Split-Plot Type Designs for Physical Prototype Testing

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ABSTRACT

Companies always seek strategies to shorten product development and reduce time-to-market. Although new technologies allow for less effort in prototyping, physical testing still remains an important step in the product development cycle. Well-planned experiments are useful to guide the decision-making process. During the design of an experiment one of the challenges is to balance limited resources and system constraints to obtain useful information. It is common that prototypes are composed of several parts, with some parts more difficult to assemble than others. And, usually, there is only one piece available of each part type and a large number of different setups. Under these conditions, designs with randomization restrictions become attractive approaches. Considering this scenario, a new and additional criterion to construct split-plot type designs is presented. Designs with a small number of setups of the more difficult parts, which are especially useful for screening purposes in physical prototype testing, are discussed. Theoretical properties of the designs are provided and it is shown how to appropriately analyze the data through a real application in testing car prototypes. As a tool to practitioners, catalogs of selected 32-run split-split-plot and split-split-split-plot designs are presented.

KEY WORDS: Fractional factorial design; Hard-to-change factor; Regular design; Restricted randomization; Screening design; Two-level design.

1. INTRODUCTION

Competition impels industry to launch innovative products fast. To achieve this goal companies search for practices to increase innovation and also shorten the overall product development cycle. In many industries products are complex and may require several iterations of prototyping and testing. New technologies have contributed to improvements in the cost and time of prototyping (see, e.g., Thomke 1998). Despite the increasing use of virtual prototypes, physical testing still remains an important step in the product development cycle. According to Bisgaard and Steinberg (1997) experimentation is a key component in developing new products through prototype testing. Well-planned experiments can help reduce the effort in physical prototype testing.

Experimentation is a basic way to acquire knowledge, which may lead to the development and/or improvement of products, processes and services. Moreover, it can also provide means to save resources in research and development, to obtain quick understanding of systems and to reduce time to launch new products and services to market. Thomke (2003) points out that the corporation´s ability towards innovation depends upon experimentation, but its cost generally works as a barrier to its use. Hence, we face the challenge to develop statistical methodology to acquire information with minimum experimental effort.

One of the most important steps in an experiment is its planning. It is at this point that the practitioner deals with the tradeoff among limited resources, system constraints and the production of useful data. Thus, it is desirable that researchers develop efficient designs. It is common in many applications that the treatments are comprised of a factorial structure. The costs and the time pressure to launch new products to market turn unreplicated two-level fractional factorial designs with as few runs as possible a convenient and simple design choice for physical prototype testing experiments.

Prototypes are usually composed of several parts, with only one piece of each part type available to carry out the comprehensive testing of a large number of possible setup configurations. In addition, some parts may be more difficult to assemble than others. This characteristic is fairly common in industrial experimentation and it is called in the literature as designs with hard-to-change factors (see, e.g., Ju and Lucas 2002, and Bingham *et al*. 2008). Under these conditions, experimental designs with randomization restrictions are an attractive and suitable choice to reduce experimental effort. Split-plot type experiments are the most common examples. They are useful solutions, and consequently, there has been an increase in the number of publications in this area, which can be found, for example, in Bailey (1983); Box and Jones (1992); Bisgaard, Fuller and Barrios (1996); Huang, Chen and Voelkel (1998), Ju and Lucas (2002); Goos and Vanderbroek (2003); Vivacqua and Bisgaard (2004, 2009); Vining, Kowalski and Montgomery (2005); Jones and Goos (2007); Anbari and Lucas (2008); Cheng and Tsai (2009); Jones and Nachtsheim (2009). Bisgaard (2000) points out that split-plot designs have been used in prototype testing where they are also called inner and outer array designs.

Although interest in experiments with randomization restrictions has been growing, most of the cases consider only two degrees of difficult for changing the factor levels (hard-to-change and easy-to-change factors). Useful experiments for these situations are split-plot designs (see, e. g., Box and Jones 1992). There is less attention towards scenarios with three or more degrees of level change difficulty. Some few exceptions are Schoen (1999), Trinca and Gilmour (2001), Brien and Bailey (2006), Bingham *et al.* (2008), Jones and Goos (2009), and Castillo (2010).

Bisgaard (2000) points out that split-plot type designs are more common in practice than they appear in engineering literature. Ganju and Lucas (1999) recommended that the randomization be planned according to the operational restrictions instead of making it for convenience of the analysis procedure. Moreover, it is important to acknowledge that a frequent and undesirable feature found in industrial experimentation is the lack of consonance between the analysis and the way the experiment is carried out. In other words, it is common to find split-plot type designs analyzed as completely randomized designs, as pointed out by Bisgaard, Fuller and Barrios (1996). This misunderstanding can cause serious consequences in the identification of active effects, and so, less than optimum conditions may be chosen to run the process.

This paper contributes to the design of experiments with different degrees of difficulty in the setup. In section 2, the experiment motivating this paper is described. It is the assembly of a Baja car prototype. The application involving the Baja car contains all the characteristics typically found in experimentation with prototype testing: a severe time restriction, only one prototype car and one piece of each part type to perform all different assembly configurations, low budget and hard-to-change factors. Considering all these features, Section 3 presents a split-split-split-plot (here denoted by split^3-plot to simplify notation) design for the Baja experiment taking into account that the four parts considered have different degrees of assembly difficulty. In the literature there are some criteria to choose experimental designs. The most known is maximum resolution (see, e.g., Fries and Hunter 1980). However, to better discriminate the designs it is necessary to consider other criteria as minimum aberration and clear effects (see Bingham and Sitter 1999a, 1999b, 2003; Bingham, Schoen and Sitter 2004; Yang *et al.* 2006, Xu 2009). Here, we introduce an additional criterion: minimum number of changes (MNC) at each stratum. In section 4 a general framework of MNC designs is described. For this aim, the first step is the determination of the number of generators for each stratum. Section 5 shows the analysis of the Baja experiment. In section 6, minimum aberration MNC designs are cataloged, considering sixteen and thirty-two runs, three and four degrees of difficulty, and seven to eleven factors. We highlight that these catalogs may not be unique, which means that other non-isomorphic designs may present the same properties. Finally, some conclusions and future research are described in section 7.

2. EXAMPLE: BAJA COMPETITION EXPERIMENT

The Society of Automotive Engineers (SAE) promotes the development of college students through car competitions all over the world. Near an upcoming Baja SAE Competition, a university team is again faced with the challenge of building its car prototype. The goal is to specify a setup of a number of auto parts in the assembly of a Baja car (customary based on Beatle chassis). In previous competitions, the choice was made considering a series of trial-and-error tests. For the first time, this team is executing a planned experiment to optimize resources and guide the decision-making process. The objective of the experiment is to maximize the performance of the vehicle on two tests carried out on a paved street with an asphalt layer. The first one, called acceleration test, evaluates the time that the vehicle takes to cover a distance of 30 meters starting from a complete stop. The second one, called velocity test, measures the final velocity reached by the Baja at the 100 meters mark. Figure 1 illustrates these two tests. The ideal setup is the one that simultaneously provides the maximum velocity at 100 meters mark and the minimum time to cover the first 30 meters.

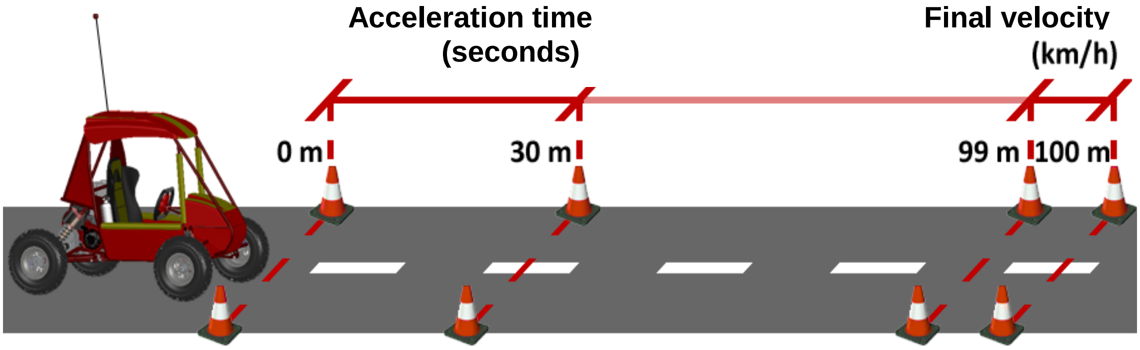

Figure 1. Illustration of the acceleration and velocity tests.

Auto parts considered in the experiment are: ceasefire plate, driven pulley, driver pulley, and tires. The function of the ceasefire plate is to protect the pilot. From previous experience, it is believed that the ceasefire plate has little effect on the performance on the acceleration and velocity tests; however the possibility of interactions with the other auto parts has never been evaluated. Two different types of ceasefire plates are available for testing. Figure 2 illustrates a driven pulley. The cam is responsible to guide the plate of the driven pulley when it is in movement. Variations in its angle may provide more or less time for the displacement of the plates. The factors related to the driven pulley included in this study are: angle of the cam, material of the driven pulley, type and pressure of a spring fixed between the cam and the opposite-cam. The factors related to the driver pulley are: material of the driver pulley cap; and masses and types of springs of the driver pulley. Also two different levels for pressure of the tires are considered.

The nine factors identified by the team of engineering students are summarized in Table 1. Two levels for each factor are chosen by the team but are left undisclosed due to confidentiality. There are a total of 512 possible combinations to assemble the Baja if a full factorial design is considered. Unfortunately the team only had about a week to perform the experiment. In addition to the time constraint, the team has available only one piece of each part type. Due to the approach of the competition date, the team reached a consensus that would be able to execute a total of 32 runs.

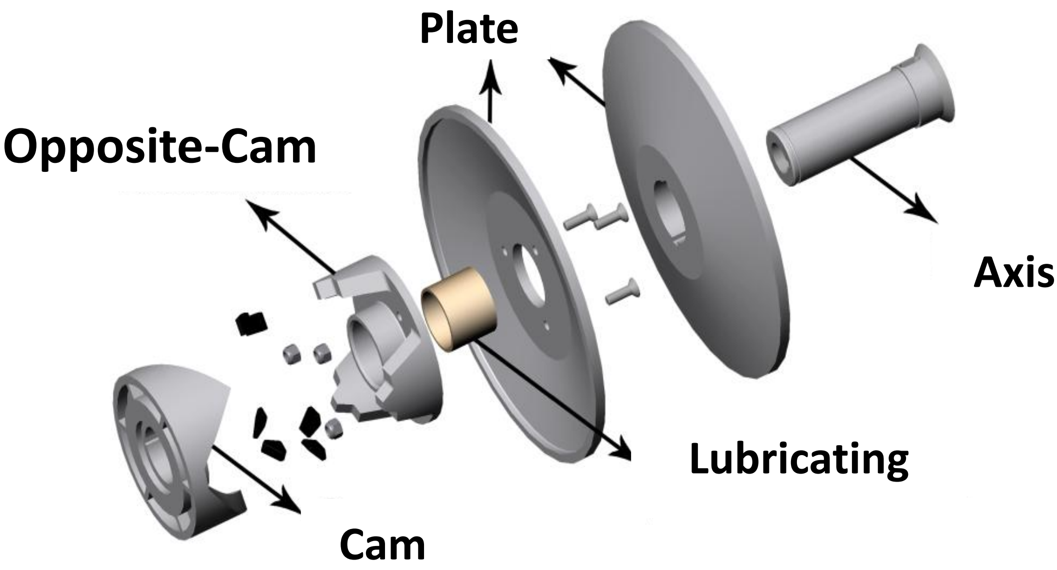

Figure 2. Baja driven pulley.

Table 1. Factors for the Baja competition experiment

Factor Label	Description of the Factor
A	Type of ceasefire plate
B	Driven pulley cam angle
C	Driven pulley material
D	Driven pulley spring type
E	Driven pulley spring pressure
F	Material of driver pulley cap
G	Driver pulley mass
H	Driver pulley spring type
J	Tire pressure

Moreover, switching different auto parts is associated with distinct levels of difficulty and also distinct times to complete the task. For instance, switching between levels of tire pressure (factor J) is a relatively easy operation when compared to switching different driver pulley case caps, masses and springs (factors F, G and H), which turns out to be a lot easier than assembling a driven pulley (factors B, C, D and E). A change of the ceasefire plate (factor A) requires almost a full disassemble of the prototype and is a very complex and time-consuming operation. In summary, the Baja experiment involves nine factors, which can be arranged in four groups according to the difficulties in changing their levels, as shown in Table 2. The design of the Baja competition experiment is described on the next section.

Table 2. Factors according to their degrees of difficulty in changing their levels

+ difficult	→		**+ easy**
Group 1	Group 2	Group 3	Group 4
Ceasefire plate setting (A)	Driven pulley cam angle (B) Driven pulley material (C) Driven pulley spring (D) Driven pulley spring pressure (E)	Driver pulley cap (F) Driver pulley masses (G) Driver pulley springs (H)	Tire pressure (J)

3. SELECTING A DESIGN FOR THE BAJA COMPETITION EXPERIMENT

This car prototype experiment includes nine two-level factors and thirty-two runs are considered a reasonable number to be executed. One alternative is to run a completely randomized 2^{9-4} fractional factorial design, choosing high order interaction effects to generate four of the nine factors. For example: a design of resolution IV with generators F=BCDE; G=ACDE; H=ABDE; J=ABCE (see, for example, Box, Hunter, Hunter, 2005, p. 272) might be applied. Since there is only one part of each type, the different prototypes are assembled and tested sequentially. Therefore, a completely randomized design requires thirty-two complete assemblies and disassemblies of the vehicle.

The nine factors are classified in four groups according to a decreasing degree of difficulty in changing their levels: one factor in the first group; four factors in the second; three in the third and one in the fourth, as shown in Table 2. Operationally, it is more rational to execute all runs under the same ceasefire plate (factor A) and then switch the other one to carry out the remaining runs. Therefore, a completely randomized design is disregarded as a plausible design. Instead, a split^3-plot experiment is used.

Then, we need to choose a set of four generators for the design, which would lead to a design with good properties. Some statistical criteria usually used to select a split-plot type design are maximum resolution, minimum aberration and clear effects. Here, we introduce an additional criterion: minimum number of changes at each stratum. Due to the operational characteristic of the assembly of the prototype and the large number of factors to screen, a design with a small number of setups of the more difficult parts is desirable. For this aim, the first step is the determination of the number of generators for each stratum. We would need more generators associated with the first strata. It is important to note that to preserve the split-plot type structure the generators associated with a specific stratum should only involves factors from that stratum or previous strata. In the Baja competition experiment, there are a total of five factors in the first two strata (one in the first and four factors in the second stratum), so, to obtain at least a resolution III design, at most two generators should be associated with the second stratum. The generators may be, for example, D=AB and E=AC. The other two generators should be assigned to the third stratum, for example, G=AF and H=BCF. Therefore, we have a $2 \times 2^{4-2} \times 2^{3-2} \times 2$ split^3-plot design with two, eight, sixteen and thirty-two changes at each stratum, respectively. This design has resolution III and attends the minimum aberration criterion.

4. A GENERAL FRAMEWORK

We now introduce notation for regular two-level split-plot type designs with minimum number of changes in each stratum. Suppose there is a total of *k* factors each at two levels and these factors can be divided into *s* groups according to the degree of difficulty in changing their levels. Each group has k_i, *i* = 1, 2, …, *s*, factors such that $\sum_{i=1}^{s} k_i = k$. The groups of factors are arranged in decreasing order according to the degree of difficulty in such a way that the first group is the most difficult to change and the last group is the easiest to change.

Split-plot type experiments involve several strata, leading to different kinds of experimental units. The number of strata is the same number of the groups of factors (*s*). Here, we denote the design as a split^{s-1}-plot to simplify notation. The number of experimental units in each stratum is equal to the number of treatment changes considered in the respective stratum. Furthermore, the total number of runs of the experiment is equal to the number of treatment changes in the last stratum.

Hence we need to distinguish among the number of treatment changes for each stratum, $N_i, i = 1,2,..., s,$ and the total number of runs N involved in a single replicate of the experiment. In general, in unreplicated experiments, N_i also represents the number of experimental units associated with the *i*-th stratum and N , the number of experimental units associated with the combined design.

Now suppose that for the first stratum there are k_1 factors each at two levels. We then use a $2^{k_1-p_1}$ fractional factorial design with $N_1 = 2^{(k_1-p_1)}$ treatments. Similarly for the second stratum we have k_2 factors each at two levels. The combined design used up to the second stratum is a $2^{(k_1-p_1)} \times 2^{(k_2-p_2)}$ fractional factorial split-plot design with

$N_2 = 2^{(k_1 - p_1)} \times 2^{(k_2 - p_2)} = \prod_{i=1}^{2} 2^{(k_i - p_i)} = 2^{\sum_{i=1}^{2}(k_i - p_i)}$ treatments, and so on. Note that $\sum_{i=1}^{s} p_i$ represents the number of generators of the fractionated design, and we denote this number by *p*. The total number of runs of the complete combined design is $N = N_s = 2^{(k_1 - p_1)} \times 2^{(k_2 - p_2)} \times \ldots \times 2^{(k_s - p_s)} = \prod_{i-1}^{s} 2^{(k_i - p_i)} = 2^{\sum_{i=1}^{s}(k_i - p_i)}$. Note that the same number of runs can be obtained, for example, by a $2^{(k-p)}$ completely randomized fractional factorial design or a $2^{(k_1 - p_1)} \times 2^{\sum_{i=2}^{s}(k_i - p_i)} = 2^{(k_1 - p_1)} \times 2^{(k - k_1) - (p - p_1)}$ fractional factorial split-plot design. However, the first design involves only one randomization step, and therefore each replicate of the experiment needs $N = 2^{(k-p)}$ treatment changes. The second has two randomizations steps with k_1 hard-to-change factors and k - k_1 easy-to-change factors, and therefore each replicate of the experiment needs $N_1 = 2^{(k_1 - p_1)}$ changes of the k_1 hard-to-change factors and $N = N_2 = 2^{(k-p)}$ changes of the (k - k_1) easy-to-change factors, $N_1 < N_2 = N$. So, there are fewer changes associated with the hard-to-change factors, which is desirable in practice to reduce experimental effort. Therefore, the notation $2^{(k_1 - p_1)} \times 2^{(k_2 - p_2)} \times \ldots \times 2^{(k_s - p_s)}$ is employed to emphasize the number of randomization steps.

The combined design can be constructed using regular fractional factorial designs. For example, maximum resolution and minimum aberration provide common criteria to choose the design generators. Nevertheless, in the presence of factors with different degrees of changing difficulty an additional selection criterion is of practical relevance: minimum number of changes of the levels of the factors in each stratum. Given the number of strata (s), the task is to determine the number of generators in each stratum that would provide the minimum number of changes. The following algorithm describes this task.

ALGORITHM FOR THE DETERMINATION OF # OF GENERATORS AT EACH STRATUM

INPUT:

k = # of factors

$N = 2^{k-p}$ = # of runs

p = # of generators

s = # of strata

k_i = # of factors at stratum i, I = 1, 2, …, s

$k_1, k_2, \cdots, k_s : 0 \leq k_i \leq k; \sum_{i=1}^{s} k_i = k$.

Calculate $F_i = \sum_{j=1}^{i} k_j ; i = 1, \cdots, s$, which represents the number of factors up to stratum i.

For i = 1 to s by 1

$$j = \text{int}\left(\frac{\ln F_i}{\ln 2}\right) = \text{int}(\log_2 F_i)$$

$$\textit{if } 2^j - 1 \leq F_i \leq 2^j \textit{ then } G_i = \min(2^j - 1 - j; p)$$
$$\textit{else if } 2^j + 1 \leq F_i \leq 2^{j+1} \textit{ then } G_i = \min(F_i - (j+1); p)$$

End

$p_1 = G_1$

For i=1 to s-1 by 1

$$p_{i+1} = G_{i+1} - G_i$$

End

OUTPUT:

$p_1, p_2, \ldots, p_s$: $0 \le p_i \le p$; $\sum_{i=1}^{s} p_i = p$; p_i= # of generators at stratum *i*.

The algorithm provides the number of generators for each stratum and the minimum number of changes in each stratum is $2^{k_1 - p_1}$: $2^{(k_1 - p_1)+(k_2 - p_2)}$: ...: $2^{\sum_{j=1}^{s}(k_j - p_j)}$.

For the Baja competition experiment:

k = 9 factors
N = 32 runs
p = 4 generators
s = 4 strata
k_1 =1; k_2 =4; k_3 =3; k_4 =1.

Using the algorithm, we obtain the following number of generators for each stratum: p_1 =0; p_2 =2; p_3 =2; p_4 =0. Therefore, the MNC design is a $2^{1-0} \times 2^{4-2} \times 2^{3-2} \times 2^{1-0}$ with two changes in the first stratum, eight changes in the second stratum, sixteen changes in the third stratum and thirty-two changes in the fourth stratum.

The algorithm considers designs in which $N_1 \le N_2 \le \ldots \le N_s$. For example, if

k = 6 factors
N = 32 runs
p = 1 generator
s = 4 strata
k_1 =1; k_2 =1; k_3 =1; k_4 =3,

the algorithm returns p_1 =0; p_2 =0; p_3 =1; p_4 =0, leading to a $2^{1-0} \times 2^{1-0} \times 2^{1-1} \times 2^{3-0}$ design with two changes in the first stratum, four changes in the second stratum, four changes in the third stratum and thirty-two changes in the fourth stratum. Since the second and third strata have the same number of changes, these two strata represent only one stratum, and, in fact, the design reduces to a split^2-plot design $2 \times 2^{2-1} \times 2^3$. We call cases like this as degenerated designs.

5. ANALYSIS OF THE BAJA COMPETITION EXPERIMENT

We now discuss the analysis of the Baja competition experiment. For this experiment, we have executed thirty-two runs according to a $2\times2^{4-2}\times2^{3-2}\times2$ split^3-plot design with generators *D=AB, E=AC, G*=AF, *H=BCF*. The ordinary least squares estimates of all possible effects are the same as if the experiment were a completely randomized fractional factorial design. Frequently normal probability plots have been used to identify active main effects and interactions from unreplicated completely randomized two-level fractional factorial designs. However, this tool needs adjustments to analyze data from experiment with randomization restrictions as the Baja competition experiment. A consequence of the restricted randomization is that not all effects have the same variance. Therefore, one alternative to identify active effects is to use normal probability plots, keeping in mind that only effects with the same variance should be plotted on the same normal or half-normal plot. Therefore, the analysis should be conducted on a stratum-by-stratum basis.

An important aspect of the analysis of split-plot type designs is to study the error structure of the contrasts. Bisgaard (2000) presents a clear discussion of the standard error of contrasts for split-plot designs, including a straightforward rule to identify which effects are testable by each of the two error types. Here we use an extension of his results to accommodate split$^{s\text{-}1}$-plot designs with several strata, which have as many error types as the number of strata (*s*).

In a split^3-plot design there are four strata. The effects in each stratum have the same variance expression, which is different from the variance expression of the effects in the other strata. Thus, we divide the effects into groups according to their associated variances. The correct classification of the effects is crucial. For unreplicated designs, the number of effects associated with the first stratum is $\boldsymbol{E_1 = N_1 - 1}$ and for the stratum *i* is

$$\boldsymbol{N_i - 1 - \sum_{j=1}^{i-1} E_j}, \qquad \boldsymbol{i = 2, 3, \ldots, s.}$$

There are 32 runs in the Baja competition experiment and hence 31 degrees of freedom. Therefore, there are one, six, eight and sixteen effects associated with the first, second, third and fourth stratum, respectively. Since there is only one effect in the first stratum, a normal probability plot for this stratum is meaningless. This is the drawback of MNC designs: the first strata have few associated effects and in some cases the evaluation of active effects for these strata may be impossible. Therefore, such designs are useful when the main interest is interaction effects rather than main effects for these strata, as is the case of the ceasefire plate (factor A) of the Baja experiment.

The effect associated with the first stratum is *A=BD=CE=FG.* The second stratum contains the main effects of the factors B, C, D, E + their interactions + interactions of (B, C, D, E) × A + their aliases. Thus, the effects B=AD, C=AE, BC=DE=FH, D=AB, E=AC and ABC=GH are associated with the second stratum and have the same variance, but different from the variance of A. Figures 3(a) and 3(b) show half-normal plots of these effects for acceleration time and velocity, respectively. The driven pulley material (Factor C) seems to be an active effect on the acceleration time while the driven pulley spring pressure (Factor E) for the velocity.

The main effects of factors (F, G, H) + interactions of these factors + interactions (F, G, H) x (A, B, C, D, F) + their aliases are evaluated in group three. Figures 3(c) and 3(d) show half-normal plots for the third stratum. Interaction between driven pulley material and driver pulley masses (C×G) is a common active effect for acceleration time and velocity.

Finally, the effects of the factor J + interactions (A, B, C, D, E, F, G, H) × J + their aliases are evaluated in group four. Figures 3(e) and 3(f) show half-normal plots for this stratum.

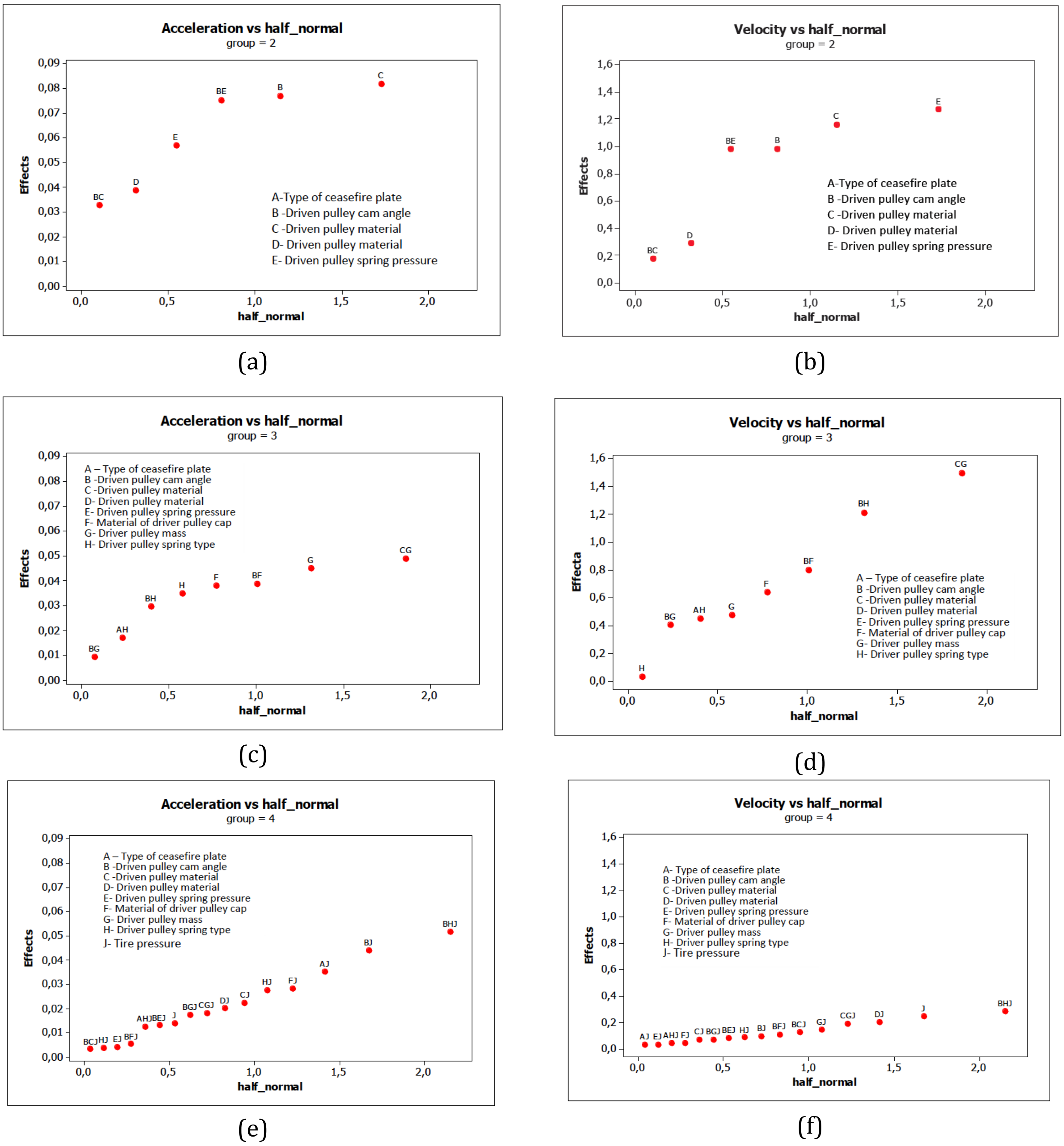

Figure 3. Half-normal plots of effects of (a) second stratum for acceleration time, (b) second stratum for velocity, (c) third stratum for acceleration time, (d) third stratum for velocity, (e) fourth stratum for acceleration time, (f) fourth stratum for velocity .

Recommended levels of the factors are identified through an analysis of interaction plots of possible active effects. Table 3 summarizes these findings. Note that the design has resolution III and to reach these results, we assume that some interactions are negligible. So, we propose confirmatory runs before the choice of the prototype for the competition. It is worth noting that, using these recommended levels, the team obtained the best performance ever achieved since started participating in the Baja competition.

Table 3. Recommended levels

Factor	Description of the factor	Recommendation
A	Ceasefire plate setting	-1 or +1
B	Driven pulley cam angle	-1
C	Driven pulley material	-1
D	Driven pulley spring	-1
E	Driven pulley pressure	+1
F	Drive pulley case cap	-1 or +1
G	Drive pulley masses	-1
H	Drive pulley springs	-1
J	Tire pressure	-1

6. CATALOGS OF DESIGNS WITH MINIMUM NUMBER OF CHANGES

Catalogs of selected 32-run split^2-plot and split^3-plot designs with minimum number of changes at each stratum from seven up to eleven factors are presented, respectively, in Tables 4 and 5. In each table, information as the number of factors, the number of the generators and the number of changes in each stratum are included in the first three columns. The next columns show the generators in each stratum employing the codes of Table 6, according to the notation used in Chen, Sun and Wu (1993). Additionally, the word pattern length (WLP), the resolution (R), the number of clear main effects (C1) and the number of clear two-factor interactions (C2) of the designs are shown in the last four columns. The selected designs in Tables 4 and 5 have the properties: $p_1 \geq 1$ (at least one generator in the first stratum) and $N_1 < N_2 < \cdots < N_s$. The design marked by * is used in the Baja example and is included in Table 5 for illustration purpose.

Table 4. Minimum number of changes 32-run two-level split^2-plot designs

	# factors			# of generators			# changes			Design			WLP			
#	1	2	3	1	2	3	1	2	3	stratum 1	stratum 2	stratum 3	3 4 5 6 7 8 9 10	R	C1	C2
7	3	1	3	1	0	1	4	8	32	3		29	1 0 1 1	3	4	18
7	3	2	2	1	1	0	4	8	32	3	5		2 1 0 0 0	3	2	11
7	3	3	1	1	1	0	4	16	32	3	13		1 1 1 0 0	3	4	12
7	4	2	1	1	1	0	8	16	32	7	11		0 3 0 0 0	4	7	6
7	5	1	1	2	0	0	8	16	32	3 5			2 1 0 0 0	3	2	11
8	3	1	4	1	0	2	4	8	32	3		13 22	1 2 3 1 0	3	5	13
8	3	2	3	1	1	1	4	8	32	3	5	30	2 1 2 2 0	3	3	18
8	3	3	2	1	2	0	4	8	32	3	5 6		4 3 0 0 0	3	2	13
8	3	4	1	1	2	0	4	16	32	3	5 14		2 3 2 0 0	3	3	9
8	4	3	1	1	2	0	8	16	32	7	11 13		0 7 0 0 0	4	8	7
8	5	2	1	2	1	0	8	16	32	3 5	14		2 3 2 0 0	3	3	9
8	6	1	1	3	0	0	8	16	32	3 5 6			4 3 0 0 0	3	2	13
9	3	1	5	1	0	3	4	8	32	3		13 21 26	1 5 6 2 1	3	6	9
9	3	2	4	1	1	2	4	8	32	3	5	14 25	2 4 6 2 0 1	3	4	11
9	3	3	3	1	2	1	4	8	32	3	5 6	31	4 3 3 4 0 0 1	3	3	21
9	3	4	2	1	3	0	4	8	32	3	5 6 7		7 7 0 0 1	3	2	15
9	3	5	1	1	3	0	4	16	32	3	5 9 14		3 7 4 0 1	3	2	9
9	4	4	1	1	3	0	8	16	32	7	11 13 14		0 14 0 0 0 1	4	9	8
9	5	3	1	2	2	0	8	16	32	3 5	9 14		3 7 4 0 1	3	2	9
9	6	2	1	3	1	0	8	16	32	3 5 6	15		4 6 4 0 0 1	3	3	8
9	7	1	1	4	0	0	8	16	32	3 5 6 7			7 7 0 0 1	3	2	15
10	3	1	6	1	0	4	4	8	32	3		13 21 25 30	1 10 11 4 3 1 1	3	7	8
10	3	2	5	1	1	3	4	8	32	3	5	14 22 25	2 8 12 4 2 3	3	5	4
10	3	3	4	1	2	2	4	8	32	3	5 6	9 30	5 6 7 8 3 1 1	3	2	13
10	3	4	3	1	3	1	4	8	32	3	5 6 7	25	7 8 3 4 5 3 1	3	3	18
10	3	5	2	1	3	1	4	16	32	3	5 9 14	31	3 8 11 4 1 3 1	3	3	12
10	3	6	1	1	4	0	4	16	32	3	5 9 14 15		4 14 8 0 4 1	3	1	9
10	4	4	2	1	3	1	8	16	32	7	11 13 14	19	0 18 0 8 0 5	4	10	0
10	4	5	1	1	4	0	8	16	32	3	5 9 14 15		4 14 8 0 4 1	3	1	9
10	5	3	2	2	2	1	8	16	32	3 5	9 14	31	3 8 11 4 1 3 1	3	3	12
10	5	4	1	2	3	0	8	16	32	3 5	9 14 15		4 14 8 0 4 1	3	1	9
10	6	2	2	3	1	1	8	16	32	3 5 6	9	30	5 6 7 8 3 1 1	3	2	13
10	6	3	1	3	2	0	8	16	32	3 5 6	9 14		6 10 8 4 2 1	3	1	9
10	7	1	2	4	0	1	8	16	32	3 5 6 7		25	7 8 3 4 5 3 1	3	3	18
10	7	2	1	4	1	0	8	16	32	3 5 6 7	9		8 10 4 4 4 1	3	0	0
11	3	1	7	1	0	5	4	8	32	3		5 14 22 25 31	2 14 22 8 6 9 2	3	6	0
11	3	2	6	1	1	4	4	8	32	3	5	14 22 25 31	2 14 22 8 6 9 2	3	6	0
11	3	3	5	1	2	3	4	8	32	3	5 6	15 23 25	4 14 16 8 12 9	3	5	4
11	3	4	4	1	3	2	4	8	32	3	5 6 7	9 26	8 12 10 12 12 7 2	3	2	10
11	3	5	3	1	3	2	4	16	32	3	5 9 14	22 26	3 16 13 12 13 3 3	3	4	4
11	3	6	2	1	4	1	4	16	32	3	5 9 14 15	22	4 18 12 8 12 5 4	3	2	2
11	3	7	1	1	5	0	4	16	32	3	5 6 9 14 15		8 18 16 8 8 5	3	1	10
11	4	4	3	1	3	2	8	16	32	7	11 13 14	19 21	0 26 0 24 0 13	4	11	0
11	4	5	2	1	4	1	8	16	32	3	5 9 14 15	22	4 18 12 8 12 5 4	3	2	2
11	4	6	1	1	5	0	8	16	32	3	5 6 9 14 15		8 18 16 8 8 5 0	3	1	10
11	5	3	3	2	2	2	8	16	32	3 5	9 14	22 26	3 16 13 12 13 3 3	3	4	4
11	5	4	2	2	3	1	8	16	32	3 5	9 14 15	22	4 18 12 8 12 5 4	3	2	2
11	5	5	1	2	4	0	8	16	32	3 5	6 9 14 15		8 18 16 8 8 5 0	3	1	10
11	6	2	3	3	1	2	8	16	32	3 5 6	15	23 25	4 14 16 8 12 9	3	5	4
11	6	3	2	3	2	1	8	16	32	3 5 6	9 14	31	6 12 16 12 6 7 4	3	2	10
11	6	4	1	3	3	0	8	16	32	3 5 6	9 14 15		8 18 16 8 8 5 0	3	1	10
11	7	1	3	4	0	2	8	16	32	3 5 6 7		9 26	8 12 10 12 12 7 2	3	2	10
11	7	2	2	4	1	1	8	16	32	3 5 6 7	9	26	8 12 10 12 12 7 2	3	2	10
11	7	3	1	4	2	0	8	16	32	3 5 6 7	9 10		10 16 12 12 10 3	3	1	10
11	9	1	1	6	0	0	8	16	32	3 5 6 9 14 15			8 18 16 8 8 5 0	3	1	10

Table 5. Minimum number of changes 32-run two-level split^3-plot designs

	# factors for stage				# generators for stage				# changes for stage				Design				WLP										
#	1	2	3	4	1	2	3	4	1	2	3	4	stratum 1	stratum 2	stratum 3	stratum 4	3	4	5	6	7	8	9	10	R	C1	C2
7	3	1	2	1	1	0	1	0	4	8	16	32	3		13		1	1	1	0					3	4	12
7	3	2	1	1	1	1	0	0	4	8	16	32	3	5			2	1	0						3	2	11
7	4	1	1	1	2	0	0	0	4	8	16	32	3	5			2	1	0						3	2	11
8	3	1	3	1	1	0	2	0	4	8	16	32	3		5 14		2	3	2	0	0				3	3	9
8	3	2	2	1	1	1	1	0	4	8	16	32	3	5	14		2	3	2	0	0				3	3	9
8	3	3	1	1	1	2	0	0	4	8	16	32	3	5 6			4	3	0	0	0				3	2	13
9*	**1**	**4**	**3**	**1**	**0**	**2**	**2**	**0**	**2**	**8**	**16**	**32**		**3 5**	**9 14**		**3**	**7**	**4**	**0**	**1**				**3**	**2**	**9**
9	3	1	4	1	1	0	3	0	4	8	16	32	3		5 9 14		3	7	4	0	1				3	2	9
9	3	2	3	1	1	1	2	0	4	8	16	32	3	5	9 14		3	7	4	0	1				3	2	9
9	3	3	2	1	1	2	1	0	4	8	16	32	3	5 6	15		4	6	4	0	0	1			3	3	8
9	3	4	1	1	1	3	0	0	4	8	16	32	3	5 6 7			7	7	0	0	1				3	2	15
10	3	1	4	2	1	0	3	1	4	8	16	32	3		5 9 14	31	3	8	11	4	1	3	1		3	3	12
10	3	1	5	1	1	0	4	0	4	8	16	32	3		5 9 14 15		4	14	8	0	4	1			3	1	9
10	3	2	3	2	1	1	2	1	4	8	16	32	3	5	9 14	31	3	8	11	4	1	3	1		3	3	12
10	3	2	4	1	1	1	3	0	4	8	16	32	3	5	9 14 15		4	14	8	0	4	1			3	1	9
10	3	3	2	2	1	2	1	1	4	8	16	32	3	5 6	9	30	5	6	7	8	3	1	1		3	2	13
10	3	3	3	1	1	2	2	0	4	8	16	32	3	5 6	9 14		6	10	8	4	2	1			3	1	9
10	3	4	1	2	1	3	0	1	4	8	16	32	3	5 6 7		25	7	8	3	4	5	3	1		3	3	18
10	3	4	2	1	1	3	1	0	4	8	16	32	3	5 6 7	9		8	10	4	4	4	1			3	1	9
11	3	1	4	3	1	0	3	2	4	8	16	32	3		5 9 14	22 26	3	16	13	12	13	3	3		3	4	4
11	3	1	5	2	1	0	4	1	4	8	16	32	3		5 9 14 15	22	4	18	12	8	12	5	4		3	2	2
11	3	1	6	1	1	0	5	0	4	8	16	32	3		5 6 9 14 15		8	18	16	8	8	5			3	1	10
11	3	2	3	3	1	1	2	2	4	8	16	32	3	5	9 14	22 26	3	16	13	12	13	3	3		3	4	4
11	3	2	4	2	1	1	3	1	4	8	16	32	3	5	9 14 15	22	4	18	12	8	12	5	4		3	2	2
11	3	2	5	1	1	1	4	0	4	8	16	32	3	5	6 9 14 15		8	18	16	8	8	5			3	1	10
11	3	3	2	3	1	2	1	2	4	8	16	32	3	5 6	15	23 25	4	14	16	8	12	9			3	5	4
11	3	3	3	2	1	2	2	1	4	8	16	32	3	5 6	9 14	31	6	12	16	12	6	7	4		3	2	10
11	3	3	4	1	1	2	3	0	4	8	16	32	3	5 6	9 14 15		8	18	16	8	8	5			3	1	10
11	3	4	1	3	1	3	0	2	4	8	16	32	3	5 6 7		9 26	8	12	10	12	12	7	2		3	2	10
11	3	4	2	2	1	3	1	1	4	8	16	32	3	5 6 7	9	26	8	12	10	12	12	7	2		3	2	10
11	3	4	3	1	1	3	2	0	4	8	16	32	3	5 6 7	9 10		10	16	12	12	10	3			3	1	10

Table 6. Matrix for 16 and 32-run designs used to build Tables 4 and 5

1	**2**	3	**4**	5	6	7	**8**	9	10	11	12	13	14	15	**16**
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

NOTE: The independent columns are in bold and numbered as 1, 2, 4, 8 and 16. The 16-run designs use the first four rows; the 32-run designs use the five rows.

7. CONCLUSION

In physical prototype testing experiments with factors which levels have different degrees of difficulty to change, split-plot type designs represent a cost-effective method for the generation of information to guide the decision-making process. The basic steps for planning these experiments are (1) identification of the factors and the corresponding degrees of difficulty to change their levels, (2) grouping of the factors with similar degrees of difficulty, and (3) choice of a convenient design. The analysis should be conducted on a stratum-by-stratum basis.

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