Hierarchical Optimisation and Equilibrium Problems in Electricity Systems: Challenges and Status Quo

Isaac Newton Institute - Workshop

Electricity systems of the future: incentives, regulation and analysis for efficient investment.

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About me
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Research interests:
Operations research, energy storage, bilevel programming, investment under uncertainty.

- PhD in Power Systems (2013)
  Comillas Pontifical University, Spain
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Areas of Research

UC Models with Fatigue Impact

Improving long-term Models with high Renewable Penetration

Bilevel Programming (GEP, TEP)

\[
\begin{align*}
\forall i \left\{ \begin{array}{l}
\max_{x_i, q_i} t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \\
\text{s.t.} \\
\max_{x_i, q_i} t(p(q_i, q_{-i}) - \delta)q_i \\
\text{s.t.} \\
0 \leq q_i \leq x_i \\
d = q_i + q_{-i}, d = D_0 - \alpha p
\end{array} \right.
\end{align*}
\]
Outline

Introduction

Current Bilevel Solution Techniques

Applications in Electricity System

Gap in the Literature & Challenges

Conclusions
Introduction

Current Bilevel Solution Techniques

Applications in Electricity System

Gap in the Literature & Challenges

Conclusions
The liberalization of the electricity sector and the introduction of electricity markets have greatly complicated the organization of the electricity sector, especially for generation companies.

Under a centralized framework a central planner took decisions maximizing social welfare, whereas in electricity markets the responsibility of taking many decisions lies with public and private entities that interact.

From a game-theoretic point of view many decision-making problems in a liberalized power sector can be regarded and analyzed as sequential Stackelberg-type games among different players.
Introduction

Motivation

The sequence in which decisions are taken, can convert simple equilibrium games into complicated hierarchical/bilevel optimisation or equilibrium problems whose outcomes can diverge significantly depending on the type of game.

This talk discusses existing solution methods for bilevel problems and then focusses on two applications of such hierarchical games in electricity markets: strategic storage investment and transmission expansion.
Introduction
Basic Concepts

Bilevel Problem
• A bilevel programming problem is a hierarchical optimization problem which is constrained by another optimization problem.

MPEC
• Mathematical Program with Equilibrium Constraints – this is a bilevel optimisation problem

EPEC
• Equilibrium Problem with Equilibrium Constraints – this is a bilevel equilibrium problem
We have two identical firms with perfectly substitutable products, facing either a one-stage or a two-stage competitive situation.

**One-stage situation (open loop/ single-level model)**

Investment and operation decisions are made simultaneously.

**Two-stage situation (closed loop/ bilevel model)**

First, firms choose capacities that maximize their profit anticipating the second stage, where...

...quantities and prices are determined by a conjectured price response market equilibrium.
Single-Level GEP Investment Equilibrium

All GENCOs simultaneously maximize their total profits (market revenues minus investment costs minus production costs) subject to lower and upper bounds on production and a demand balance.

\[
\text{Max}_{\{x_t, q_t\}} \text{Total Profits}_i \\
\text{s. t. } 0 \leq q_t \leq x_t + K_i
\]

---

Market Clearing Demand-Price Function
GEP
SL Investment Formulation

Concept:
\[ \forall i \left\{ \max_{x_i, q_i} \ t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \right\} \]
\[ \text{s.t.} \]
\[ q_i \leq x_i \]
\[ d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}) \]

KKT-conditions:
\[ \forall i \left\{ \right. \]
\[ \frac{\partial L_i}{\partial q_i} = tp(q_i, q_{-i}) - t\theta q_i - t\delta - \lambda_i = 0 \]
\[ \frac{\partial L_i}{\partial x_i} = \beta - \lambda_i = 0 \]
\[ q_i \leq x_i \]
\[ \lambda_i \geq 0 \]
\[ \lambda_i(x_i - q_i) = 0 \]
\[ d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}) \]
This model assists one GENCO in taking capacity decisions while considering the competitors’ investments as fixed.

This model is an MPEC.

In the upper level investment decisions of firm i* are taken.

The lower level corresponds to the previously defined market equilibrium.

MPEC Model of Firm i*

Upper Level
Max_{X_i^*} \text{Total Profits}_{i^*}
\text{s. t.}

Lower Level
Market Equilibrium
\{p, q_1, \ldots, q_{i^*}, \ldots, q_I\}
This model **assists ALL GENCOs** in taking capacity decisions.

This problem is an **EPEC**: all GENCOs simultaneously face an **MPEC**.
First Stage (Investment):

\[ \forall i \left\{ \max_{x_i} \left( t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \right) \right. \]

s.t.

Second Stage

Second Stage (Production):

\[ \forall i \left\{ \max_{q_i} \left( t(p(q_i, q_{-i}) - \delta)q_i \right) \right. \]

s.t.

\[ q_i \leq x_i \]

\[ d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}), \]
Intro & Overview of Applications

Current Bilevel Solution Methods

Applications in Electricity System

Gap in the Literature & Challenges

Conclusions
Overview

Solution Methods

A
- Bilevel Optimisation Problems & MPECs
  - Parametric Programming
  - Single-level Reduction Methods
  - Enumeration
  - Descent Methods
  - Penalty Methods
  - Trust-region Methods
  - Evolutionary Algorithms
  - Computationally efficient Solution/Approximation Methods

B
- Bilevel Equilibrium Problems & EPECs
  - Single-level reduction of Bilevel Equilibrium
  - Diagonalisation
  - Computationally efficient Solution/Approximation Methods
Classification of Solution Methods

A. Bilevel Optimisation Problems & MPECs

MPEC Model of Firm $i^*$

Upper Level
$\max_{p_i,q_i} \text{Total Profits}_i$
$s.t.$

Lower Level
Market Equilibrium
$\{p, q_i, ..., q_{i-1}, ..., q\}$

B. Bilevel Equilibrium Problems & EPECs

EPEC Model of all Firms

MPEC Model of Firm $i^*$

Upper Level
$\max_{p_i,q_i} \text{Total Profits}_i$
$s.t.$

Lower Level
Market Equilibrium
$\{p, q_i, ..., q_{i-1}, ..., q\}$
Bilevel Optimisation Problems & MPECs

- Parametric Programming
- Single-level Reduction Methods
- Enumeration
- Descent Methods
- Penalty Methods
- Trust-region Methods
- Evolutionary Algorithms
- Computationally efficient Solution/Approximation Methods

Solved directly as a non-convex optimisation problem
- Transformed into a MILP
- Vertex enumeration
- Branch and bound
- Nested Methods
- Single-level reductions
- Metamodeling-based Methods

Global optimality guaranteed.

Only applies to linear BPP (and bilinear in LL primal and dual variables).

Not available in commercial software.

Not clear how search of neighboring bases in LL scales for large-scale problems.
Bilevel Optimisation Probs (A)  
Single-level Reduction Methods (I)

1. Replacing the LL by its KKT conditions

2. Replacing the LL by primal and dual feasibility constraints, plus the strong-duality constraint.

3. Given that the LL is convex (and satisfies some constraint qualification).
Bilevel Optimisation Probs (A)
Single-level Reduction Methods (II)

Bilevel Optimisation Problems & MPECs

Single-level Reduction Methods

Transformed into a MILP

Solved directly as a non-convex optimisation problem

- the actual (exact) problem is solved (no approximations, discretizations, BigMs).
- Non-linear solvers return (at most) a local optimum.
- Depending on the problem, solvers might not even find such a point and get stuck in infeasibility.

- global optimality is achieved.
- slow CPU time (due to B&B tree).
- Solutions might have been cut off (due to discretization).
- Numerical problems due to Big Ms (Pineda and Morales, 2019)

Source: Ruiz and Conejo 2009, Baringo and Conejo 2012, Dvorkin et al 2017

Source: Wogrin et al 2011
Bilevel Optimisation Probs (A) Enumeration Methods

Bilevel Optimisation Problems & MPECs

Branch and bound

Source: Bard and Moore 1990

It solves the single-level reformulation of the BPP using a binary tree, and continues until the subproblems corresponding to all ending nodes are infeasible or have an objective value larger than the current upper bound.

Enumeration

Vertex enumeration

Source: Candler and Townsley 1982, Bialas and Karwan 1984

the Kth best method computes global solutions of BPP by enumerating the extreme points of the polyhedral constraint region

• global optimality.

• do not scale for large problems
Bilevel Optimisation Probs (A)
Descent, Penalty, Trust-region Methods

Bilevel Optimisation Problems & MPECs

- Descent Methods
- Penalty Methods
- Trust-region Methods

Decrease UL objective function value while remaining feasible. Can be challenging to calculate gradient of UL because LL optimality is required.
Source: Kolstad and Lasdon, Savard and Gauvin 1994

There exist several versions, but usually one single-level optimization problem with a penalty term in the objective function is solved. Correct choice of penalty term is questionable.
Source: Aiyoshi and Shimizu 1981, Ishizuka and Aiyoshi 1992

Iteratively approximate a certain region of the objective function locally with a model function.
Source: Liu et al 1998, Colson et al 2005

- ??
- ??
Bilevel Optimisation Probs (A)
Evolutionary Algorithms

Bilevel Optimisation Problems & MPECs

Evolutionary Algorithms

Metamodelling-based Methods
Source: Sinah various

meta-model or surrogate model is an approximation of the actual model that is relatively quicker to evaluate.

Nested Methods
Source: Mathieu, 1994

Many applications of different heuristics (genetic algorithm, particle swarm optimisation, ant colony optimisation etc). Methods are non-scalable to large-scale problems.

Single-level reductions
Source: Wan et al 2013

apply heuristic optimisation methods to solve the NLP. Global optimum not guaranteed.

• ¿?

• ¿?
Bilevel Optimisation Problems & MPECs

Source: Pineda, Bylling and Morales 2018

- Computational efficiency.
- Very successful to obtain global optimum (randomly chosen test cases).

- Heuristic method and global optimum is not guaranteed.
- Only works for linear bilevel optimisation (A) problems.

Iterative algorithm that combines advantages of obtaining good estimates of big M constants and regularization.

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Algorithm that combines MIP & regularization (Pineda et al 2018):

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Small (n = 50)</th>
<th>Medium (n = 100)</th>
<th>Large (n = 200)</th>
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<tbody>
<tr>
<td></td>
<td>#opt</td>
<td>#inf</td>
<td>Time (s)</td>
</tr>
<tr>
<td>B&amp;B</td>
<td>92</td>
<td>2</td>
<td>2761</td>
</tr>
<tr>
<td>SOS1</td>
<td>98</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>FA-5</td>
<td>11</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>FA-10</td>
<td>72</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>FA-20</td>
<td>95</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>FA-50</td>
<td>98</td>
<td>2</td>
<td>5</td>
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<td>97</td>
<td>2</td>
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<td>FA-200</td>
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<td>2</td>
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<td>FA-500</td>
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<td>2</td>
<td>36</td>
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<tr>
<td>FA-1000</td>
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<td>2</td>
<td>55</td>
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<tr>
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<td>2</td>
<td>80</td>
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<td>FA-10000</td>
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<td>36</td>
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<td>61</td>
<td>2</td>
<td>0</td>
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<td>PEN</td>
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<tr>
<td>REG-FA-2</td>
<td>94</td>
<td>2</td>
<td>2</td>
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<tr>
<td>REG-FA-5</td>
<td>98</td>
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</tr>
<tr>
<td>REG-FA-10</td>
<td>98</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

Good tradeoff between achieving optimality and CPU time.
Bilevel Equilibrium Problems (B)

Current Solution Methods Overview

- **Bilevel Equilibrium Problems & EPECs**
  - Single-level reduction of Bilevel Equilibrium
  - Diagonalisation
  - Computationally efficient Solution/Approximation Methods

- **EPEC Model of all Firms**
  - Formulated as Complementarity Problem

- **MPEC Model of Firm i**
  - Transformed into MIP
  - Open-loop approx. of closed-loop equilibria
  - Closed-form solution
  - Column generation
Bilevel Equilibrium Probs (B)
Single-level Reduction

**Bilevel Equilibrium Problems & EPECs**

Single-level reduction of Bilevel Equilibrium

Formulated as Complementarity Problem

- No discretization of variables.
- Solve the exact problem.

- Solvers might not find feasible solution.
- At most local equilibrium guaranteed.

Transformed into MIP

- MIP yields global solution.

- Only guarantee to find stationary point (not necessarily equilibrium).
- Potential loss of optimal solution/equilibrium.

Source: Dirkse and Ferris 95, Ferris and Munson 99
Source: exact (Ruiz, Conejo and Smeers 2012) / discretized (Wogrin et al 2013a)

*Single-level Reduction of Bilevel Equilibrium refers to writing out the KKT conditions of each underlying MPEC or MPCC resulting in a non-convex system of equations*
Bilevel Equilibrium Problems & EPECs

- Easy to implement (only requires solving MPECs) and computationally efficient.
- If it converges, then it yields an equilibrium.
- Not guaranteed to find a solution (even if it exists).
- There may be many equilibria.

Source: Hu and Ralph 2007

It is a kind of fixed-point iteration in which players cyclically or in parallel update their strategies while treating other players’ strategies as fixed.
Bilevel Equilibrium Problems & EPECs

Computationally efficient Solution/Approximation Methods

Open-loop approx. of closed-loop equilibria

Source: Wogrin et al. 2013b

- Computational efficiency
- Works well only if market behavior is close to Cournot.
- Only applies to specific problem.

Proposes a process where two sequential single-level equilibria (one Cournot, and one with conjecture) are solved in order to approximate a bilevel capacity expansion equilibrium (B) problem.

Closed-form solution
Column generation
Open-loop approx. of closed-loop equilibria (Wogrin et al. 2013b):

- The approximation scheme (AP) works very well when close to Cournot.
- The approximation scheme outperforms the naïve (SL) approach.
- The AP still is two orders of magnitude faster than diagonalization.
Bilevel Equilibrium Probs (B)  
Computationally Efficient Solution / Approximation Methods IV

Proposes a closed-form solution of a bilevel merchant storage optimization problem (A), and a capacity expansion equilibrium (B) problem.

- Computational efficiency
- A priori the binding number of load levels is not known.
- Closed-form solution depends on it.
- Only for specific problem (usually toy problems).

Source: Wogrin, Hobbs, Ralph, Barquin and Centeno 2013 (B)  
Siddiqui, Sioshansi and Conejo 2019 (A)
Bilevel Equilibrium Probs (B)  
Computationally Efficient Solution / Approximation Methods V

Closed-form solution (Wogrin et al. 2013c):

This is an extension of (Kreps and Scheinkman, 1983).

**Theorem**

*Let there be two identical firms with perfectly substitutable products and one load period and let the affine price $p(d)$ and the parameters be as previously defined. When comparing the open and closed loop competitive equilibria for two firms, we find the following: The open loop Cournot solution, is a solution to the closed loop conjectured price response equilibrium for any choice of the conjectured price response parameter $\theta$ from perfect competition to Cournot competition.*
Closed-form solution (Wogrin et al. 2013c): 2 load period example

Comparison Single-Level (SL) vs Bilevel (BL) - 2 load periods

- **Capacity (MW)**
  - SL: Black line
  - BL: Red dashed line

- **Profits (M Euro)**
  - SL: Black line
  - BL: Red dashed line

- **Production (MW)**
  - SL: Black line
  - BL: Red dashed line
  - Load Period 1: Red line
  - Load Period 2: Blue line

Strategic Market Behavior θ: Perfect Competition, Allaz-Vila, Cournot
Bilevel Equilibrium Problems & EPECs

Computationally efficient Solution/Approximation Methods

Column generation

Source: Pozo, Sauma and Contreras (2017)

- Optimal solution guaranteed.
- Allows to study optimistic and pessimistic TEPGEP.
- 2nd level solution space needs to be discretizable.
Bilevel Applications in Electricity Systems

Transmission Expansion Planning (TEP)

Generation Expansion Planning (GEP)

Strategic Bidding

Others: Natural Gas, PEV, etc.

Energy Storage

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Storage Strategic Investment I

• Upper level: Merchant storage investors / TSO / DSO / DR aggregators deciding: siting, sizing, bidding/offering, reliability

• Lower level: Market clearing / Economic dispatch / OPF / energy-reserve market

• Technical characteristics:
  • Representative days (1 target year), or weeks, or (limited) chronological periods
  • Ramping constraints, upper and lower bound on operation
  • Storage SOC chronological constraints
  • DC power flow

• Applied solution methods: MIP single-level reduction, heuristic methods (GA), column generation, decompositions (Benders), diagonalisation, cutting plane method, etc.
Problem size & CPU Examples:

a. 8-zone ISO New England testbed. MPEC as MIP. (600,000 constraints, 130,000 binaries). Mipgap 0.01%. 2.4h Benders.

b. 8-zone ISO New England testbed. EPEC using diagonalisation. 15 representative days 10h (50 repr. days - 30h cpu time).

c. 8-zone ISO New England testbed. MPEC. 5 representative days. 3,633,153 constraints, 947,057 continuous variables, and 29,760 binary variables. 72 h cpu.

d. 240-bus, 448-line model of the WECC interconnection. 3-level model. Column generation. 5 representative days. Max cpu 72 h. Average 12.6h.

e. 240-bus model of the WECC system. MPEC using Cutting plane method. Between 15 minutes and 1 h.

f. IEEE 118-bus transmission. 33-bus distribution test system. 5 representative days. Max cpu 808 s.

g. Real-life data from Alberta’s electricity market. MPEC using Benders. Max almost 6h.
Storage
Strategic Investment III

(Some) people working on this:

14 detailed references at the end.
Storage
Other Applications

• Assess arbitrage potential of ESS considering wind power and LMP smoothing effect (Cui et al 2018).
• Optimal scheduling (Fang et al 2015, Ju et al 2016).
• Multi hydro reservoir operation (Guo et al 2012).
• ESS strategic behavior and ownership impacts on flexibility (Hartwig & Kockar 2016).
• **Upper level**: TEP-GEP (max social welfare, min cost), proactive / reactive

• **Lower level**: Strategic Market Equilibrium (ME), Pool-based ME, Market clearing, OPF

• **Technical characteristics:**
  - 2 or sometimes even 3 levels
  - Power transfer distribution factor (PTDF), DC-OPF
  - Hourly load curve, load duration curve (no chronology between time steps), only 1 work with representative days.
  - Energy and reserve

• **Applied solution methods**: MIP/MIQCP single-level reductions (exact and discretized) Primal/dual, big M; Moore Bard Algorithm; Diagonalisation; Decompositions (e.g., Benders); Column Generation; Regularization.

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TEP/GEP
Problem Size & CPU

• Stylized representation of the main Chilean power network. MIP. Max: 53,000 binary variables, 230,000 constraints. Between 9h-22h CPU.

• IEEE RTS (24 buses, 32 generators, 38 transmission lines). Average CPU **14.37 min**. IEEE 118-bus Test System average CPU **12.25h**.


• IEEE 118-bus test system. MIP + Moore–Bard algorithm. CPU **46-68h**.
TEP/GEP
(Some) authors

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S. Pineda, J. M. Morales
I. Gonzalez-Romero, S. Wogrin, T. Gomez
Introduction

Current Solution Techniques

Applications in Electricity System

Gap in Literature & Challenges

Conclusions
Storage
Strategic Investment Gap

- Incorporate binary Unit Commitment (UC) constraints (start-up/shut-down) in operational level (usually LL); or in UL for EPECs.
- If UC is considered, it is energy-based, and not power-based. This can potentially over-estimate system/storage flexibility.
- Most works are DCOPF. Incorporate ACOPF and reactive power.
- Representative days are not linked. Disregards: potential UC constraints; and operation of large hydro reservoirs.
- Introduce stochasticity.
TEP/GEP
Technical gap in literature

• Many works lack representation of storage technologies.
• Introduce (linked) representative days.

• Incorporate binary Unit Commitment (UC) constraints (start-up/shut-down) in operational level (usually LL), or in UL for EPECs.
• If UC is considered, it is energy-based, and not power-based. This can potentially over-estimate system/storage flexibility.
• Most works are DCOPF. Incorporate ACOPF and reactive power.
Challenges
Bilevel Problems in Power Systems

• Binary UC variables in LL cause loss of convexity (Problem with existence of KKT conditions of LL). Remedy?

• AC-OPF is a non-convex problem. If OPF is in the LL, KKT conditions would only yield stationary point. Remedy?

• Solving equilibria with binary variables efficiently (Huppmann and Siddiqui 2018).

• Large-scale computations due to stochasticity. Need for efficient numerical methods.

• As for linking representative days & power-based UC:
Each day is solved independently and has a weight in the objective function.

Short-term/intraday storage is modeled within each representative period.

Long-term (e.g. hydro) storage evolution or UC cannot be modeled because there is no relationship among representative periods.

Hydro representation is generally modeled as available production within the representative period.

Enhanced Representative Periods
(Tejada, Domeshek, Wogrin and Centeno 2018)

Representative Periods with Transition Matrix and Cluster Index

We include a transition matrix and cluster index into the representative periods model, so that it is possible to link chronological information among the representatives such as storage levels or unit commitments.

INTER-day constraints: UC & hydro storage balance

INTRA-day constraints: UC & BESS
Example of superposing inter- and intra-day in ESS balance

Cluster Index

<table>
<thead>
<tr>
<th>days</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
<th>d4</th>
<th>d5</th>
<th>d6</th>
<th>d7</th>
</tr>
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<tbody>
<tr>
<td>h=1</td>
<td>rp1</td>
<td>rp1</td>
<td>rp1</td>
<td>rp1</td>
<td>rp1</td>
<td>rp2</td>
<td>rp2</td>
</tr>
</tbody>
</table>

Intra-day balance equations per rp (only for STESS)

**representative day 1 (rp1)**

\[ storage_{rp1,1} = storage_{rp1,0} + charge_{rp1,1} - discharge_{rp1,1} \]
\[ storage_{rp1,2} = storage_{rp1,1} + charge_{rp1,2} - discharge_{rp1,2} \]
\[ \vdots \]
\[ storage_{rp1,24} = storage_{rp1,23} + charge_{rp1,24} - discharge_{rp1,24} \]

**representative day 2 (rp2)**

\[ storage_{rp2,1} = storage_{rp2,0} + charge_{rp2,1} - discharge_{rp2,1} \]
\[ storage_{rp2,2} = storage_{rp2,1} + charge_{rp2,2} - discharge_{rp2,2} \]
\[ \vdots \]
\[ storage_{rp2,24} = storage_{rp2,23} + charge_{rp2,24} - discharge_{rp2,24} \]

Inter-day balance equations per rp (for STESS and LTESS)

\[ storage_{h=168} = storage_{h=0} + 5 \sum_{i=1}^{24} (charge_{rp1,i} - discharge_{rp1,i}) + 2 \sum_{i=1}^{24} (charge_{rp2,i} - discharge_{rp2,i}) \]
\[ storage_{h=336} = storage_{h=168} + \ldots \]

(Tejada, Domeshek, Wogrin and Centeno 2018)
Example of transition matrix use in representative periods

Cluster Index

\[
\begin{array}{c|cc}
\text{Cluster Index} & days & representative days \\
\hline
rp1 & d1 & rp1 \\
rp2 & d2 & rp1 \\
rp1 & d3 & rp1 \\
rp2 & d4 & rp1 \\
rp1 & d5 & rp1 \\
rp2 & d6 & rp2 \\
rp2 & d7 & rp2 \\
\end{array}
\]

Transition Matrix

\[
\begin{array}{c|cc}
\text{Transition Matrix} & rp1 & rp2 \\
rp1 & 4 & 1 \\
rp2 & 0 & 1 \\
\end{array}
\]

(Tejada, Domeshek, Wogrin and Centeno 2018)
Enhanced Representative Periods
(Tejada, Domeshek, Wogrin and Centeno 2018)

• Computationally speaking, it has the same advantages as “plain vanilla” representative days.
• Allows us to model (and optimise) both short- and long-term storage.
• It also accounts for UC links between days.

- What about calculating an hourly value of storage (“water value”)?
- What about modeling more complicated hydro subsystems?
  (Tejada, Wogrin, Siddiqui and Centeno 2019 under review in Energy)
In the linked repr. periods we have two storage balance equations: 

### Inter-day storage balance (among an aggregation of hours, e.g. month):

\[ R_{m-1,r}^{\text{inter},\omega} - R_{m,r}^{\text{inter},\omega} + \sum_{ci(p,rp,k)\in mp(m,p)} i_{\omega}^{rp,k,r} - S_{\omega}^{rp,k,r} + \sum_{r'\in up(r')} S_{\omega}^{rp,k,r'} \]

\[ + \sum_{h\in up(r')} P_{\omega}^{rp,k,h} / c_h - \sum_{h\in dw(r')} P_{\omega}^{rp,k,h} / c_h \]

\[ + \sum_{h\in up(r')} C_{\omega}^{rp,k,h} / c_h = 0 : \mu_{m,r}^{\text{inter},\omega} \]

\[ \forall \omega, m, r \; \omega' \in a(\omega) \]

### Intra-day storage balance (inside the representative period):

\[ R_{\omega}^{\text{inter},\omega} - R_{\omega}^{\text{inter},\omega} + \sum_{ci(p,rp,k)\in mp(m,p)} i_{\omega}^{rp,k,r} - S_{\omega}^{rp,k,r} + \sum_{r'\in up(r')} S_{\omega}^{rp,k,r'} \]

\[ + \sum_{h\in up(r')} P_{\omega}^{rp,k,h} / c_h - \sum_{h\in dw(r')} P_{\omega}^{rp,k,h} / c_h \]

\[ + \sum_{h\in up(r')} C_{\omega}^{rp,k,h} / c_h = 0 : \mu_{rp,k,r}^{\text{intra},\omega} \]

\[ \forall \omega, r_p, k, r \; \omega' \in a(\omega) \]

We have two dual variables, one for each balance equation (a short- and a long-term signal for storage).

\( rp \): representative period, \( k \): hour inside a \( rp \), \( ci(p,rp,k) \): cluster index, \( mp(m,p) \): relation among hours and months

(Tejada, Wogrin, Siddiqui and Centeno 2019)
Storage Value ("value of water") using Linked Representative Periods

The storage value is obtained using both dual variables:

\[
\mu_{pr}^\omega = \sum_{(rp,k) \in ci(p,rp,k)} \sum_{m \in mp(m,p)} \frac{1}{p_m^\omega} \left( \frac{\mu_{rp,k,r}^{intra,\omega}}{w_{rp}} + \mu_{m,r}^{inter,\omega} \right)
\]

Therefore, we can obtain hourly the storage/water value for short- and long-term storage using the linked representative periods formulation, which allows us to determine the interaction between BESS and hydro reservoir in a stochastic hydrothermal coordination model.

\(p_m^\omega\): scenario probability at uncertainty node \(m\) (aggregation of hours)

\(w_{rp}\): weight of representative period \(rp\)

(Tejada, Wogrin, Siddiqui and Centeno 2019)
Stylized Spanish Case for 2030

(Tejada, Wogrin, Siddiqui and Centeno 2019)

- Time scope: 1 year (2030)
- Hourly demand profile was taken from Vision 1 in Ten-Year Network Development Plan 2016 of ENTSO-E
- Hourly wind and solar production profiles. The total renewable penetration is 37%
- 8 generation technologies are considered:
  - Nuclear: 1 unit
  - Coal: 4 units
  - CCGT: 4 units
  - OCGT: 3 units
  - Fuel oil gas: 1 unit
  - Run of river: 1 unit
  - Hydro: 3 plant in a hydro subsystem (Basin)
  - BESS: 1 battery energy storage systems (1 cycle per day: 4h = 800MWh/200MW)

- Scenario tree representing monthly uncertainty on hydro inflows
Operational Planning Results
Error on Hydro Reservoir Level

Average error over all the scenarios

(Tejada, Wogrin, Siddiqui and Centeno 2019)
Energy and power
(Tejada, Morales-España, Wogrin, and Centeno, submitted IEEE TPWRS 2019)

- **Power (MW)** is the *instantaneous* value of electric demand, generation production, or transmission flow.
- **Energy (MWh)** is proportional to the *average* value of Power in a period of time (e.g., 1h).

Two main things are drawn from this example:

1. Average values could be tricky, e.g., you have one hand at 100°C and the other at -100°C, therefore, on average your body is at 0°C, but how do you feel?

2. The **flexibility problem** (e.g., load ramps) is a power problem not energy problem.

BTW: Current models around the world are developed using the energy-based concepts.
### Dutch Case Study

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<td>24.0</td>
<td>68.9</td>
<td>94.3</td>
<td>45.5</td>
</tr>
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</table>

Notes: 1) these results are the expected values in a week, 2) the operating cost is obtained from a 5-min simulation

- The investment cost is **13.8%** higher in the EB model.
- Despite the EB invests more, the operating cost is **2.0%** higher than the obtained in the PB model. Moreover, the EB operates with more CO2 emissions (**14.2%**) and renewable curtailment (**5.3%**).
- The Power-based model invests less, operates at lower cost, and it is more environmentally friendly.
Introduction

Current Solution Techniques

Applications in Electricity System

Gap in Literature & Challenges

Conclusions
Conclusions

Hierarchical equilibrium models are important when analyzing liberalized electricity markets.

They provide dynamic insight that single-level models cannot capture.

There are many applications of bilevel problems in power systems, e.g., storage investment, and TEP/GEP.

**Challenges:** Require efficient numerical techniques to handle integrality (UC), non-convexity (AC-OPF), stochasticity.
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References


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