Phase plots visualize a complex function $f$ by depicting its color-coded argument (phase) as an image on the domain. Though meromorphic functions are uniquely determined by their phase plots up to a positive constant, more sophisticated color schemes help to make the reconstruction easier.

The phase plot of the Euler Gamma function (top left) is enhanced by equidistant contour lines of $\log \Gamma$ and $\arg \Gamma$. Most “tiles” generated by the shading have an almost square shape, which reflects the conformality of the mapping $z \mapsto \Gamma(z)$. The points where all colors meet are poles of $\Gamma$.

The image top right depicts a special solution to the 2D Ginzburg-Landau equation

$$\frac{\partial u}{\partial t} = (1 + i\alpha) \Delta u + \lambda u - (1 + i\beta) |u|^2 u$$

Introduced to model the phenomenon of superconductivity, equations of this type also describe oscillating chemical reactions, Bose-Einstein condensates, liquid crystals, pattern formation, and self-organizing systems.

In the lower left corner we see an approximation of the inverse tangent function proposed by Isaac Newton. Starting from the Maclaurin series of $\tan^{-1} t$ for $-1 < t \leq 1$, Newton derived the representation

$$\tan^{-1} z = \frac{z}{1 + z^2} \sum_{k=0}^{\infty} \frac{(2k)!!}{(2k + 1)!!} \left( \frac{z^2}{1 + z^2} \right)^k$$

valid in the domain $(\text{Im } z)^2 - (\text{Re } z)^2 < 1/2$. This is a pioneering example of analytic continuation. Depicted is the 20th partial sum of the series.

The fourth image (lower right) is an enhanced phase of a Cauchy integral with constant density along the black spiral $S$ with endpoints $a$ and $b$. The function is a well-defined continuous branch of

$$f(z) = c \log \frac{z - a}{z - b}$$

in the complement of $S$. It has an analytic extension to a Riemann surface with logarithmic branch points at $a$ and $b$.

For more information on phase plots, see Visual Complex Functions, Springer, 2012 by Elias Wegert.