Uncertainty elicitation and quantification from experts

Kevin Wilson

School of Mathematics, Statistics & Physics, Newcastle University, UK

Application: MPhil work of
Sara Graziadio, NIHR Newcastle MIC

Wednesday 10th June 2020
Contents

Introduction
    Infectious disease modelling

Eliciting a single unknown
    From a single expert
    From multiple experts

Elicitation of multiple unknowns

Back to infectious disease modelling
Modelling choices

- **Mathematical modelling** has had, and will continue to have, an important role in the COVID-19 pandemic.
Modelling choices

- Mathematical modelling has had, and will continue to have, an important role in the COVID-19 pandemic.
- In the UK, the most influential modelling was probably in:

  Impact of non-pharmaceutical interventions (NPIs) to reduce COVID-19 mortality and healthcare demand
Modelling choices

- **Mathematical modelling** has had, and will continue to have, an important role in the COVID19 pandemic.
- In the UK, the most influential modelling was probably in:
  
  **Impact of non-pharmaceutical interventions (NPIs) to reduce COVID-19 mortality and healthcare demand**

- All modelling involves specifying parameter values:
  - “We assumed an incubation period of 5.1 days”
  - “We assume that 30% of those that are hospitalised will require critical care”
  - “We make a baseline assumption that $R_0 = 2.4$ but examine values between 2.0 and 2.6”
Mathematical modelling has had, and will continue to have, a
important role in the COVID19 pandemic.
In the UK, the most influential modelling was probably in:

Impact of non-pharmaceutical interventions (NPIs) to reduce
COVID-19 mortality and healthcare demand

All modelling involves specifying parameter values:
- “We assumed an incubation period of 5.1 days”
- “We assume that 30% of those that are hospitalised will
  require critical care”
- “We make a baseline assumption that $R_0 = 2.4$ but examine
  values between 2.0 and 2.6”
- “Based on expert clinical opinion, we assume that 50% of
  those in critical care will die”
Eliciting a single unknown

- Suppose we have some unknown $\theta$ in a model.
Eliciting a single unknown

- Suppose we have some unknown $\theta$ in a model.
- We wish to encode an expert’s knowledge and uncertainty about $\theta$.
Eliciting a single unknown

- Suppose we have some unknown $\theta$ in a model.
- We wish to encode an expert’s knowledge and uncertainty about $\theta$.
- We can ask for quantiles of $\theta$, e.g., the $100\alpha\%$ quantile of $\theta$ is the value for which

$$\Pr(\theta \leq q_\alpha) = \alpha.$$
Eliciting a single unknown

- Suppose we have some unknown $\theta$ in a model.
- We wish to encode an expert’s knowledge and uncertainty about $\theta$.
- We can ask for quantiles of $\theta$, e.g., the $100\alpha\%$ quantile of $\theta$ is the value for which

$$\Pr(\theta \leq q_\alpha) = \alpha.$$ 

- The median (50% quantile) is the value for which the expert thinks $\theta$ is equally likely to be above or below this value.
Eliciting a single unknown

- Suppose we have some unknown $\theta$ in a model.
- We wish to encode an expert’s knowledge and uncertainty about $\theta$.
- We can ask for quantiles of $\theta$, e.g., the $100\alpha\%$ quantile of $\theta$ is the value for which
  \[ \Pr(\theta \leq q_{\alpha}) = \alpha. \]
- The median (50% quantile) is the value for which the expert thinks $\theta$ is equally likely to be above or below this value.
- Typically, an expert might be asked for 3 quantiles: the median, one quantile above the median and one below.
Eliciting a single unknown

- Suppose we have some unknown $\theta$ in a model.
- We wish to encode an expert’s knowledge and uncertainty about $\theta$.
- We can ask for quantiles of $\theta$, e.g., the $100\alpha\%$ quantile of $\theta$ is the value for which
  \[ \Pr(\theta \leq q_\alpha) = \alpha. \]
- The median (50% quantile) is the value for which the expert thinks $\theta$ is equally likely to be above or below this value.
- Typically, an expert might be asked for 3 quantiles: the median, one quantile above the median and one below.
- These are used to define a probability distribution for $\theta$. 
Example

Provide your 5%, 50% and 95% quantiles for the distance between:

1. Glasgow and Edinburgh
2. Istanbul and Ankara
Example

Provide your 5%, 50% and 95% quantiles for the distance between:

1. Glasgow and Edinburgh: 41 miles (67km)
2. Istanbul and Ankara: 217 miles (319km)
Ability of experts
Ability of experts

- Based on the TU Delft database of 45 expert judgement studies.
- Over 67,000 probability distributions assessed.
- Experts asked for 5%, 50% and 95% quantiles for unknowns of seed variables.
Eliciting a single unknown

- We need to choose a probability distribution which matches the expert's quantiles.

E.g. consider the proportion of individuals with a virus when symptomatic.

The expert has given (25%, 50%, 75%) quantiles of (0.4, 0.45, 0.55).

We want to fit a Beta distribution to the expert's judgements.

This will involve compromise, as the Beta distribution only has two parameters.

We fit using least squares.
Eliciting a single unknown

▶ We need to choose a probability distribution which matches the expert's quantiles.
▶ E.g. consider the proportion of individuals with a virus when symptomatic.
Eliciting a single unknown

- We need to choose a probability distribution which matches the expert's quantiles.
- E.g. consider the proportion of individuals with a virus when symptomatic.
- The expert has given (25%, 50%, 75%) quantiles of (0.4, 0.45, 0.55).
Eliciting a single unknown

- We need to choose a probability distribution which matches the expert’s quantiles.
- E.g. consider the proportion of individuals with a virus when symptomatic.
- The expert has given (25%, 50%, 75%) quantiles of (0.4, 0.45, 0.55).
- We want to fit a Beta distribution to the expert’s judgements.
Eliciting a single unknown

- We need to choose a probability distribution which matches the expert's quantiles.
- E.g. consider the proportion of individuals with a virus when symptomatic.
- The expert has given (25%, 50%, 75%) quantiles of (0.4, 0.45, 0.55).
- We want to fit a Beta distribution to the expert's judgements.
- This will involve compromise, as the Beta distribution only has two parameters.
Eliciting a single unknown

- We need to choose a probability distribution which matches the expert's quantiles.
- E.g. consider the proportion of individuals with a virus when symptomatic.
- The expert has given (25%, 50%, 75%) quantiles of (0.4, 0.45, 0.55).
- We want to fit a Beta distribution to the expert’s judgements.
- This will involve compromise, as the Beta distribution only has two parameters.
- We fit using least squares.
The fitted quartiles are: (0.39, 0.46, 0.54).
The fitted quartiles are: (0.39, 0.46, 0.54).
Aggregation of expert judgements

- We typically consult multiple experts for their judgements.

- We may wish to coerce these judgements into a single view.

- This can be done via a mathematical rule, a behavioural method, or a combination of the two.
Aggregation of expert judgements

- We typically consult multiple experts for their judgements.
- We may wish to coerce these judgements into a single view.
Aggregation of expert judgements

- We typically consult multiple experts for their judgements.
- We may wish to coerce these judgements into a single view.
- This can be done via a mathematical rule, a behavioural method, or a combination of the two.
The SHeffield ELicitation Framework (SHELF)

- Consensus method

http://www.tonyohagan.co.uk/shelf/
The SHeffield ELicitation Framework (SHELF)

- Consensus method
- Two stages: individual elicitation and group elicitation

http://www.tonyohagan.co.uk/shelf/
The SHeffield ELicitation Framework (SHELF)

- Consensus method
- Two stages: individual elicitation and group elicitation
- Discussion between experts
The SHffield ELicitation Framework (SHELF)

- Consensus method
- Two stages: individual elicitation and group elicitation
- Discussion between experts
- Eliciting the opinions of a “Rational Impartial Observer” (RIO)

http://www.tonyohagan.co.uk/shelf/
Application I

A company has developed a new diagnostic test for Chronic Obstructive Pulmonary Disease (COPD).
A company has developed a new diagnostic test for Chronic Obstructive Pulmonary Disease (COPD).

They wish to identify the best role for the test in the NHS via a mathematical model.
A company has developed a new diagnostic test for Chronic Obstructive Pulmonary Disease (COPD).

They wish to identify the best role for the test in the NHS via a mathematical model.

To do so, they required parameter estimates for key parameters on patient and clinician actions.
A company has developed a new diagnostic test for Chronic Obstructive Pulmonary Disease (COPD).

They wish to identify the best role for the test in the NHS via a mathematical model.

To do so, they required parameter estimates for key parameters on patient and clinician actions.

In the absence of adequate data, we performed an expert elicitation.
Application II

Pre-workshop tasks

► **Evidence dossier:** Reports of COPD in the literature, data from test development and data on COPD from national databases.
Application II

Pre-workshop tasks

▶ **Evidence dossier**: Reports of COPD in the literature, data from test development and data on COPD from national databases.

▶ **Elicitation team**: The client is the company. The roles of co-ordinator, recorder and analyst were fulfilled by Sara, the roles of advisor and facilitator by Kevin.
Pre-workshop tasks

- **Evidence dossier:** Reports of COPD in the literature, data from test development and data on COPD from national databases.

- **Elicitation team:** The client is the company. The roles of co-ordinator, recorder and analyst were fulfilled by Sara, the roles of advisor and facilitator by Kevin.

- **Identification and recruitment:** the Respiratory Programme Lead for the Academic Health Society Network and a GP, Clinical Advisor for Newcastle Hospitals and Senior Partner in a Surgery in Newcastle.
Application II

Pre-workshop tasks

▶ **Evidence dossier:** Reports of COPD in the literature, data from test development and data on COPD from national databases.

▶ **Elicitation team:** The client is the company. The roles of co-ordinator, recorder and analyst were fulfilled by Sara, the roles of advisor and facilitator by Kevin.

▶ **Identification and recruitment:** the Respiratory Programme Lead for the Academic Health Society Network and a GP, Clinical Advisor for Newcastle Hospitals and Senior Partner in a Surgery in Newcastle.

▶ **Experts’ evidence:** Details of the Newcastle primary care co-operative COPD audit 2014/15, data from the quality and outcomes framework and data of patients in a GP surgery with COPD.
Tasks during workshop

- **Training the experts:** The experts were trained in subjective probabilities, the basic structure of influence diagrams and conditional probabilities.
Tasks during workshop

- **Training the experts:** The experts were trained in subjective probabilities, the basic structure of influence diagrams and conditional probabilities.

- **The elicitation session:** During the elicitation, for each QoI, the experts were asked to provide a best estimate and “surprise limits”. 

<table>
<thead>
<tr>
<th>Condition</th>
<th>Diagnosis</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test: Yes</td>
<td>Exacerbation: Yes</td>
<td>0.9 (0.8, 0.97)</td>
</tr>
<tr>
<td></td>
<td>Exacerbation: No</td>
<td>0.1 (0.03, 0.2)</td>
</tr>
<tr>
<td>Test: No</td>
<td>Exacerbation: Yes</td>
<td>0.1 (0.03, 0.3)</td>
</tr>
<tr>
<td></td>
<td>Exacerbation: No</td>
<td>0.9 (0.7, 0.97)</td>
</tr>
<tr>
<td>Test: No</td>
<td>Exacerbation: Yes</td>
<td>0.7 (0.6, 0.75)</td>
</tr>
<tr>
<td></td>
<td>Exacerbation: No</td>
<td>0.3 (0.25, 0.4)</td>
</tr>
</tbody>
</table>
Tasks during workshop

- **Training the experts:** The experts were trained in subjective probabilities, the basic structure of influence diagrams and conditional probabilities.

- **The elicitation session:** During the elicitation, for each QoI, the experts were asked to provide a best estimate and “surprise limits”.

<table>
<thead>
<tr>
<th>Conditional probabilities for $X_2$: Patient Diagnosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_2$: Patient diagnosis</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>Test: Yes</td>
</tr>
<tr>
<td>Test: Yes</td>
</tr>
<tr>
<td>Test: No</td>
</tr>
<tr>
<td>Test: No</td>
</tr>
</tbody>
</table>
Eliciting multiple unknowns

▶ We can use expert to develop probabilistic models.
Eliciting multiple unknowns

- We can use experts to develop **probabilistic** models.
- This is typically a **qualitative process**, and involves discussion and mapping.

**Bayesian networks**
Eliciting multiple unknowns

- We can use expert to develop **probabilistic models**.
- This is typically a **qualitative process**, and involves discussion and mapping.

**Bayesian networks**

- A **Bayesian Network** (BN) is a collection of nodes and arcs that form a Directed Acyclic Graph.
Eliciting multiple unknowns

- We can use expert to develop probabilistic models.
- This is typically a qualitative process, and involves discussion and mapping.

**Bayesian networks**

- A Bayesian Network (BN) is a collection of nodes and arcs that form a Directed Acyclic Graph.
- Nodes represent variables and arcs represent relationships.
Eliciting multiple unknowns

We can use expert to develop probabilistic models. This is typically a qualitative process, and involves discussion and mapping.

Bayesian networks

A Bayesian Network (BN) is a collection of nodes and arcs that form a Directed Acyclic Graph. Nodes represent variables and arcs represent relationships. The direction of the arc represents the causal direction of the relationship.
Trace back graph

Figure: A trace back graph identifying the variables which are relevant to the decision making process for the management of COPD patients.
Figure: The final reduced influence diagram
Discussion: infectious disease modelling

- Uncertainty quantification is critical in real-time infectious disease modelling, where there is much we do not know.
Discussion: infectious disease modelling

- Uncertainty quantification is critical in real-time infectious disease modelling, where there is much we do not know.
- This should go further than a “simple” sensitivity analysis.
Discussion: infectious disease modelling

- Uncertainty quantification is critical in real-time infectious disease modelling, where there is much we do not know.
- This should go further than a “simple” sensitivity analysis.
- Decisions need to take into account the understanding of the uncertainty in the modelling.
Discussion: infectious disease modelling

- Uncertainty quantification is critical in real-time infectious disease modelling, where there is much we do not know.
- This should go further than a “simple” sensitivity analysis.
- Decisions need to taken with an understanding of the uncertainty in the modelling.
- In the absence of high quality, comprehensive data, expert judgement has a role to play in this.
Discussion: infectious disease modelling

- Uncertainty quantification is critical in real-time infectious disease modelling, where there is much we do not know.
- This should go further than a “simple” sensitivity analysis.
- Decisions need to taken with an understanding of the uncertainty in the modelling.
- In the absence of high quality, comprehensive data, expert judgement has a role to play in this.
- Uncertainty analysis can be based on Monte Carlo simulation or full Bayesian methods.
Discussion: infectious disease modelling

- Uncertainty quantification is critical in real-time infectious disease modelling, where there is much we do not know.
- This should go further than a “simple” sensitivity analysis.
- Decisions need to be taken with an understanding of the uncertainty in the modelling.
- In the absence of high quality, comprehensive data, expert judgement has a role to play in this.
- Uncertainty analysis can be based on Monte Carlo simulation or full Bayesian methods.
- Parameter uncertainty, as considered here, is only one element of uncertainty relevant to these models.
Further resources

- Probabilistic judgements e-learning course: http://www.tonyohagan.co.uk/shelf/ecourse.html
- SHELF (The SHeffield ELicitation Framework): http://www.tonyohagan.co.uk/shelf/
- Short videos for online course Structured Expert Judgment TU Delft: http://rogermcooke.net/