Isaac Newton Institute for Mathematical Sciences

Developments in Quantitative Finance

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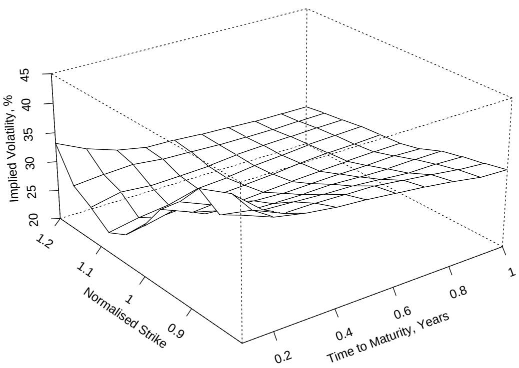
Organisers: Darrell Duffie (Stanford); David Hobson (Bath); Chris Rogers (Cambridge); José Scheinkman (Princeton)

There is a long history, dating back to at least Bachelier, of using stochastic processes to describe the unpredictable movements of stock prices. This link between finance and mathematics was reinforced in the 1970s by the realisation that models from probability could help answer some of the fundamental questions in economics about fair prices of securities. The link has broadened and deepened, so that now an understanding of stochastic calculus is a prerequisite for many roles in the City of London, and many financial institutions are reliant on such expertise for their success.

The most celebrated result in financial mathematics is the Black-Scholes formula for the price of a call option. A call option gives the holder the right, but not the obligation, to buy a unit of stock on some future date (the maturity), at a specified price (the strike). Under certain assumptions the Black-Scholes formula gives the unique fair price for a contingent claim. Given this fair price as initial fortune it is possible for an investor to replicate the payoff of the option if she follows a prescribed hedging strategy. It follows that dynamic hedging strategies can be used to remove the risks associated with options, and the development of a theory for the management of risk facilitated the explosive growth in option trading that has lasted for three decades.

A measure of the success of the Black-Scholes-Merton theory is that it provides the language used by the industry for the pricing of derivatives. Option prices are invariably quoted in terms of an implied volatility, and implied volatility provides a natural measure for comparing different options. Under the Black-Scholes model all options should trade at the same volatility. Unfortunately this is rarely the case — in the figure we plot the implied volatilities of a family of call options with different strikes and maturities. It is a stylised fact that these implied volatilities display a smile effect.



A Volatility 'Smile'

The fact that implied volatilities are not constant implies that some of the underlying assumptions must fail. In fact many would argue that all of the Black-Scholes assumptions are violated in practice — the underlying price process has jumps, trading is not instantaneous, trades must occur in discrete amounts, agents are not price-takers, there are transaction costs and taxes. One of the main aims of the programme is to improve our understanding of the impact of the breakdown of the standard assumptions on option pricing.