Efficient Storage of Nonclassical Light in Parallel Quantum Memory Based on Λ-Scheme of Atomic Levels

Ksenia Samburskaya, Sammy Ragy, Antony R Lee, Gerardo Adesso, T. Golubeva

St. Petersburg State University, Peterhof, St. Petersburg, 198504 Russia

Saint Petersburg State University www.spbu.ru

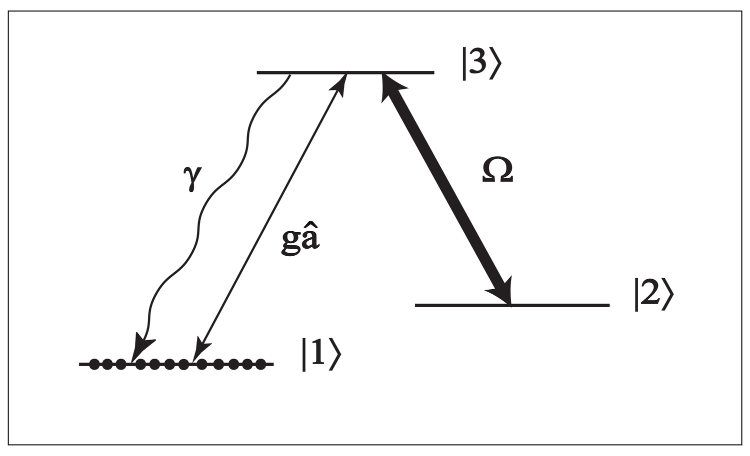
The University of Nottingham UNITED KINGDOM · CHINA · MALAYSIA

Abstract

Present study focuses on the development of the theory in continuous variables for efficient storage and retrieval of multimode squeezed and entangled light.
We consider quantum memory protocol based on atoms with **Λ**-configuration of energy levels.
We estimate degree in which light preserves squeezing during the full process of writing and read-out of a light pulse.
We demonstrate that proposed scheme preserves squeezing in the retrieved light.
The goal is to devise theoretically optimized and experimentally realizable schemes for the transmission of squeezed states of light, and assess their performance against the set benchmarks under realistic conditions.

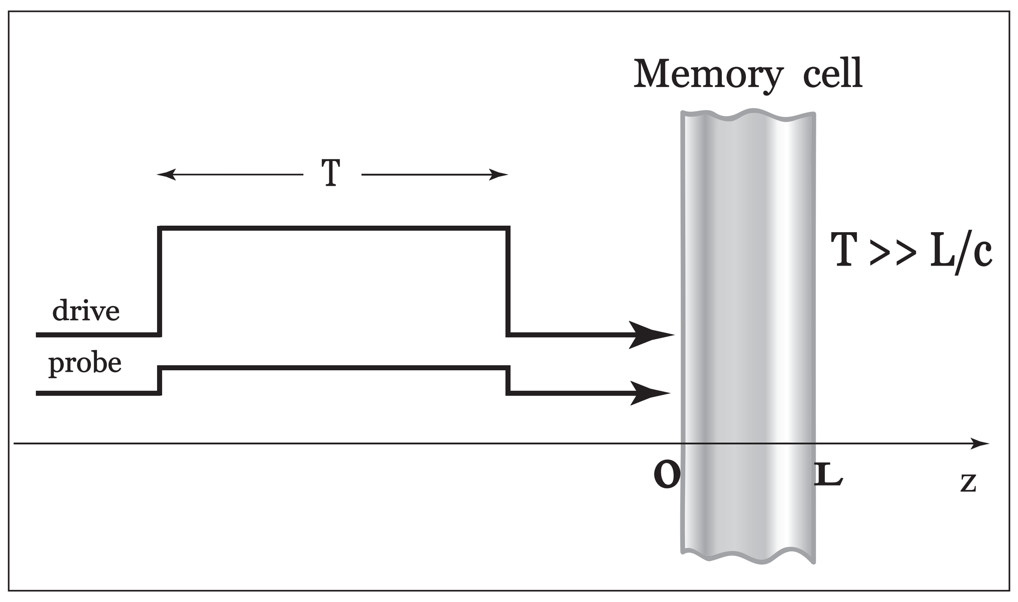
Physical Model

Atomic medium has **Λ**-scheme energy configuration.



$$|\Omega|^2 \gg g^2 n$$

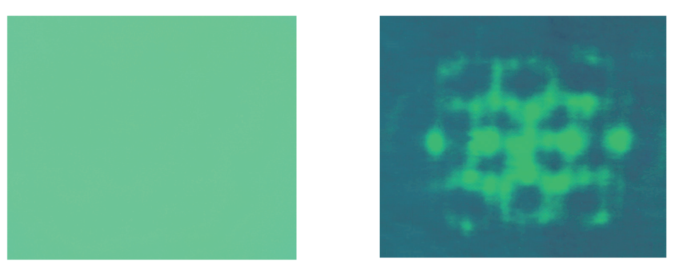
Pulses of driving and signal fields are propagating along the positive direction of z-axis. The pulse duration T is the same for the driving and signal fields, it is long enough in comparison to the length of an atomic layer. We can neglect the time intervals of wave front propagation through the medium. Therefore we consider driving field as constant during the interaction.



T – time of interaction, $T \gg L/c$
L – length of the atomic layer. $T \gg 1/\gamma$
c is a speed of light

Quantum information contained in a transverse spatial profile of the signal field is going to be stored on the atomic coherency of long-lived lower levels.

driving field signal field



Commutation relation

$$[\hat{a}(z,\vec{\rho},t), \hat{a}^{+}(z,\vec{\rho}',t')] = \delta^2(\vec{\rho}-\vec{\rho}')\delta(t-t')$$

$$[\hat{a}(z,\vec{\rho},t), \hat{a}^{+}(z',\vec{\rho}',t)] = c\left(1-\frac{i}{k_0}\frac{\partial}{\partial z}-\frac{1}{2k_0^2}\frac{\partial^2}{\partial \rho^2}\right)\delta^3(\vec{r}-\vec{r}')$$

here $\hat{a}^{+}(z,\vec{\rho},t)$ and $\hat{a}(z,\vec{\rho},t)$ - are operators of photon creation and annihilation in the signal field.
Operators of collective coherences and populations:

$$\hat{\sigma}_{mn}(\vec{r},t) = \sum_j |m\rangle\langle n|_j \delta^3(\vec{r}-\vec{r}_j), \quad m,n = 1,2,3, \quad m \neq n,$$

$$\hat{N}_m(\vec{r},t) = \sum_j |m\rangle\langle m|_j \delta^3(\vec{r}-\vec{r}_j). \quad j \text{ – number of atom.}$$

Commutation relation reads:

$$\left[\hat{\sigma}_{mn}(\vec{r},t), \hat{\sigma}_{nm}(\vec{r}\,',t)\right] = \left(\hat{N}_m(\vec{r},t) - \hat{N}_n(\vec{r},t)\right)\ \delta^3(\vec{r}-\vec{r}\,').$$

Publications

1. K. Tikhonov, K. Samburskaya, T. Golubeva, and Yu. Golubev, Storage and retrieval of squeezing in multimode resonant quantum memories, Phys. Rev. A 89, 013811 (2014)
2. K. Samburskaya, T. Golubeva, V. Averchenko, and Yu. Golubev, Quadrature Squeezing in an Isolated Pulse of Light, Optics and Spectroscopy Volume 113, Number 1, pp 86-95, 2012
3. K. Samburskaya, T. Golubeva, Yu. Golubev and E. Giacobino, Quantum holography upon resonant adiabatic interaction of fields with an atomic medium in a Λ-configuration, Optics and Spectroscopy Volume 110, Number 5, pp 775-787, 2011.
4. Tatiana Golubeva, Yuri Golubev, Ksenia Samburskaya, Claude Fabre, Nicolas Treps, Mikhail Kolobov. Entanglement measurement of the quadrature components without homodyne detection in the bright, spatially multimode far field, Phys. Rev. A, 2010, 81(1),013831.

Adiabatic approximation $T \gg 1/\gamma$

Since the interaction occurs on time intervals that are significantly longer than lifetime of an excited state an adiabatic approximation is applicable.

$$\gamma >> \Omega >> ga$$

Heisenberg-Langevien equations in adiabatic approximation can be written as:

$$\left(\frac{\partial}{\partial z} - \frac{i}{2k_s}\Delta_\perp\right)\hat{a} = -\frac{2g}{\gamma}[g\hat{a}N + \Omega\hat{\sigma}_{12}] + \hat{F}_a,$$

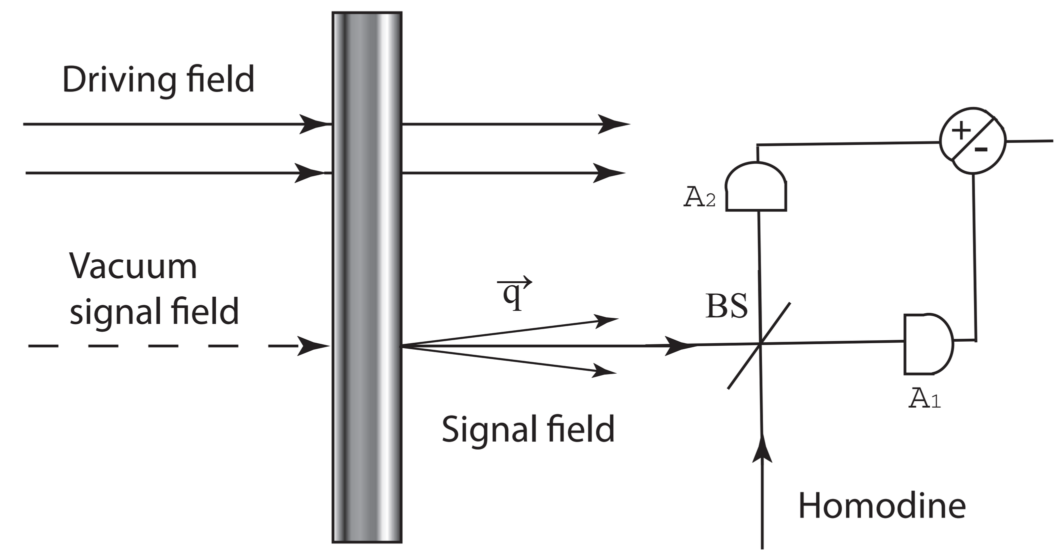
$$\frac{\partial}{\partial t}\hat{\sigma}_{12} = -\frac{2\Omega^*}{\gamma}[g\hat{a}N + \Omega\hat{\sigma}_{12}] + \hat{F}_{12}.$$

These equations are written in the Fourier domain relative to the transverse co-ordinates ($\vec{r} = (\vec{\rho}, z)$ and $\vec{\rho} \to \vec{q}$).

SETUP

We consider the following thought experiment: there is a stationary source resonator generates a quantum of light with transverse spatial distribution. (e. g. degenerate optical parametric oscillator and sub-Poissonian injection locked laser). Time gate is located at the output mirror of the generator. It cuts the pulse in time interval $[0, T]$. Pulse reaches the surface of the memory cell, stored in the cell and can be read by request.

Read-out scheme



BS – beamsplitter 50:50, A_1, A_2 – detectors
We can set the phase of the Homodine H that $H = H_X = H_X^*$

Squeezing in X and Y quadratures

$$\langle \hat{x}^{out}_{\vec{q},\omega}\hat{x}^{out}_{\vec{q}',\omega'}\rangle = \frac{1}{4}\Big[\delta^{T_R}(\omega+\omega')\delta_{\vec{q},-\vec{q}'} - f_{\vec{q}}f_{\vec{q}'}\frac{\tilde{\kappa}p}{2}\frac{1-\mu}{1-\mu/2}\frac{1}{T_R}\int_0^L\int_0^L dzdz'F^W(z,z')\,e^{-2L}\int_0^{T_R}\int_0^{T_R}dtdt'e^{-t-t'}\times I_0(2\sqrt{t(L-z)})\,I_0(2\sqrt{t'(L-z')})e^{i\omega t+i\omega' t'}\Big],$$

$$\langle \hat{y}^{out}_{\vec{q},\omega}\hat{y}^{out}_{\vec{q}',\omega'}\rangle = \frac{1}{4}\Big[\delta^{T_R}(\omega+\omega')\delta_{\vec{q},-\vec{q}'} + f_{\vec{q}}f_{\vec{q}'}\frac{4\tilde{\kappa}p(1-\mu)}{\mu}\frac{1}{T_R}\int_0^L\int_0^L dzdz'F^W(z,z')\,e^{-2L}\int_0^{T_R}\int_0^{T_R}dtdt'e^{-t-t'}\times I_0(2\sqrt{t(L-z)})\,I_0(2\sqrt{t'(L-z')})e^{i\omega t+i\omega' t'}\Big],$$

where

$$F^W(z,z') = \int_0^{T_W}\int_0^{T_W}dtdt'\ e^{-\tilde{\kappa}\mu/2|t-t'|}\ e^{-t-t'}\ I_0(2\sqrt{tz})\ I_0(2\sqrt{t'z'}),$$

and

$$\delta^T(\omega+\omega') = \frac{\sin(\omega+\omega')T/2}{(\omega+\omega')T/2}\,e^{i(\omega+\omega')T/2}.$$

$$C_1 = \frac{g^2N}{\gamma}, \qquad C_2 = \frac{2|\Omega|^2}{\gamma}$$

Acknowledgments

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Squeezing in the retrieved pulse

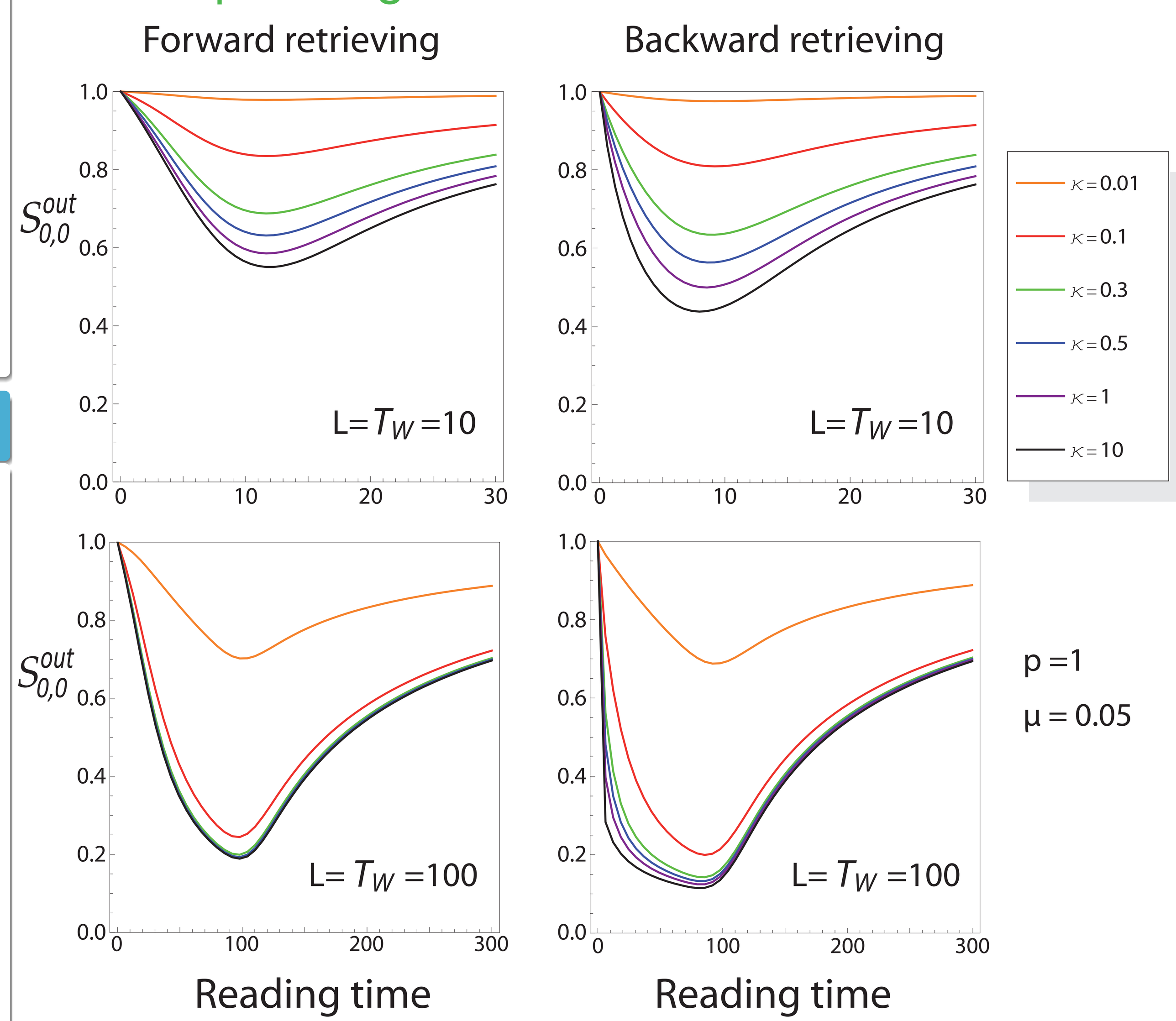
Maximum squeezing is achieved for pulse with $\kappa T \gg 1$ i.e., squeezing is limited by the laser locking. A pulse shorter than the coherence time experiences no squeezing.

$$\langle\delta\hat{x}_{out,\omega}\delta\hat{x}_{out,-\omega}\rangle = \frac{1}{4}\,e^{-r_{out}(\omega)}, \quad \langle\delta\hat{x}_{in,\omega}\delta\hat{x}_{in,-\omega}\rangle = \frac{1}{4}\,e^{-r_{in}(\omega)}$$

$r_{\omega,\vec{q}}$ – spectral parameter of squeezing for multimode light.

$$\mathcal{S}_{\omega,\vec{q}} = \exp(-r_{\omega,\vec{q}}), \qquad 0 < \mathcal{S}_{\omega,\vec{q}} < 1$$

Pulse squeezing



Benchmark

Quantum benchmarks for the transmission of general pure Gaussian states with arbitrary phase, displacement, and squeezing, randomly sampled according to realistic prior distributions, were obtained by Dr Adesso and collaborators.

Average Fidelity

$$F^{\Lambda} = \int dr d\Theta P(r,\Theta),$$

where **Λ** - ”measure and prepare”

$$F(\mathrm{R}) = \pi^N \int^N W_{in}(\mathrm{R})W_{out}(\mathrm{R})d^N\mathrm{R}$$

Wigner function for Gaussian state:

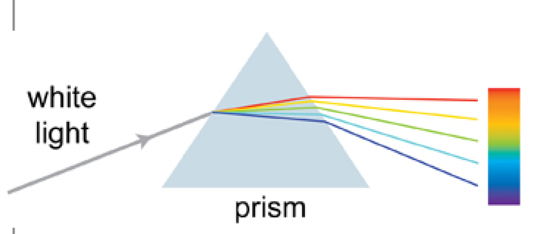
$$W_\rho(\mathrm{R}) = \frac{1}{\pi^N\sqrt{Det(\sigma)}}e^{(\mathrm{R})^T\sigma^{-1}\mathrm{R}}$$

where σ is a covariance matrix with elements $\sigma_{i,j}$

$$\sigma_{i,j} = \langle\hat{R}_i\hat{R}_j + \hat{R}_j\hat{R}_i\rangle_\rho - 2\langle\hat{R}_i\rangle\langle\hat{R}_j\rangle_\rho$$

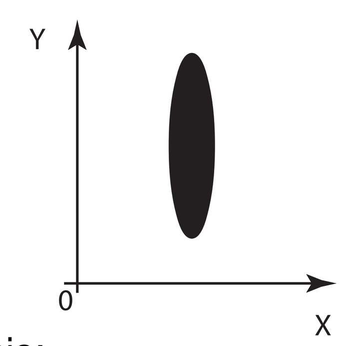
Here $\hat{R}$ is a vector of canonical operators.

Quantum Properties Of Model



$$\hat{Q}_\Omega = \frac{\hat{A}_\Omega + \hat{A}^\dagger_\Omega}{\sqrt{2}}, \qquad \hat{P}_\Omega = \frac{\hat{A}_\Omega - \hat{A}^\dagger_\Omega}{i\sqrt{2}}.$$

$$\hat{X}_\Omega = \frac{\hat{A}_\Omega + \hat{A}^\dagger_{-\Omega}}{\sqrt{2}}, \qquad \hat{Y}_\Omega = \frac{\hat{A}_\Omega - \hat{A}^\dagger_{-\Omega}}{i\sqrt{2}}.$$



Transformation from the Q-P basis to the X-Y basis:

$$\begin{bmatrix}\hat{Q}_\Omega\\ \hat{P}_\Omega\\ \hat{Q}_{-\Omega}\\ \hat{P}_{-\Omega}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & i & 1 & i\\ -i & 1 & i & -1\\ 1 & -i & 1 & -i\\ i & 1 & -i & -1\end{bmatrix}\begin{bmatrix}\hat{X}_\Omega\\ \hat{Y}_\Omega\\ \hat{X}_{-\Omega}\\ \hat{Y}_{-\Omega}\end{bmatrix} \quad \Rightarrow \quad L\sigma L^\dagger$$

Further goals

To devise a protocol suitable to teleport squeezing in an optimal way by means of a finitely entangled resource, basing on analysis of results for Fidelity of proposed memory scheme.