

Joint Reconstruction of PET-MRI by Parallel Level Sets

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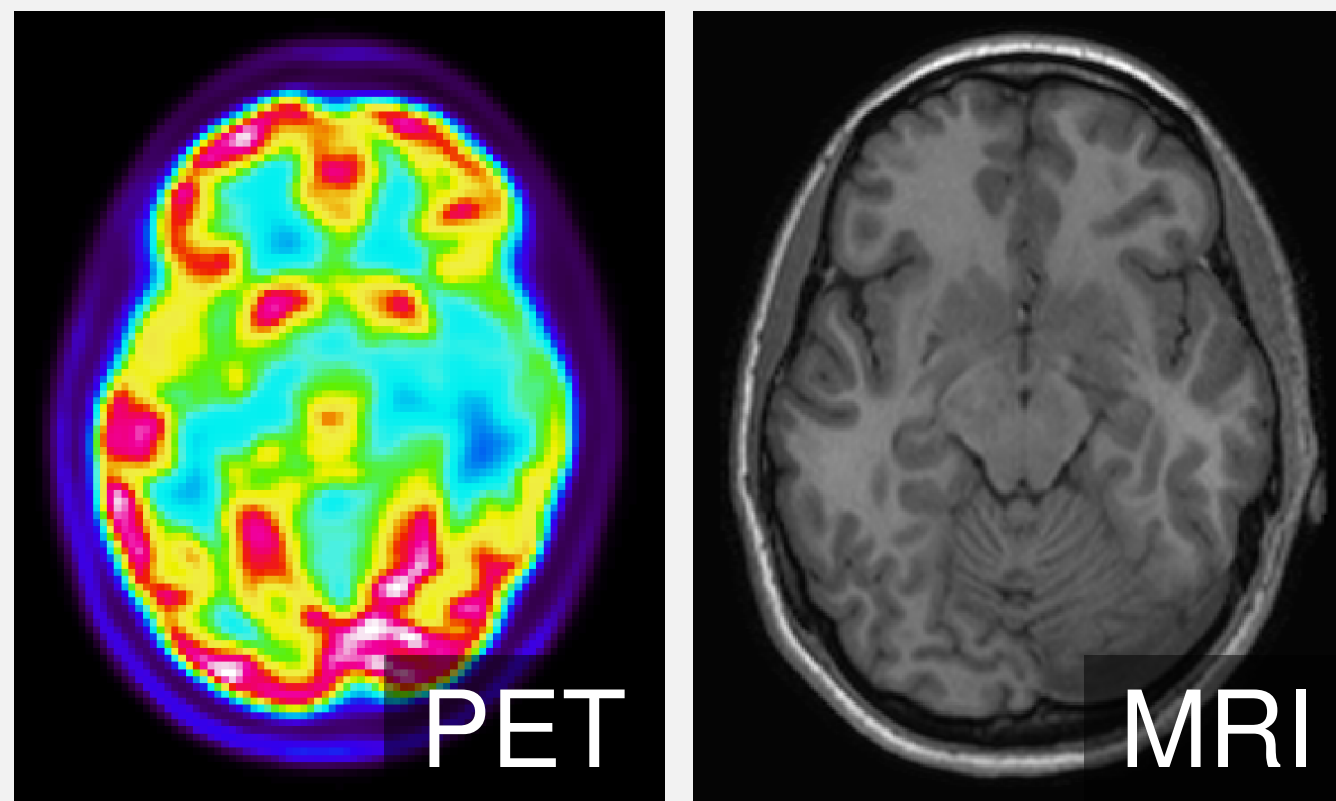
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Motivation: Similar Structures

Combined Positron emission tomography (PET) and magnetic resonance imaging (MRI) scanners acquire functional and anatomical data simultaneously [1]. We aim to exploit the expected similarity during reconstruction.



Method: Parallel Level Sets

We perform *joint reconstruction* [2] by minimizing

$$\operatorname{argmin}_{u,v} \|A(u) - f\|^2 + \|B(v) - g\|^2 + \alpha \mathcal{R}(u, v)$$

where the *parallel level sets* functional [3]

$$\mathcal{R}(u, v) := \int [\|\nabla u\| \|\nabla v\| - \langle \nabla u, \nabla v \rangle]$$

ensures the regularity and structural similarity of the solution. The smoothed norm $\llbracket x \rrbracket := (\|x/\beta\|^2 + 1)^{1/2}$ regulates which gradients should be taken into account.

This approach was successfully applied to colour image processing [3]. Similar functionals have been used in geophysics [4, 5], colour image processing [6], EIT [7] and multi-modality image registration [8].

Analysis of the Diffusion

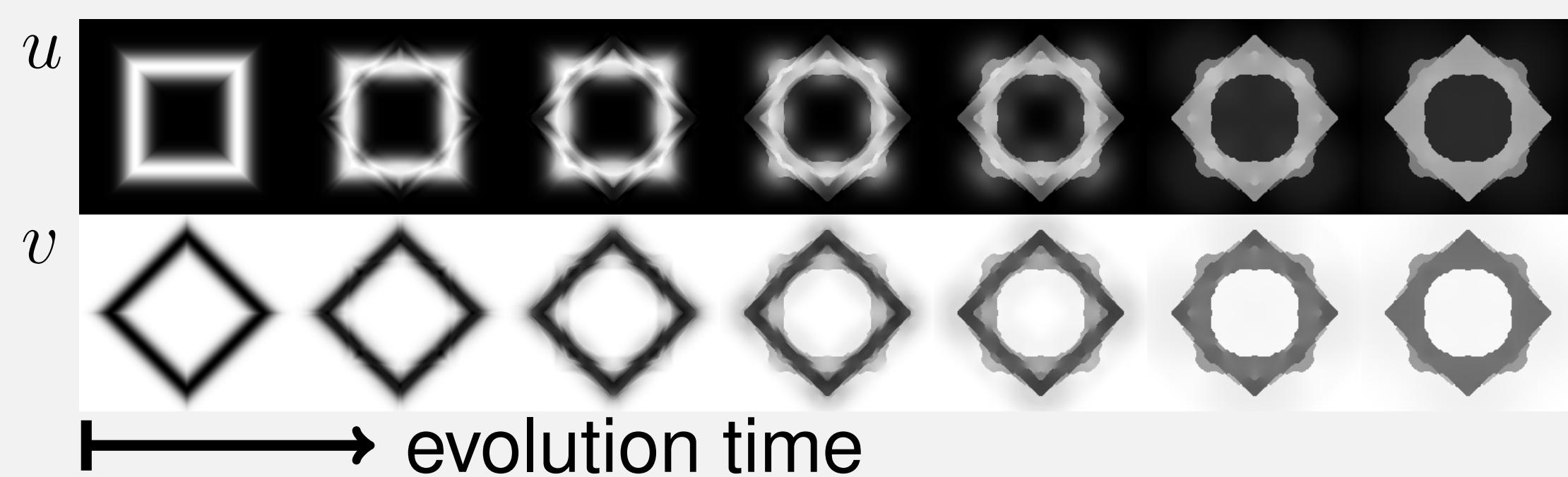
The derivative of \mathcal{R} with respect to (u, v) takes the form

$$D\mathcal{R}_{u,v} = -\operatorname{div} \begin{pmatrix} K(u, v) \nabla u \\ K(v, u) \nabla v \end{pmatrix}$$

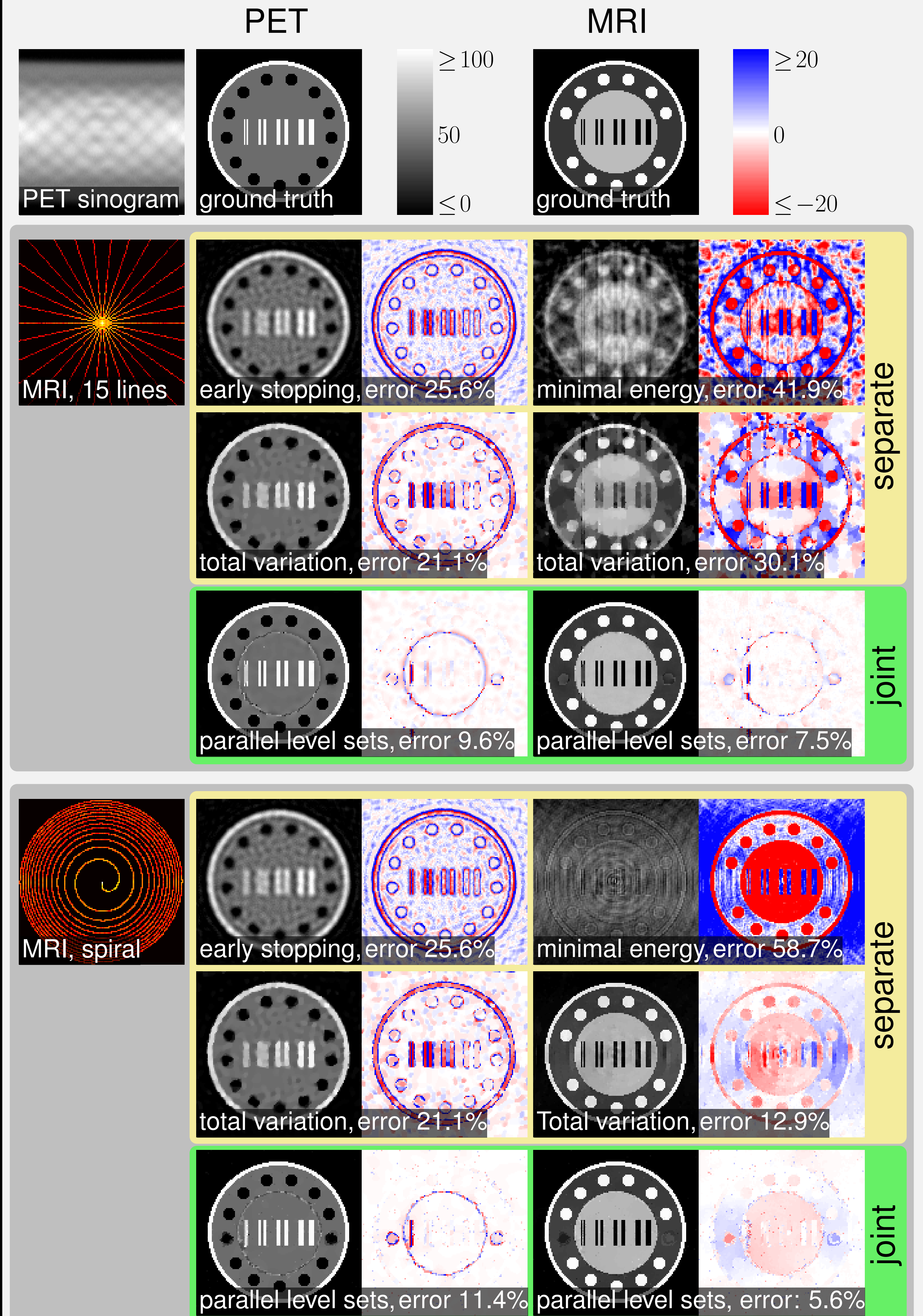
where the diffusivity

$$K(u, v) := \llbracket \nabla v \rrbracket \llbracket \nabla u \rrbracket^{-1} \operatorname{Id} - \llbracket \langle \nabla u, \nabla v \rangle \rrbracket^{-1} \nabla v \nabla v^T$$

couple the equations. This results in a flow for u where the principle directions are dictated by v and vice versa. If the edges are aligned there is no flow across the edges. The figure shows the evolution of u and v minimizing the parallel level sets functional.



Numerical Results



Conclusions

By coupling two modalities in a **joint reconstruction** we make use of more information. The images are sharper and of higher quality than **separate reconstructions**. Errors are greatly decreased in areas of joint edge information.

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References

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