Isaac Newton Institute for Mathematical Sciences

Algebraic Lie Theory

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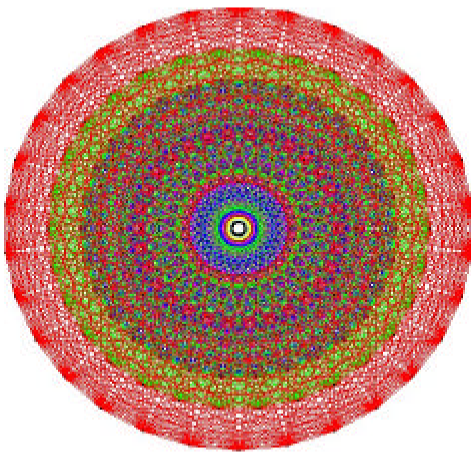
A central theme in mathematics is to study symmetries: an object or a structure is symmetrical if it looks the same after a specific type of change is applied to it. This very general concept occurs in a huge variety of situations, ranging from classical geometry to natural phenomena in physics and chemistry, e.g., crystal structures. Perhaps the most familiar symmetrical objects are the five Platonic solids, or regular polyhedra. Mathematicians study such symmetries, and their higher-dimensional analogues, by investigating the corresponding "symmetry group", that is, the possible transformations of the object which preserve the symmetry.

The programme is named after Sophus Lie (pronounced "lee"), a Norwegian mathematician who lived in the 19th century. The roots of his work lie in geometrical and analytical problems, with applications to the study of symmetries of differential equations. "Lie groups" and their "linearisations", now known as "Lie algebras", play a central role in contemporary mathematics and theoretical physics; they are fundamental tools for describing continuous symmetries of natural phenomena. In the 1950s, Lie's originally "analytic" theory was extended to an "algebraic" context, using the methods of Algebraic Geometry instead of Differential Geometry. This extension is what we mean by "Algebraic Lie Theory".

A driving force for the development of the subject has always been the abundance of challenging, yet very basic problems, especially in connection with applications. Many of these basic problems involve properties of "representations", that is, concrete realizations of the basic objects in Algebraic Lie Theory by "matrices". The entries of these matrices may be real or complex numbers (like in the original "analytic" theory), or numbers obtained by a process of reducing modulo a prime number. One of the important open problems is the determination of the "irreducible" representations - which form the building blocks for all representations - of the various structures arising in Algebraic Lie Theory.

It is remarkable that a number of such problems can be stated in relatively elementary terms, but their solution (if at all achieved!) may require a highly complex chain of theoretical arguments, or a complicated large-scale computer-aided calculation. An example is given by the recent determination of the irreducible representations of Lie groups of "type E_8" by a team of 18 mathematicians and computer scientists from the U.S. and Europe. (The figure, taken from D. A. Vogan's MIT talk on this event, shows the root system of E_8.) It is now clear that the solution of the open problems will only be achieved through the interaction of methods from various different areas.



The programme at the Newton Institute intends to make serious advances in the core area of Algebraic Lie Theory and associated representation theory. It is anticipated that the activities of the programme will lead to a focalisation and popularization of the various recent methods and advances in Algebraic Lie Theory, with a view to applications in other areas of mathematics and mathematical physics.