Existence results in interfacial flows with kinetic undercooling regularization in a time-dependent gap Hele-Shaw cell

Xuming Xie

Department of Mathematics
Morgan State University
Baltimore, Maryland, USA

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Outline of the talk

- Background
- Interfacial flows in a time-dependent gap Hele-Shaw cell
- Reformulation of the problem
- Existence of analytic solution
- A problem in multidimensional spaces
- Existence of classical solutions
- Summary

Background: Viscous fingering in a Hele-Shaw cell

- Displacement of a viscous fluid by a less viscous fluid in a Hele-Shaw cell has been extensively studied since Saffman and Taylor's seminal works in 1950's.

  P.G. Saffman and G.I.Taylor (1958) The penetration of a fluid into a porous medium of Hele-Shaw cell containing a more viscous fluid. Proc. R. Soc. London Ser. A. 245, 312-329.

- They found a family of exact solutions parameterized by the the finger width $\lambda \in (0, 1)$.

Background: Viscous fingering (cont.)

- In experiment, λ was repeatedly found to be close to $\frac{1}{2}$ except for very slow flow. Why $\lambda = \frac{1}{2}$ is selected?
- The selection problem was solved by adding the surface tension regularization.
  R. Combescot, V. Hakim, T. Dombre, Y. Pomeau and A. Pumir (1986, 1987), R. Combescot and T. Dombre (1988), Tanveer (1987, 1988), Ben Amar and Combescot (1991), S.J. Chapman (1999), Tanveer and Xie (2003).

Background: kinetic undercooling regularization

- The Hele-Shaw problem is similar to the Stefan problem in the context of melting or freezing. Besides surface tension, the physical effect most commonly incorporated in regularizing the ill-posed Stefan problem is kinetic undercooling.
- Saffman-Taylor finger problem with kinetic undercooling regularization first appeared in Romero (1981), Saffman (1986).

Background: kinetic undercooling regularization (cont.)

- Y.E. Hohlov and M. Reissig (1995), M. Reissig, D. V. Rogosin, and F. Hubner (1999) obtained Local existence of analytic solution for the classical injection (suction) Hele-Shaw problem with kinetic undercooling.
- Using exponential asymptotics, Chapman and King (2003) analyzed the selection problem of determining the discrete set of widths of a traveling finger for varying kinetic undercooling.

Background: kinetic undercooling regularization (cont.)

- Dallaston and McCue (2014) obtained a continuum of corner-free traveling fingers numerically for any finger width above a critical value.
- Gardiner et al (2015) computed a discrete set of analytic fingers, as predicted by Chapman and King (2003).

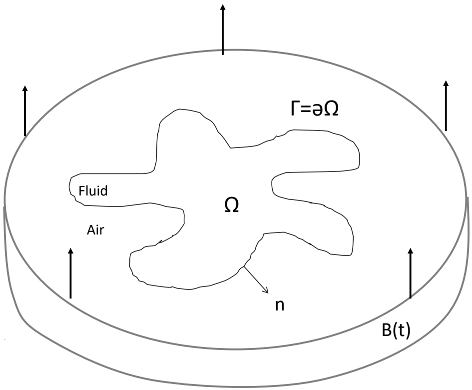
Background: kinetic undercooling regularization (cont.)

- Anjos et al (2015) revealed the physical connection between the kinetic undercooling effect and the action of the dynamic contact angle.
- X. Xie (2019) obtained some rigorous results in existence and selection of Saffman-Taylor fingers by kinetic undercooling.

Background: time-dependent gap Hele-Shaw cell

- Besides the classical Hele-Shaw setup, there are several variants related to the viscous fingering problem.
- One of the variants is interfacial flows in a Hele-Shaw cell where the top plate is lifted uniformly at a prescribed speed and the bottom plate is fixed (lifting plate problems). M. Shelley, F. Tian, and K. Wlodarski (1997), F. Tian (1999), C.-Y. Chen, C.-H. Chen, and J. Miranda (2005), S. Sinha, T. Dutta, and S. Tarafdar (2008), S. Sinha and S. Tarafdar (2009), E. Dias and J. Miranda (2010), J. Nase, D. Derks, and A. Lindner (2011), A.W. Woods and N. Mingotti (2015), M. Zhao et al (2018).

Lifting plate Hele-Shaw flow



Mathematical model

Following M. Shelley, F. Tian, and K. Wlodarski (1997), we have the following governing equations:

Darcey's Law

$$u = -\frac{b^2(t)}{12\mu}\nabla p(x, y, t) \text{ in } \Omega(t),$$

where u is the fluid velocity, p is the pressure, μ is the viscosity of the fluid.

For an incompressible fluid

$$\nabla u = -\frac{\dot{b}(t)}{b(t)} \text{ in } \Omega(t). \tag{1}$$

Mathematical model (cont.)

The kinematic boundary condition is

$$-\frac{b^2(t)}{12\mu}\frac{\partial p}{\partial n} = V_n \text{ on } \Gamma = \partial\Omega(t), \tag{2}$$

where V_n is the normal component of the interface Γ.

The dynamic boundary condition is

$$p = \tau V_n \text{ on } \partial\Omega(t), \tag{3}$$

where τ is a kinetic undercooling coefficient.

Mathematical model (cont.)

Non-dimensionalizing the length and time, the nondimensional version of the equations are

$$\nabla^2 p = \frac{\dot{b}(t)}{b^3(t)} \text{ in } \Omega(t) \tag{4}$$

$$-b^2(t)\frac{\partial p}{\partial n} = V_n \text{ on } \partial\Omega(t), \tag{5}$$

$$p = cV_n \text{ on } \partial\Omega(t), \tag{6}$$

where c is the nondimensional kinetic undercooling coefficient.

Exact solution

It can be seen that (4), (5) and (6) have a radially symmetric solution $\Omega(t) = \{(x,y) : r = \sqrt{x^2+y^2} < R(t)\}$, where

$$R(t) = \frac{R(0)\sqrt{b(0)}}{\sqrt{b(t)}}; \qquad (7)$$

$$p(t,r) = -\frac{cR(0)\sqrt{b(0)}\dot{b}(t)}{2b(t)\sqrt{b(t)}} - \frac{R^2(0)b(0)\dot{b}(t)}{4b^4(t)} - \frac{\dot{b}(t)}{4b^3(t)}r^2. \qquad (8)$$

Linear stability analysis

We consider the interface to be perturbed, $r(t,\alpha) = R(t) + \epsilon\delta(t)\cos(k\alpha)$, where ϵ is small, $k \geq 2$ is the perturbation mode, $\alpha \in [0, 2\pi]$ is the polar angle and $\delta(t)$ is the amplitude of the perturbation. From (4) (5) and (6), we find that

$$\frac{\dot{\delta}(t)}{\delta(t)} = \frac{(k-1)\dot{b}(t)}{2b(t)(1+kcb^{5/2}(t))}. \tag{9}$$

We can see that the perturbation grows when $\dot{b}(t)$ is positive while the perturbation decays when $\dot{b}(t)$ is negative.

Reformulation of the problem

Let $\tilde{p} = p - \frac{\dot{b}(t)}{4b^3(t)}(x^2 + y^2)$, then $\tilde{p}$ satisfies

$$\nabla^2 \tilde{p} = 0 \text{ in } \Omega(t); \tag{10}$$

$$\tilde{p} = -cV_n - \frac{\dot{b}(t)}{4b^3(t)}(x^2 + y^2) \text{ on } \partial\Omega(t), \tag{11}$$

$$V_n = -b^2(t)\frac{\partial \tilde{p}}{\partial n} - \frac{\dot{b}(t)}{2b(t)}(x, y) \cdot \mathbf{n} \text{ on } \partial\Omega(t). \tag{12}$$

Reformulation of the problem (cont.)

Let $z = f(t, \xi)$ be the conformal mapping that maps $\Omega(t)$ onto the unit disk $|\xi| < 1$. Let $W(t, \xi) = \tilde{p} + i\tilde{q}$, where $\tilde{q}$ is a harmonic conjugate of $\tilde{p}$, then $W(t, \xi)$ is analytic in $|\xi| < 1$; then (12) becomes

$$Re\left(\frac{\partial f}{\partial t}\left(\xi\frac{\partial f}{\partial \xi}\right)^*\right) = -b^2(t)Re\,[\xi W_\xi] - \frac{\dot{b}(t)}{2b(t)}Re\left(f\left(\xi\frac{\partial f}{\partial \xi}\right)^*\right) \quad \text{on } |\xi| = 1. \qquad (13)$$

Reformulation of the problem (cont.)

(11) becomes

$$Re\ W = c\frac{Re\left(\frac{\partial f}{\partial t}\left(\xi\frac{\partial f}{\partial \xi}\right)^*\right)}{|f_\xi|} - \frac{\dot{b}(t)}{4b^3(t)}|f|^2; \qquad (14)$$

where $\xi = e^{i\theta}$ on the unit circle $|\xi| = 1$.

Reformulation of the problem (cont.)

Let $\phi = \left(\frac{\partial f}{\partial \xi}\right)^{-1}$, $w(t,\xi) = \xi W_\xi(t,\xi)$ and define operator $T(\phi, w)$ as

$$T(\phi, w) = \frac{1}{2\pi i}\int_{|s|=1} |\phi|^2 Re[w] \frac{(s+\xi)ds}{s(s-\xi)}. \quad (15)$$

Reformulation of the problem (cont.)

Given that f and ϕ are analytic in $|\xi| < 1$ and continuous on $|\xi| \leq 1$, w is the solution of the following **Riemann-Hilbert Problem:**

w is analytic in $|\xi| < 1$ and continuous on $|\xi| \leq 1$, and on $|\xi| = 1, \xi = e^{i\theta}$, w satisfies

$$Im\ [w] = cb^2 \partial_\theta \left(|\phi| Re\ [w]\right) + c\frac{\dot{b}(t)}{2b(t)} \partial_\theta \left(|\phi|^{-1} Re\ [\xi^{-1} f \phi]\right) - \frac{\dot{b}(t)}{4b^3(t)} \partial_\theta \left(|f|^2\right). \quad (16)$$

Reformulation of the problem (cont.)

Using the Poisson formula, we obtain from (13)

$$f_t = -b^2 \xi f_\xi T(\phi, w) - \frac{\dot{b}(t)}{2b(t)} f \text{ in } |\xi| < 1, \tag{17}$$

$$\begin{aligned} \frac{\partial \phi}{\partial t} = & - b^2 \xi \left(\frac{\partial \phi}{\partial \xi} \right) T(\phi, w) - b^2 \phi \frac{\partial}{\partial \xi} \left(\xi T(\phi, w) \right) \\ & - \frac{\dot{b}(t)}{2b(t)} \phi \text{ in } |\xi| < 1. \end{aligned} \tag{18}$$

We impose initial conditions

$$f(0, \xi) = f_0(\xi), \phi(0, \xi) = \frac{1}{f_0'(\xi)}. \tag{19}$$

A scale of Banach spaces

Let r_1 and r be two fixed numbers such that $1 < r_1 < r$. Let $\mathcal{R}_s$ be the disk in complex ξ plane with radius $s, r_1 < s < r$, i.e. $\mathcal{R}_s = \{\xi, |\xi| < s\}$; we define function space $\mathbf{B}_s$ so that $\mathbf{B}_s = \{f(\xi) : f(\xi) \text{ is analytic in } \mathcal{R}_s \text{ and continuous on } \overline{\mathcal{R}_s}\}$ with norm $\|f\|_s = \sup_{\mathcal{R}_s} |f(\xi)|$.

Analytic continuation in $\mathcal{R}_s$

Lemma 1: *Let $f, \phi \in \mathcal{R}_s$, and w is the solution of the Riemann- Hilbert problem (16), then $w \in \mathcal{R}_s$.*

Let $\bar{\phi}(\xi) = \left(\phi(\frac{1}{\xi^*}\right)^*$ and

$$T^+(\phi, w) = \frac{1}{4\pi i} \int_{|s|=1} \phi\bar{\phi}(w + \bar{w}) \frac{(s+\xi)ds}{s(s-\xi)} + \phi\bar{\phi}(w + \bar{w}). \tag{20}$$

Then $T^+(\phi, w)$ is the analytic continuation of $T(\phi, w)$ into $\mathcal{R}_s \cap \{|\xi| > 1\}$.

An abstract Cauchy-Kowalewski Problem

The problem can be rewritten as the following abstract Cauchy-Kowalewski Problem:

$$\frac{d\mathcal{U}}{dt} = \mathcal{L}(\mathcal{U}, t), \quad \mathcal{U}(0) = 0;$$

where $\mathcal{U} = [f - f_0, \phi - \phi_0]$ and $\mathcal{L}(\mathcal{U}) = [\mathcal{L}_1(\mathcal{U}, t), \mathcal{L}_2(\mathcal{U}, t)]$;

$$\mathcal{L}_1(\mathcal{U}, t) = -b^2 \xi f_\xi T(\phi, w) - \frac{\dot{b}(t)}{2b(t)} f,$$

$$\mathcal{L}_2(\mathcal{U}, t) = -b^2 \xi \left(\frac{\partial \phi}{\partial \xi} \right) T(\phi, w) - b^2 \phi \frac{\partial}{\partial \xi} \left(\xi T(\phi, w) \right) - \frac{\dot{b}(t)}{2b(t)} \phi.$$

An abstract Cauchy-Kowalewski Problem (cont.)

The operator $\mathcal{L}$ has the following properties:
(i) For some constants $M > 0, \delta > 0$ and every pair of numbers s, s' such that $r_1 < s' < s < r$, $(u,t) \to \mathcal{L}(u,t)$ is a continuous mapping of

$$\{u \in \mathbf{B}_s \times \mathbf{B}_s : \|u\|_s < M\} \times \{t; |t| < \delta\} \text{ into } \mathbf{B}_{s'} \times \mathbf{B}_{s'} \quad (21)$$

(ii) For any $r_1 \leq s' < s \leq r$ and all $u, v \in \mathbf{B}_s \times \mathbf{B}_s$ with $\|u\|_s < M, \|v\|_s < M$ and for any t, $|t| < \delta$, $\mathcal{L}$ satisfies

$$\|\mathcal{L}(u,t) - \mathcal{L}(v,t)\|_{s'} \leq C \frac{\|u - v\|_s}{s - s'} \quad (22)$$

where C is some positive constant independent of t, u, v, s, s'.

(iii) $\mathcal{L}(0, t)$ is a continuous function of t, $|t| < \delta$ with values in $\mathbf{B}_s \times \mathbf{B}_s$ for every $r_1 < s < r$ and satisfies, with some positive constant K,

$$\|\mathcal{L}(0, t)\|_s \leq \frac{K}{(r - s)} \tag{23}$$

Existence of analytic solution

Applying Nishida-Nirenberg Theorem, we obtain the following existence result:

Theorem 1: *If* $f_0 \in \mathbf{B}_r$ *and* $f_0' \neq 0$ *in* $\mathcal{R}_r$ *, then there exists one and only one solution* $f \in C^1([0,T), \mathbf{B}_s), w \in C^1([0,T), \mathbf{B}_s), r_1 < s < r, w(0,t) = 0$ *to the problem (17) (18) and (19), where* $T = a_0(r-s)$*,* a_0 *is a suitable positive constant independent of* s*.*

A problem in R^n

We consider the following equations in $R^n, n \geq 2$:

$$\nabla^2 p = \frac{\dot{b}(t)}{b^3(t)} \text{ in } \Omega(t) \tag{24}$$

$$-b^2(t)\frac{\partial p}{\partial n} = V_n \text{ on } \partial\Omega(t), \tag{25}$$

$$p = cV_n \text{ on } \partial\Omega(t), \tag{26}$$

$$\Omega(0) = \Omega_0, \Gamma_0 = \partial\Omega(0). \tag{27}$$

We assume that $\Gamma_0 \in C^{m+\alpha}$, $m \geq 3$ is an integer, and $0 < \alpha < 1$.

Description of $\Gamma(t) = \partial\Omega(t)$ in R^n

Let ω be a variable point of Γ_0 and $n(\omega)$ the unit exterior normal to Γ_0 at ω.
Let γ_0 be a given sufficiently small positive number such that the mapping

$$x : \Gamma_0 \times [-\gamma_0, \gamma_0] \to N_0 \subset R^n, x(\omega, \lambda) = \omega + \lambda n(\omega)$$

is a $C^{m-1+\alpha}$ diffeomorphism.
We define $\Gamma_{0,T} = \Gamma_0 \times [0, T]$ and $\Omega_{0,T} = \Omega_0 \times [0, T]$. For $\rho(\omega, t) \in C([0, T], C^{m+\alpha}(\Gamma_0))$ with $|\rho|_{m+\alpha} \leq \gamma_0$. Let

$$\Gamma_{\rho,T} = \{\omega + \rho(\omega, t)n(\omega); (\omega, t) \in \Gamma_{0,T}\}$$

and $\Omega_{\rho,T}$ be the domain bounded by $\Gamma_{\rho,T}$.

A problem in R^n (cont.)

$$\nabla^2 p = \frac{\dot{b}(t)}{b^3(t)} \text{ in } \Omega_{\rho,T} \tag{28}$$

$$-b^2(t)\frac{\partial p}{\partial n} = V_n \text{ on } \Gamma_{\rho,T}, \tag{29}$$

$$p = cV_n \text{ on } \Gamma_{\rho,T}, \tag{30}$$

$$\rho(\omega, 0) = 0. \tag{31}$$

Theorem 2: *Let* $b(t) \in C^2[0, T]$, $\Gamma_0 \in C^{m+\alpha}$, $m \geq 3$ *is an integer, and* $0 < \alpha < 1$. *Then for a sufficiently small* $T \in [0, T_0]$, *there exist* $\rho(\omega, t) \in C([0, T], C^{m+\alpha}(\Gamma_0)) \cap C^1([0, T], C^{m-1+\alpha}(\Gamma_0))$ *and* $p(x, t) \in C([0, T], C^{m+\alpha}(\Omega_{\rho,T})) \cap C^1([0, T], C^{m-1+\alpha}(\Omega_{\rho,T}))$ *satisfying (28)-(31).*

Hanzawa diffeomorphism

Choose a function $\chi(\lambda) \in C^\infty(R)$ so that
(1) $\chi(\lambda) = 1$, for $|\lambda| \leq \gamma_0$.(2) $\chi(\lambda) = 0$, for $\lambda > 3\gamma_0$. (3) $|\chi'(\lambda)| \leq \dfrac{3}{4\gamma_0}$ for $\lambda \in R$.
For $\rho(\omega, t) \in C([0, T], C^{m+\alpha}(\Gamma_0)) \cap C^1([0, T], C^{m-1+\alpha}(\Gamma_0))$, with $|\rho|_{m+\alpha} \leq \delta_0$, we define Hanzawa diffeomorphism:

$$\begin{aligned} & e_\rho : R^n_y \times [0, T] \to R^n_x \times [0, T] \\ & e_\rho(y, t) = (x(\omega, \lambda + \chi(\lambda)\rho(\omega, t)), t) \\ & \qquad \text{for } (y, t) = (x(\omega, \lambda), t) \in N_0 \times [0, T] \\ & e_\rho(y, t) = (y, t) \text{ for } (y, t) \in (R^n - N_0) \times [0, T] \end{aligned} \tag{32}$$

Note that $e_\rho(\Omega_{0,T}) = \Omega_{\rho,T}$, $e_\rho(\Gamma_{0,T}) = \Gamma_{\rho,T}$.

A problem in the fixed domain

Let $v(y,t) = p(e_\rho(y,t))$, then v satisfies the following equations in the fixed domain:

$$\mathcal{L}_\rho v = \frac{\dot{b}(t)}{b^3(t)} \text{ in } \Omega_{0,T},$$

$$\partial_t \rho = -b^2(t)(S_\rho \partial_\lambda v - \sum_{k=1}^{n-1} \partial_{\omega_k}\rho |\nabla_x \omega_k(x) \circ e_\rho(\omega,t)|^2 \partial_{\omega_k} v),$$

$$v = c\frac{\partial_t \rho}{(S_\rho)^{1/2}} \text{ on } \Gamma_{0,T}. \tag{33}$$

A problem in the fixed domain (cont.)

$$\mathcal{L}_\rho = \sum_{k,j=1}^{n} A_{\rho,kj}(y,t)\partial^2_{y_k y_j} + \sum_{k=1}^{n} A_{\rho,k}\partial_{y_k},$$

$$A_{\rho,kj}(y,t) = A_{kj}(y, \rho(\omega(y),t); \partial_{\omega_1}\rho(\omega(y),t)\cdots\partial_{\omega_{n-1}}\rho(\omega(y),t)),$$

$$\text{for } k,j = 1,2\cdots n, \text{ and } (y,t)\in N_0\times[0,T],$$

$$A_{\rho,kj}(y,t) = \delta_{kj} \text{ for } k,j = 1,2\cdots n, (y,t)\in(R^n - N_0)\times[0,T]. \tag{34}$$

$$A_{\rho,k}(y,t) = A_k(y, \rho(\omega(y),t), \nabla_\omega\rho(\omega(y),t), \nabla^2_\omega\rho(\omega(y),t)); \tag{35}$$

$$S_\rho = 1 + \left|\sum_{k=1}^{n-1}\partial_{\omega_k}\rho\nabla_x\omega_k(x)\circ e_\rho(\omega,t)\right|^2. \tag{36}$$

Solving an elliptic PDE

For $\rho(\omega, t) \in C([0,T], C^{m+\alpha}(\Gamma_0)) \cap C^1([0,T], C^{m-1+\alpha}(\Gamma_0))$, with $|\rho|_{m+\alpha} \le \delta_0$, we can solve the following elliptic PDE:

$$\begin{aligned} &\mathcal{L}_\rho v = \frac{\dot{b}(t)}{b^3(t)} \text{ in } \Omega_{0,T}, \\ &v + cb^2(t) S_\rho^{1/2} \partial_\lambda v \\ &\quad - cb^2(t) S_\rho^{-1/2} \sum_{k=1}^{n-1} \partial_{\omega_k} \rho |\nabla_x \omega_k(x) \circ e_\rho(\omega, t)|^2 \partial_{\omega_k} v = 0 \text{ on } \Gamma_{0,T}. \end{aligned} \tag{37}$$

Schauder estimate for the elliptic PDE

Lemma 2: *For given* $\rho(\omega, t) \in C([0, T], C^{m+\alpha}(\Gamma_0)) \cap C^1([0, T], C^{m-1+\alpha}(\Gamma_0))$, *with* $|\rho|_{m+\alpha} \le \delta_0$, *there exists a unique solution* $v(y, t) \in C([0, T], C^{m+\alpha}(\Omega_0)) \cap C^1([0, T], C^{m-1+\alpha}(\Omega_0))$ *to (37) and*

$$|v(t, \cdot)|_{m+\alpha} \le C(1 + |\rho(t, \cdot)|_{m+\alpha}),$$

$$|\partial_t v(t, \cdot)|_{m-1+\alpha} \le C(1 + |\partial_t \rho(t, \cdot)|_{m-1+\alpha}).$$

Solving a nonlinear first order PDE

Now we need to solve the following nonlinear first order PDE:

$$\partial_t \rho = \frac{v}{c}\left(1 + \left(\sum_{k=1}^{n-1} \partial_{\omega_k}\rho \nabla_x \omega_k(x) \circ e_\rho(\omega, t)\right)^2\right)^{1/2} \tag{38}$$

$$\rho(\omega, 0) = 0.$$

Linearizing the nonlinear first order PDE

Let $\mathcal{F}(\rho) = \frac{v}{c}\left(1 + \left(\sum_{k=1}^{n-1} \partial_{\omega_k}\rho \nabla_x \omega_k(x) \circ e_\rho(\omega, t)\right)^2\right)^{1/2}$,
$\mathcal{DF}(0)$ be the Fréchet derivative of $\mathcal{F}(\rho)$ at $\rho = 0$. Then (38) can be written as

$$\partial_t \rho = \mathcal{DF}(0)\rho + \mathcal{G}(\rho), \tag{39}$$

where $\mathcal{G}(\rho) = \mathcal{F}(\rho) - \mathcal{DF}(0)\rho$.
The linearized part of (39) can be written as
$\mathcal{DF}(0)\rho = \sum_{k=1}^{n-1} B_k(\omega)\partial_{\omega_k}\rho$, where $B_k(\omega) \in C^{m-1+\alpha}(\Gamma_0)$.

Solving a linearized first order hyperbolic PDE

We can solve the linearized first order hyperbolic PDE:

$$\begin{aligned} \partial_t \rho &= \sum_{k=1}^{n-1} B_k(\omega) \partial_{\omega_k} \rho + G(\omega, t), \\ \rho(\omega, 0) &= 0. \end{aligned} \tag{40}$$

Lemma 3: *For* $G \in C([0, T], C^{m-1+\alpha}(\Gamma_0))$. *Then there exists a unique solution* $\rho(\omega, t)$ *of (40) and*
$\rho(\omega, t) \in C([0, T], C^{m-1+\alpha}(\Gamma_0)) \cap C^1([0, T], C^{m-2+\alpha}(\Gamma_0))$.

Nash-Moser implicit function theorem

Lemma 3 indicates that the solution ρ of the first order linearized equation has the same regularity as G, that means that the linearized equation can only be solved with a derivative loss. To overcome the derivative loss, we can apply Nash-Moser iteration to solve the nonlinear PDE (38); hence we can prove Theorem 2. We omit the technical details about verifying all conditions for Nash-Moser imnplicit function theorem.

Summary

- We considered an interface flow with kinetic undercooling regularization in a radial Hele-Shaw cell with a time dependent gap.Using complex analysis approach, we reduced the free boundary problem to a Riemann-Hilbert problem and an abstract Cauchy-Kovalevsky evolution problem. We obtained the local existence of analytic solution of the moving boundary problem when the initial data is analytic.
- A similar problem in multidimensional spaces was also studied; Using Hanzawa diffeomorphism, we reduce the problem to solving a nonlinear first order PDE in a fixed domain. The local existence of classical solutions can be obtained by applying Nash-Moser interation.