Isaac Newton Institute for Mathematical Sciences

Integrable Systems

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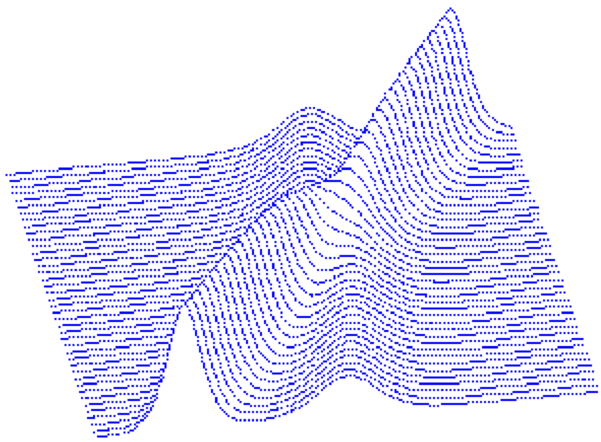
Many natural systems can be modeled by partial differential equations (PDEs), especially systems exhibiting wave-like phenomena. Such systems often have quantities that are conserved in time, common examples being energy or momentum. Often such systems are nonlinear; small changes in input can produce large changes in output, or vice versa. Mathematically, such nonlinearities make such systems difficult to study except using computer simulations.

Rather surprisingly, relatively sizeable classes of nonlinear systems are found to have an extra property, integrability, which changes the picture completely. Integrable systems have a rich mathematical structure, which means that many interesting exact solutions to these PDEs can be found. Although interesting in their own right, these systems form and archipelago of solvable models in a sea of unknown, and can be used as stepping stones to investigate properties of "nearby" non-integrable systems. Models described by integrable or near-integrabble systems are beginning to find important applications, such as more efficient ways of sending information down optical cables.

Although some mathematical examples were known in the 19th century, and experimental studies began with the famous observations of solitary waves by Scott Russell in 1834, the modern study of integrable systems dates back to pioneering work by Kruskal and co-workers in the 1960s. These researchers showed that the motion of atoms in certain model one-dimensional crystals could be described by a PDE, which originated in the study of water waves, the eponymous Korteweg-de Vries equation. Many different initial disturbances of this system resulted in the formation of one or more so-called solitary waves, isolated lumps unlike the more well-known ripples of limear wave theory.

These waves, unlike normal linear waves, had speeds that depended on the height of the wave. They had the original idea of studying the collision of such strongly nonlinear waves. The result was completely unexpected: instead of breaking up, the two waves went through a complicated collision region, then reappeared unchanged apart from a relative shift in position (see figure).



This observation, and the subsequent realisation that many other physically relevant systems had similar properties, started an explosion of interest in integrable systems that continues to the present day. Integrability is recognized by a variety of mathematical properties, any one of which usually implies the other features. However the interconnectedness of these different features is still not fully understood, and these gaps in our knowledge make it difficult to give an exhaustive classification of integrable systems. The present programme is designed to overcome many of these obstacles by bringing together leading experts in the different approaches to the study of integrable systems.