Isaac Newton Institute for Mathematical Sciences

Computation, Combinatorics and Probability

29 July – 20 December 2002

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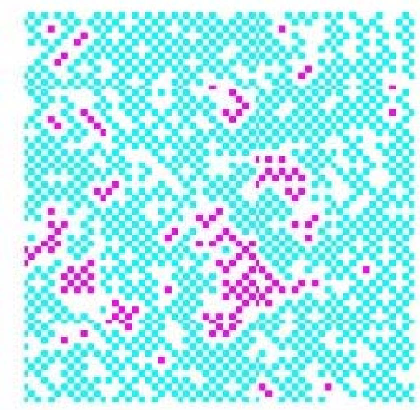
The accompanying illustration is of a configuration of the 'hard core lattice gas' model. It was created as follows. There is an underlying graph which is a square fragment of the '2-dimensional lattice'; think here of an enlarged n x n chess board. Each square of the board may contain a single particle. A configuration is an arrangement of particles that obeys the rule that no two particles may occupy adjacent squares. A k-particle configuration is assigned λ^k where λ is some real parameter. A configuration was selected at random, with probability proportional to its weight. In the illustration, the particles inherit the colour of the squares of the chessboard, here turquoise (light) and violet (dark).



Some questions:

- How many configurations are there as a function of n, the side of the chess board? Is there a closed form solution or, failing that, an asymptotic expression?
- In the absence of a closed form solution, is there an efficient algorithm for counting configurations?
- Is there an efficient (randomised) algorithm for sampling config-urations according to their weights?

Actually, we lied a little. The illustration is of a finite square fragment of a conceptually infinite random configuration.

Some further questions:

- When is it possible to make sense of the idea of 'random configuration' when the space of configurations is infinite?
- When it is possible, is the probability distribution on configurations uniquely defined? In the illustration, there seems to be a preponderance of turquoise (light) particles; by symmetry there should also be configurations with a preponderance of violet (dark) particles. Thus the answer seems to be 'no' in this case.
- Can one algorithmically sample an infinite configuration so that any finite region is
- completely determined within a bounded number of steps?
- Can a (finite window onto a) configuration be sampled efficiently, and is this question related to the one about uniqueness of the distribution?

This example illustrates the connections between computation, probability and combinatorics that will be explored in this programme. The particular example is inspired by statistical physics, but we could equally have chosen examples from, say, the theory of random graphs, or communication networks. In fact, the programme will include all interdisciplinary work that touches on the three keywords in the title.