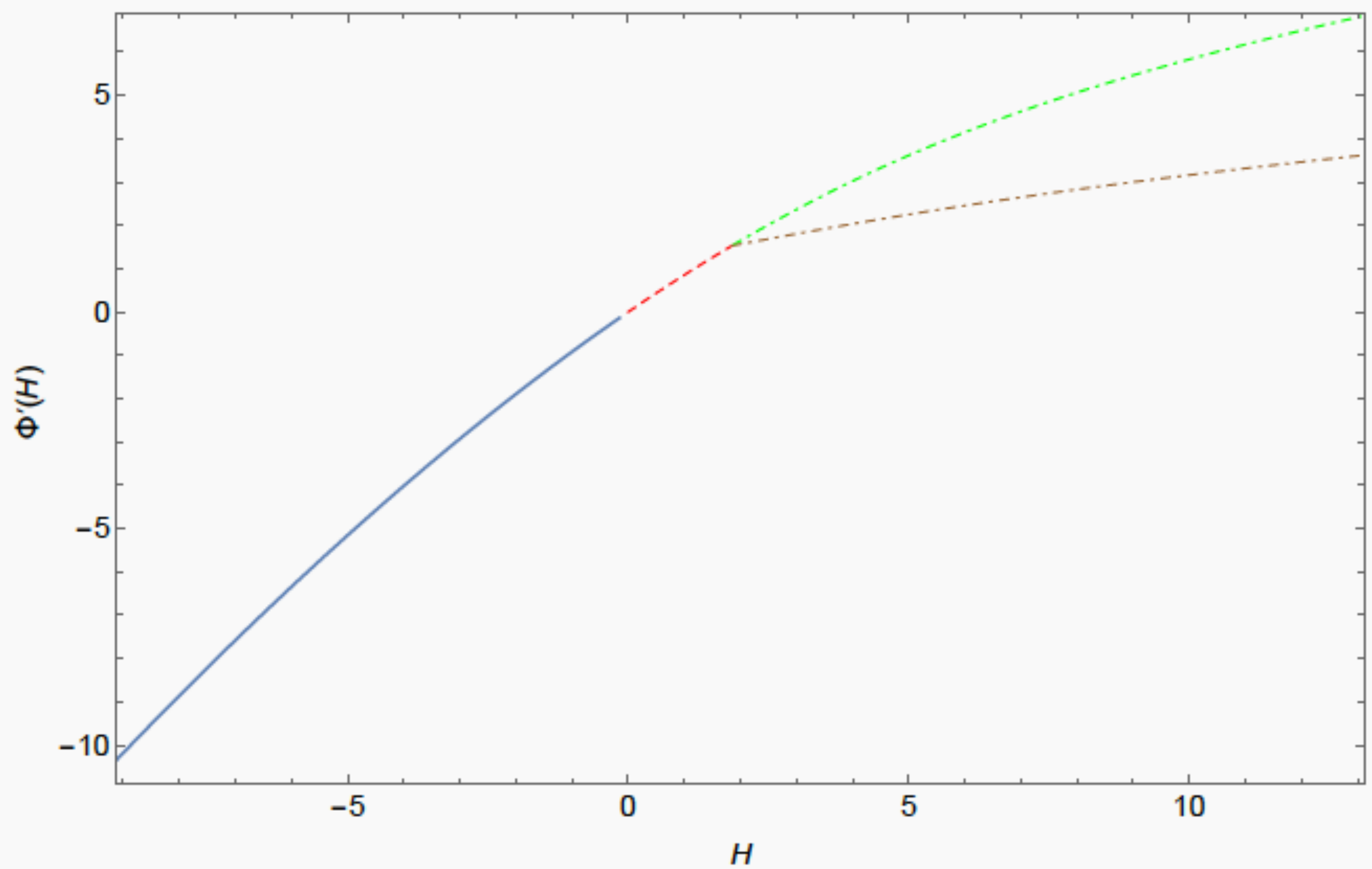


Figure: The function $\Phi'(H)$. The blue line corresponds to the $H < 0$ solution, the red line to the first continuation for $0 < H < H_c$, the green line to the analytic branch $H_c < H$ and the brown line to the non-analytic branch for $H_c < H$. Note the singularity for the brown line.

z is in $I_1 = [0, +\infty],\ I_2 = [0, e^{-1}],\ I_3 =]0, e^{-1}]$

H is in $J_1 = [-\infty, H_c(0)],\ J_2 = [H_c(0), H_{c2}(0)],\ J_3 = [H_{c2}(0), +\infty]$

$$H_c(0) = 0$$

$$H_{c2}(0) = 2\ln(2e - \Psi_0'(e^{-1})) - 1$$

$$\simeq 1.85316$$

z is in $I_1 = [0, +\infty],\ I_2 = [0, e^{-1}],\ I_3 =]0, e^{-1}]$

H is in $J_1 = [-\infty, H_c(0)],\ J_2 = [H_c(0), H_{c2}(0)],\ J_3 = [H_{c2}(0), +\infty]$

relation between H and z in these intervals

$$H_c(0) = 0$$

$$H_{c2}(0) = 2\ln(2e - \Psi_0'(e^{-1})) - 1$$

$$\simeq 1.85316$$

$$e^H = z\Psi'(z)^2 \quad \text{for} \quad z \in I_1 \text{ and } H \in J_1$$

$$e^H = z[\Psi'(z) + \Delta_0'(z)]^2 \quad \text{for } z \in I_2 \text{ and } H \in J_2$$

For $z \in I_3$ and $H \in J_3$ there are two distinct relations

$$e^H = z[\Psi'(z) + \Delta'_{-1}(z)]^2 \qquad (analytic)$$

$$e^H = z[\Psi'(z) + \frac{\Delta'_{-1}(z) + \Delta'_0(z)}{2}]^2 \ (non\ analytic)$$

$$\Delta_0(z) = \frac{4}{3}[\tilde{w}^2 - W_0(-ze^{\tilde{w}^2})]^{\frac{3}{2}} - 4[\tilde{w}^2 - W_0(-ze^{\tilde{w}^2})]^{\frac{1}{2}} + 2\tilde{w}\ \ln(\frac{\tilde{w} + [\tilde{w}^2 - W_0(-ze^{\tilde{w}^2})]^{\frac{1}{2}}}{|\tilde{w} - [\tilde{w}^2 - W_0(-ze^{\tilde{w}^2})]^{\frac{1}{2}}|})$$

$$\Delta_{-1}(z) = \frac{4}{3}[\tilde{w}^2 - W_{-1}(-ze^{\tilde{w}^2})]^{\frac{3}{2}} - 4[\tilde{w}^2 - W_{-1}(-ze^{\tilde{w}^2})]^{\frac{1}{2}} + 2\tilde{w}\ \ln(\frac{\tilde{w} + [\tilde{w}^2 - W_{-1}(-ze^{\tilde{w}^2})]^{\frac{1}{2}}}{|\tilde{w} - [\tilde{w}^2 - W_{-1}(-ze^{\tilde{w}^2})]^{\frac{1}{2}}|}) \qquad (27)$$

z is in $I_1 = [0, +\infty],\ I_2 = [0, e^{-1}],\ I_3 =]0, e^{-1}]$

H is in $J_1 = [-\infty, H_c(0)],\ J_2 = [H_c(0), H_{c2}(0)],\ J_3 = [H_{c2}(0), +\infty]$

relation between H and z in these intervals

$$H_c(0) = 0$$

$$H_{c2}(0) = 2\ln(2e - \Psi_0'(e^{-1})) - 1 \simeq 1.85316$$

$$e^H = z\Psi'(z)^2 \quad \text{for} \quad z \in I_1 \text{ and } H \in J_1$$

$$e^H = z[\Psi'(z) + \Delta_0'(z)]^2 \quad \text{for } z \in I_2 \text{ and } H \in J_2$$

For $z \in I_3$ and $H \in J_3$ there are two distinct relations

$$e^H = z[\Psi'(z) + \Delta_{-1}'(z)]^2 \qquad (analytic)$$

$$e^H = z[\Psi'(z) + \frac{\Delta_{-1}'(z) + \Delta_0'(z)}{2}]^2 \ (non\ analytic)$$

$$\Delta_0(z) = \frac{4}{3}[\tilde{w}^2 - W_0(-ze^{\tilde{w}^2})]^{\frac{3}{2}} - 4[\tilde{w}^2 - W_0(-ze^{\tilde{w}^2})]^{\frac{1}{2}} + 2\tilde{w}\ln\left(\frac{\tilde{w} + [\tilde{w}^2 - W_0(-ze^{\tilde{w}^2})]^{\frac{1}{2}}}{|\tilde{w} - [\tilde{w}^2 - W_0(-ze^{\tilde{w}^2})]^{\frac{1}{2}}|}\right)$$

relation between $\Phi(H)$ and z

$$\Delta_{-1}(z) = \frac{4}{3}[\tilde{w}^2 - W_{-1}(-ze^{\tilde{w}^2})]^{\frac{3}{2}} - 4[\tilde{w}^2 - W_{-1}(-ze^{\tilde{w}^2})]^{\frac{1}{2}} + 2\tilde{w}\ln\left(\frac{\tilde{w} + [\tilde{w}^2 - W_{-1}(-ze^{\tilde{w}^2})]^{\frac{1}{2}}}{|\tilde{w} - [\tilde{w}^2 - W_{-1}(-ze^{\tilde{w}^2})]^{\frac{1}{2}}|}\right) \qquad (27)$$

$$\Phi(H) = \Psi(z) - 2z\Psi'(z) \quad \text{for } z \in I_1$$

$$\Phi(H) = \Psi(z) - 2z\Psi'(z) + \frac{4}{3}[-W_0(-z)]^{\frac{3}{2}} \text{ for } z \in I_2$$

For $z \in I_3$ there exist two branches for $\Phi(H)$

$$\Phi(H) = \Psi(z) - 2z\Psi'(z) + \frac{4}{3}[-W_{-1}(-z)]^{\frac{3}{2}} \quad (analytic)$$

$$\Phi(H) = \Psi(z) - 2z\Psi'(z) + \frac{2}{3}[-W_0(-z)]^{\frac{3}{2}} + \frac{2}{3}[-W_{-1}(-z)]^{\frac{3}{2}}$$

A. K. Hartmann, P. Le Doussal, S. N. Majumdar, A. Rosso, G. Schehr. "High-precision simulation of the height distribution for the KPZ equation" arXiv:1802.02106

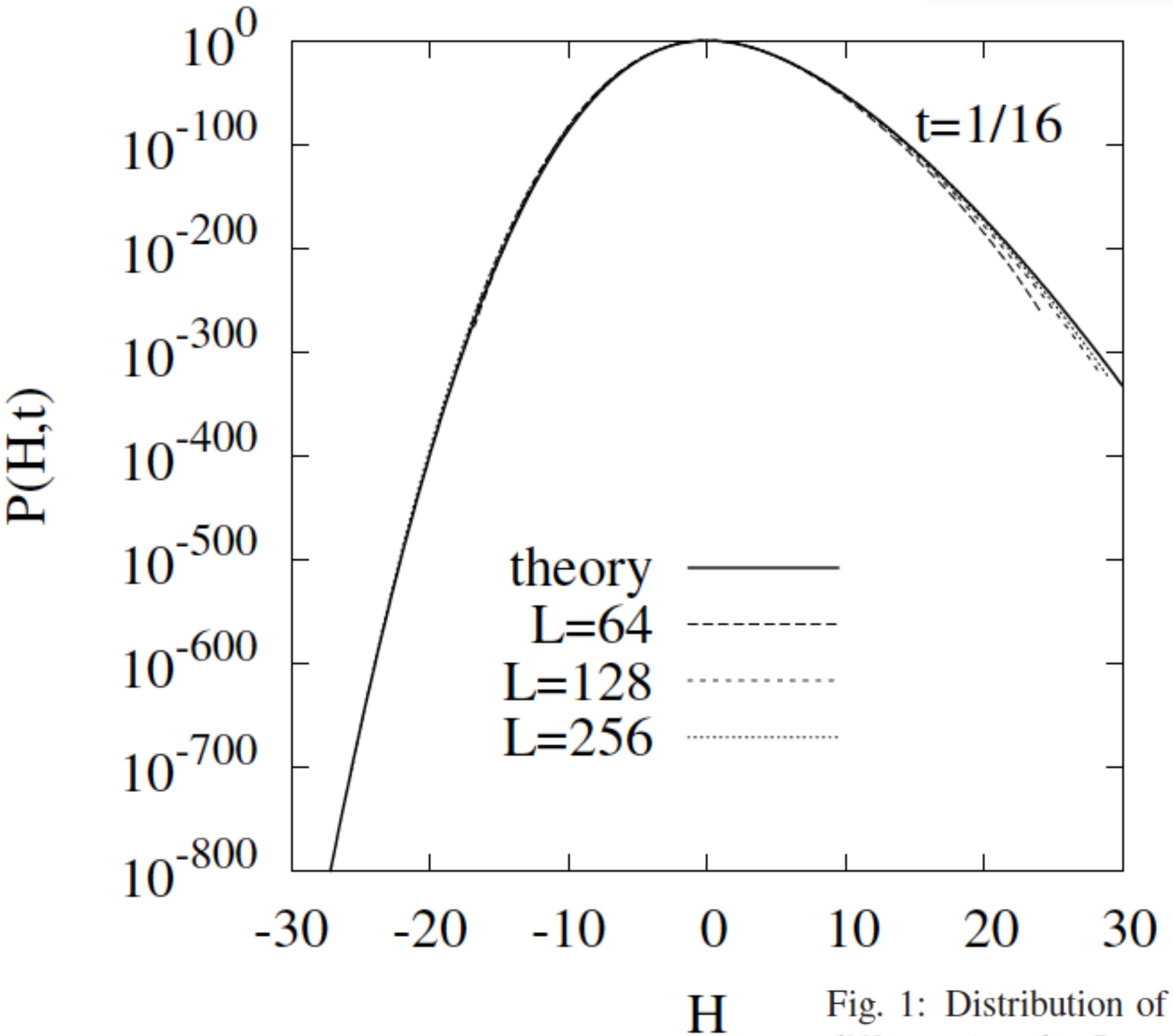
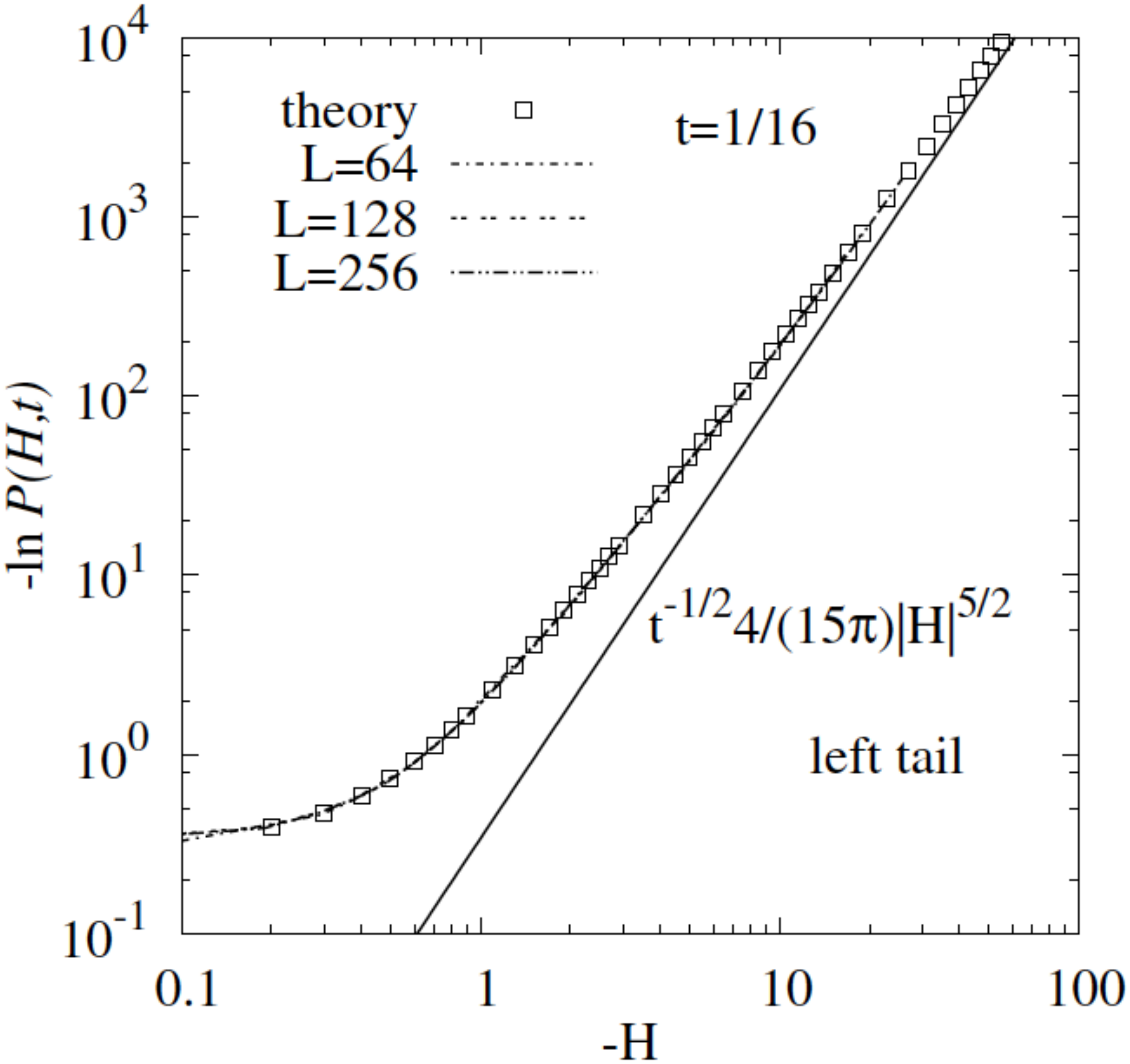
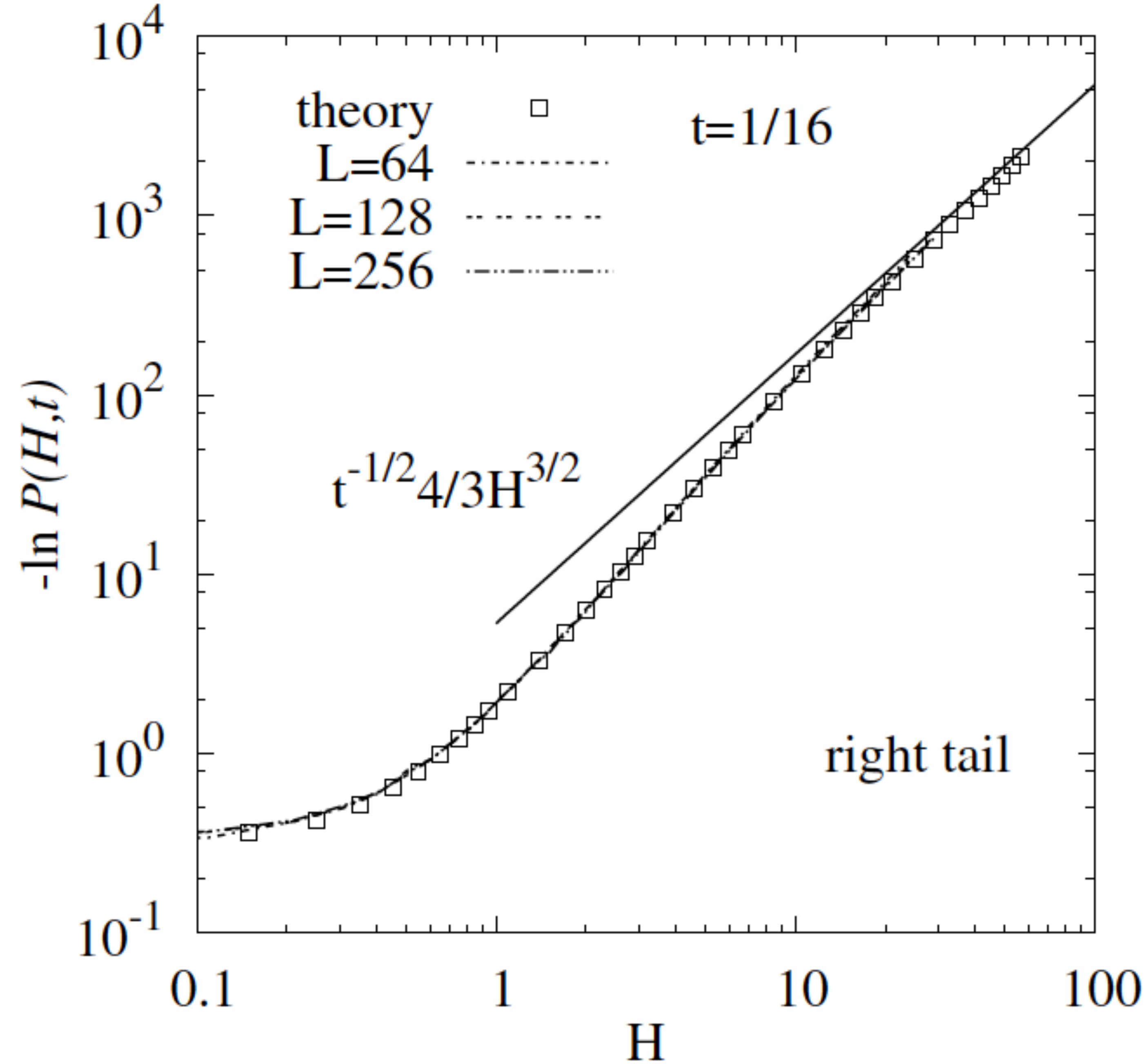


Fig. 1: Distribution of $P(H,t)$ for a short time $t = 1/16$ for three different lengths $L = 64$, $L = 128$ and $L = 256$. The solid line indicates the analytical result in Eq. (2) obtained in Ref. [21]. The agreement between numerical and analytical results is extremely good (on the left tail, down to values of the order 10^{-800}).



theory
t=1/16
L=64
L=128
L=256
$t^{-1/2}4/(15\pi)|H|^{5/2}$
left tail
-ln $P(H,t)$
-H



theory
t=1/16
L=64
L=128
L=256
$t^{-1/2}4/3H^{3/2}$
right tail
-ln $P(H,t)$
H

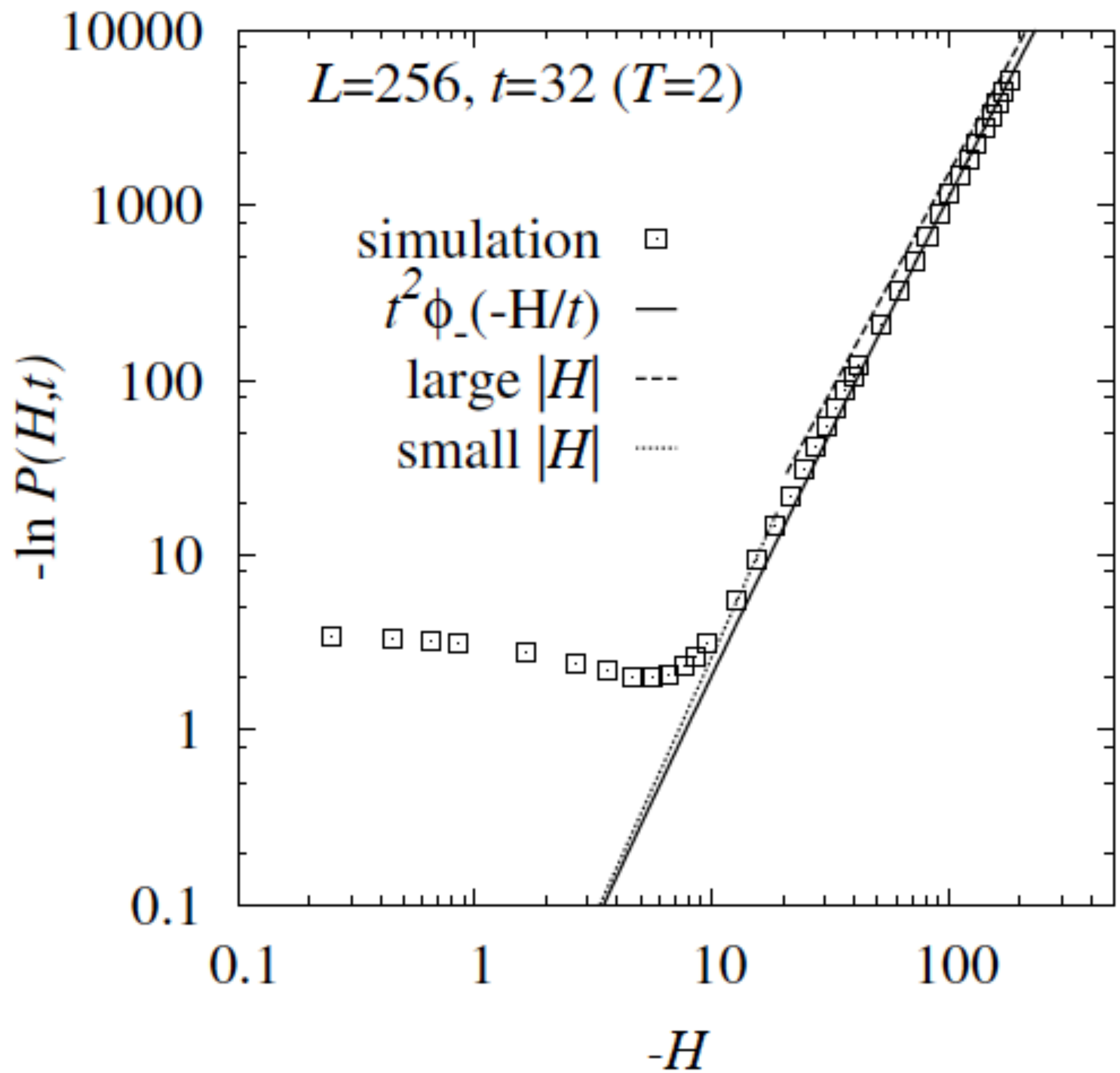


Fig. 5: Logarithm of the left tail of $P(H,t)$ for longer time ($t = 32$) and for the longest length $L = 256$, shown in double-logarithmic scale. The solid line shows the analytical prediction of Eq. (6). The broken line shows the resulting limiting power-law: $|H|^3/(12t)$ for very large H, and $\frac{4}{15\pi}|H|^{5/2}/\sqrt{t}$ for moderate large H.