Isaac Newton Institute for Mathematical Sciences

Spaces of Kleinian Groups and Hyperbolic 3-Manifolds

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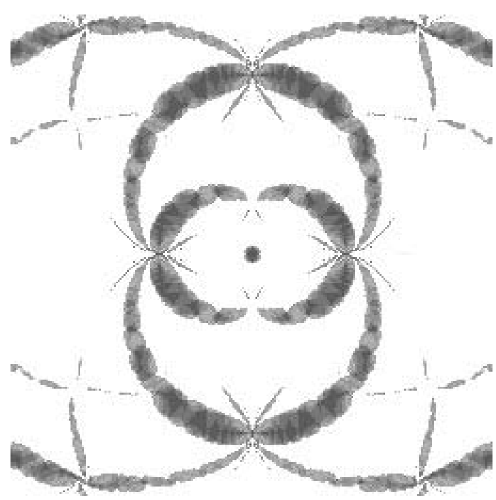
Kleinian groups lie at the meeting point of several parts of mathematics which, superficially, are quite different. One approach is via iteration theory: repeat some simple operation many times and study the result. Kleinian groups are made by iterating linear fractional transformations, in which the complex number z is replaced by the new number $(az + b) / (cz + d)$. The fixed complex numbers a, b, c, d are called the parameters. The final outcome of the iteration depends delicately on the initial choice of parameters. The picture here records what happens in parameter space when you iterate several linear fractional transformations at once. It is a close cousin of that much more familiar object called the Mandelbrot set, a concise visual record in c-space of what happens when one iterates the tranformation which replaces z by $z^2 + c$. Such pictures are nowhere near as widely known as the Mandelbrot set, but in recent years a combination of sophisticated computer explorations and rapidly developing theory have revealed fascinating new phenomena which will be closely investigated during this meeting.

Why should one care about iterating linear fractional transformations? In 1881 Poincaré had the great insight that in another guise, these transformations are the rigid motions of 3-dimensional hyperbolic or non-Euclidean geometry, geometry in which the parallel postulate fails because there are many lines parallel to a given line through another point. The study of Kleinian groups is nothing other than non-Euclidean crystallography.

Poincaré's insight is a very powerful tool. Problems in complex analysis, particularly about Riemann surfaces and their moduli, can be transformed into questions about rigid 3-dimensional geometry, and vice versa. Interesting and beautiful as they are, however, Kleinian groups may at first sight seem too special to have enormous significance. Around 1977, Thurston stunned the mathematical world by showing that, far from being isolated examples, spaces with the symmetries of a Kleinian group (more simply said, non-Euclidean crystals) are actually precise geometrical models for a very large class of 3-manifolds. These revolutionary new ideas have engendered a huge output of work whose conclusion is yet far from reached.

Recent efforts have deepened our understanding of how these different strands fit together. One major success has been the complete classification of 3-manifolds whose symmetries correspond to the fundamental group of a once-punctured torus. The present moment is seeing very rapid progress, as many of these rather special results are currently being extended to much wider classes of manifolds. Disseminating and discussing the implications of these ongoing developments will be the hot topic of the forthcoming meeting.



Parameter space of a once-punctured torus group in the plane of a Bers slice.

This picture results from joint work of Y. Komori, T. Sugawa, M. Wada, and Y. Yamasita.
Further details are available at
http://www.kusm.kyoto-u.ac.jp/complex/Bers/