POSSIBLE EMERGENCE OF FUNDAMENTAL CONSTANTS FROM A GENERALLY COVARIANT MODEL OF QUANTUM MECHANICS

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Abstract. By making a radical change to the textbook interpretation of quantum mechanics, I obtain a generally covariant model in which many of the fundamental parameters of the standard model of particle physics can be seen. The fact that most of these parameters are covariant rather than invariant provides a robust mechanism for calculating their values, at least to a first approximation.

I estimate the values of eight fundamental parameters, all accurate to better than 1%, or experimental uncertainty, if greater. These parameters are the mass ratios of the electron, proton and neutron, the pion and kaon mass ratios, the kaon/eta mass ratio, the Weinberg angle, the Cabibbo angle and the CP-violating phase. The model also throws light on the causes of the chirality of the weak force, neutrino oscillations and the CP-violation of neutral kaons, the existence of three generations of electrons, and on the measurement problem.

1. Introduction

1.1. **The standard model.** The standard model of particle physics [1], put together in the 1970s, has withstood the tests of time remarkably well, requiring only one or two apparently minor adjustments in the ensuing four decades. Nevertheless, it does not explain everything about fundamental particles, but requires something in the region of 20 external parameters, that can only be determined by experiment. Most of these parameters are not Lorentz invariant, since they are experimentally measured to 'run' with the energy scale. The standard model is therefore not Lorentz-invariant, but only Lorentz-covariant.

A full understanding of this covariance requires an allocation of the fundamental parameters to representations of the Lorentz group. But there is a bigger problem, namely that for compatibility with general relativity a model must also be generally covariant, that is covariant under the action of the full general linear group $GL(4,\mathbb{R})$. (Note that other meanings of the term 'generally covariant' are often used in the literature [2]. This particular usage is reasonably common and is the most appropriate for our purposes.)

The standard model as presently constituted is very far from being generally covariant. All the gauge groups, which form the algebraic core of the model, are supposed to be generally *invariant*. Yet the relationships between the gauge groups are expressed by mixing parameters which are not even Lorentz-invariant, let alone generally invariant. It is my belief that the existence of this mathematical contradiction is the essential problem that needs to be solved before real progress can be made in understanding the genesis of the fundamental parameters.

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1.2. **A proposed generally covariant model.** In order to make a start on this problem, I proposed in [3] a generally covariant model for quantum mechanics. This was obtained by embedding the Dirac group $SL(2,\mathbb{C})$ as a subgroup of $SL(4,\mathbb{R})$. The property of general covariance then permits an analysis of parameters that are generally invariant, as well as parameters that are generally covariant.

For the purposes of this paper it is convenient to define a parameter to be *invariant* if it is invariant under the special linear group $SL(4,\mathbb{R})$. This allows one to ignore an overall scale factor, and to talk about masses being invariant when only mass *ratios* can (strictly speaking) be generally invariant. Indeed, a quantum theory cannot, by definition, be scale-invariant, and therefore it makes no sense to include the scalars of $GL(4,\mathbb{R})$ in the group of symmetries.

I proposed [3] two specific mass parameters that appear to be invariant, and that give rise to two equations that are both consistent with experiment, and predictive. The most impressive is a prediction of the mass of the tau lepton to 8 significant figures, that agrees with experiment to within $.1\sigma$, where the relative value of σ is around 10^{-4}. I believe, therefore, that these equations lend strong support to the model.

1.3. **Classification of parameters.** Fundamental covariant parameters must belong to small representations of $SL(4,\mathbb{R})$. The available small dimensions (see [4, 5]) are 4 (two such representations, dual to each other), 6, 10 (a dual pair), 15 and 20 (five of these, one self-dual and two dual pairs). The parameters as used in the standard model do not necessarily lie in irreducible representations, and it may be useful to consider in particular $6+10a$, $6+10b$ and $1+15$, all three of which factorise as the product of two 4-dimensional representations.

The representations of $SL(4,\mathbb{R})$ divide naturally into faithful (or 'fermionic') representations, in which the negative identity matrix acts as -1, and unfaithful (or 'bosonic') representations, in which it acts as $+1$. The representation theory exhibits a fundamental division of the representations into four types, distinguished by the rank modulo 4 of the tensors they belong to. Two of these are bosonic, and two are fermionic, and may have some relationship to the left- and right-handed spins in the standard model. Let us denote these four types by a *quasi-spin*, taking the values 0 and 1 for bosons, and $\pm 1/2$ for fermions. These quasi-spins correspond via exponentiation to complex scalars 1, -1 and $\pm i$ respectively.

This concept of quasi-spin has an intricate and not necessarily obvious relationship to the (at least five) different concepts of spin and isospin in the standard model, so caution is required in interpretation. The fermionic representations are those of dimension 4, and the two dual pairs of dimension 20. The bosonic representations split into quasi-spin 1 (dimensions 6 and 10) and quasi-spin 0 (dimensions 1, 15 and the self-dual 20).

1.4. **Covariant supersymmetry.** In particular, an observer can interact with an elementary particle either bosonically (for example, using a photon) or fermionically (for example, using an electron or a neutrino). A fermionic interaction leads to interchanging the fermionic representations with the bosonic representations. Thus observed properties (as opposed to intrinsic properties) of fermions can appear in bosonic representations, and vice versa. On the one hand, this requires us to be very careful to distinguish intrinsic from observed properties, but on the other hand it provides a powerful tool to help us make this distinction.

As we shall see, the dividing line between intrinsic and observed properties appears in the new model in a rather different place from where it is generally assumed to be in the standard model. This issue is discussed in depth, but at a non-technical level, in [6], where it is placed in a wider philosophical context including possible approaches to the measurement problem and quantum gravity.

In a sense, this mapping between fermionic and bosonic representations is a kind of 'supersymmetry', but it is a covariant supersymmetry, not an invariant supersymmetry. Thus it is quite different from the supposed invariant supersymmetry that is a guiding principle of more conventional attempts to go beyond the standard model [7, 8]. Covariant supersymmetry of this kind can best be described in terms of a Clifford algebra.

The bosonic representations are, by definition, those in which -1 acts trivially, so that they are representations of the quotient group of $SL(4,\mathbb{R})$ by the group of overall signs. This quotient group is $SO^+(3,3)$, that is the connected component of the identity in the Lie group $O(3,3)$. Its defining representation is the 6-dimensional representation given above. The above bosonic representations form the Clifford algebra of $SO(3,3)$, which is a 64-dimensional representation that underlies various novel theories [9]. On the one hand it is the tensor product of $4a+4b$ with itself, and on the other hand it breaks up into 7 graded pieces (grades 0 to 6) of dimensions $1+6+15+(10a+10b)+15+6+1$. The even-numbered grades have quasi-spin 0, and the odd-numbered grades have quasi-spin 1.

2. Measurement of parameters

2.1. **Local versus global.** In the proposed model, the internal symmetries of an elementary particle are described by one copy of $SL(4,\mathbb{R})$, and macroscopic symmetries of spacetime by another copy. Measurement of a particle involves setting up an isomorphism between these two copies. Now the group of all isomorphisms of $SL(4,\mathbb{R})$ with itself is a Lie group with four connected components. There are therefore four different types of isomorphism, which correspond to four different types of measurement. In [3] I proposed a provisional mapping of these components to the four fundamental forces, although the complicated 'mixing' of the different forces in the real world implies that this is a vastly over-simplified picture.

For a fixed connected component, there are 15 independent parameters of such an isomorphism. It is clear that not all of them correspond to observables. But equally, some parameters do correspond to observables, or the particle could not have any macroscopic properties at all. The mass equation referred to above [3] has four independent masses, namely the three generations of electron and the proton, to which should be added three dimensions for momentum (of the neutrinos, for example), making 7 in all. This suggests that roughly half the parameters of the isomorphism are observable. This corresponds closely to the standard approach to quantum mechanics, in which the adjoint representation of $GL(4,\mathbb{R})$ is given a complex structure which reduces the number of parameters from 16 to 8.

2.2. **Complex structure.** In [3], the three generations of electron and the proton were associated with four different directions in internal spacetime, that correspond roughly to three directions of space and one of time. If we restrict to a single generation of electron, then we have chosen a single direction of internal space to associate with internal time.

The electromagnetic interactions of electrons and protons are then described by first using the gauge group $U(1)$ to put a complex structure on this 2-dimensional spacetime. This complex structure converts internal spacetime into a 2-dimensional complex space. General covariance allows us to transfer this complex structure to the macroscopic world, where it has a similar interpretation. The essential point is that the mass of an electron is not an internal property of the electron, but a property of the interaction of the electron with the environment, and in particular with the local gravitational field. The measurement of the mass can be carried out *either* in the traditional way with the complex structure attached to the internal copy of spacetime, *or* with the complex structure attached to the macroscopic copy of spacetime. Mathematically the two processes are equivalent, so if the model is valid, then they must give the same answer.

2.3. **Electron, proton and neutron.** The model [3] therefore implies that the mass ratios of electron, proton and neutron can be determined by imposing a sensible complex structure onto macroscopic spacetime. In effect we have transferred the internal properties of particle spins to macroscopic properties of the spinning of the environment. This seems absurd and impossible, but it is an inevitable consequence of general covariance.

So we need a complex structure that allows us to describe the relationship between the experimental frame of reference and an (approximately) inertial frame. Since the gravitational effects of the Earth, Sun and Moon are all significant, we need to go up at least to the Solar System scale, but it seems reasonable to treat the rest frame of the Solar System as near enough to an inertial frame. On this scale there is an obvious preferred direction in space, perpendicular to the Earth's orbit around the Sun. This direction defines the complex structure, up to choice of the sign of i, that is, up to complex conjugation.

Complex conjugation in the (z,t) plane corresponds to time-reversal, while complex conjugation in the (x,y) plane corresponds to parity-reversal. We cannot have one without the other, since physical reality requires transformations with positive determinant.

In the model [3], the proton and neutron are supposed to be independent of space direction, at least to a first approximation. Therefore their mass ratio can only depend on time, and therefore on a frequency ratio. There is only one plausible frequency ratio to use, namely the ratio of the lengths of the day and the year. These particles are fermions, so they have to rotate through 720° before reaching their initial state. The same property transfers mathematically to the macroscopic situation, so that the parameter that we must actually use is the number of days in two years.

Hence we obtain the following estimate for the mass ratio:

$$\begin{aligned} m(n)/m(p) &\approx 1 + 1/730.5 \\ &\approx 1.0013689. \end{aligned} \qquad (1)$$

This compares to the measured value

$$\begin{aligned} m(n)/m(p) &\approx 939.565/938.272 \\ &\approx 1.0013781. \end{aligned} \qquad (2)$$

The estimate is therefore as good as one could possibly hope for, given the various approximations that have been made along the way.

Now let us consider the electron. The electron has a definite association with a particular direction in (internal) space. Our mathematical procedure has transferred this direction from the internal copy of spacetime to the macroscopic spacetime. There is only one plausible direction to use, and that is the direction of the Earth's axis, relative to its orbit. The angle between the Earth's axis of rotation and its axis of revolution is approximately 23.44°.

The electron represents the charged part of the difference between the neutron and the proton, that is, the part that remains in place after beta decay while the neutrino takes much of the remainder of the energy out of the Solar System altogether. The mass of the electron should therefore increase as the axis tilts further from the z direction towards the (x, y) plane. Since the mass ratio can only be a simple ratio of distances, the only possible formula is

$$\begin{aligned} m(e)/m(p) &\approx \sin 23.44^\circ/730.5 \\ &\approx .00054454. \end{aligned} \tag{3}$$

This compares to a measured value of

$$\begin{aligned} m(e)/m(p) &\approx .51100/938.272 \\ &\approx .00054462. \end{aligned} \tag{4}$$

This estimate is accurate to 1/60 of one percent, which is considerably more accurate than I could have reasonably hoped for by this procedure.

2.4. **On the nature of mass.** These two predictions together, I suggest, provide strong evidence that the proposed generally covariant model of quantum mechanics captures something important about the nature of mass that is missing from the standard model. In the rest of this paper I investigate how other mass ratios, and other parameters of the standard model, can be similarly transferred from the internal to the macroscopic symmetry group of spacetime.

3. Three angles

3.1. **The CP-violating phase.** In the above discussion the parity symmetry P corresponds to negating one direction within the plane of the Earth's orbit, effectively reversing the direction of the orbit. It is probably better to consider the P symmetry to negate the axis of revolution instead, that is the direction perpendicular to the ecliptic. The introduction of charge into the proton/electron system corresponds to the rotation of the Earth, and therefore the charge symmetry C corresponds to reversing the axis of this rotation. If the axis of the Earth's rotation were perpendicular to the ecliptic, then the P and C symmetries would be equal, and CP symmetry would be a property of the standard model. But in fact there is an angle of 23.44° between the directions of the C and P symmetries.

This is not exactly the angle that appears as the CP-violating phase in the standard model, because physicists generally introduce extra factors of i to make matrices Hermitian, when it is often more natural mathematically to use anti-Hermitian matrices. This introduces an extra component of 90°, and the angle that appears explicitly in the standard model is the complementary angle, estimated here as 66.56°. Experimentally it is measured at $1.20 \pm .08$ radians, that is around $(68.8 \pm 4.6)^\circ$, so this prediction is accurate to $.5\sigma$.

3.2. **Chirality of the weak force.** The original experimental observation of the chirality of the weak force [10] was an observed correlation between the internal properties of the spin of a cobalt-60 nucleus, and the direction of the momentum of the ejected electron. As such it falls somewhere in the middle between the internal symmetries and the macroscopic symmetries. In the standard model, however, this is expressed entirely in terms of internal symmetries. Our usual mathematical procedure then translates the phenomenon of chirality into macroscopic spacetime.

Chirality is not exhibited by a system with only two components of uniform rotation, but requires three independent rotations. The orbit of the Moon around the Earth has a sufficiently large effect on the motion of the laboratory that it is a plausible source of the observed chirality. Indeed, the requirement of general covariance implies that it is the *only* significant source of chirality in particle physics. The proposed model therefore explains the chirality of the weak force as being due to the chirality of the macroscopic motion of the laboratory. Moreover, measurements of this chirality must correspond to measurements of properties of the Moon's orbit.

In particular, the Weinberg angle [11] is a mixing angle between the weak force (which we have just seen, is related to the orbit of the Moon) and the electromagnetic force (which we earlier identified as depending on the rotation of the Earth). Now the angle between these two rotations is not constant, and cannot directly correspond to the Weinberg angle.

But if we take out the angle of 23.44° from the electromagnetic interaction, so that we eliminate the effect of the Earth's rotation, then we are left with an angle related to the Moon's orbit alone. The Moon's orbit is inclined at an average angle of 5.14° to the ecliptic, varying on a predictable 347-day cycle between approximately 4.99° and 5.30°. This suggests that the Weinberg angle can be made by adding these two angles together. This gives an estimate of

$$\theta_W \approx (28.58 \pm .15)^\circ \tag{5}$$

in which the error bar is not a statistical uncertainty, but a predictable systematic oscillation. This is consistent with the variety of different experimental measurements in the range between 28° and 29°.

This proposed decomposition of the Weinberg angle into two components has interesting consequences for the masses of the intermediate vector bosons, that is the Z and W bosons, whose mass ratio according to the standard model is $\cos\theta_W$. If, as I suggest, this mass ratio can be decomposed into smaller pieces, then perhaps this can throw some light on the internal structure of the Z and W bosons. In particular, it may be more enlightening to consider them as composite particles rather than as elementary particles. In order to do this, however, it is first necessary to examine what the new model has to say about the way quarks combine in other particles, such as pseudoscalar mesons and/or baryons. This issue will be addressed in detail in Section 4.

3.3. **The Cabibbo angle.** There is one further angle in the standard model that contributes significantly to observed effects in particle physics, and that is the Cabibbo angle [12]. This is one parameter of the 9 that appear in the Cabibbo–Kobayashi–Maskawa (CKM) matrix [13] in the standard model, and it relates the strange quark to the up and down quarks. The other parameters in the CKM matrix, apart from the CP-violating phase already considered, are very small.

The Cabibbo angle can be considered a parameter of the weak force, in the sense that the generations of quarks defined by the weak force are not the same as those defined by the strong force, which implies that the weak force does not only act on up and down quarks, but involves at least the strange quark as well. This is not exactly how the mixing of weak and strong forces is usually described in the standard model, but it corresponds more closely to the original insights obtained from Gell-Mann's eightfold way [14].

Therefore the Cabibbo angle must be related to the orbit of the Moon in some way, and it measures some kind of 'neutral charge', or 'strangeness', that distinguishes the strange quark from the down quark. By analogy with the charge on the proton versus the uncharged neutron, it would appear that it is the orbital period that determines the crucial parameter. The only plausible angle is the angle traversed by the Moon in its orbit in one day.

This angle can be measured in slightly different ways, depending on what type of day and what type of month are used. There is a significant difference between the lunar month and the sidereal month, for example. It is not entirely obvious which is required, but it seems likely that the relationship to an inertial frame is the important issue, so let us take the length of a sidereal month as approximately 27.397 sidereal days. This gives an estimate for the Cabibbo angle of

$$\begin{aligned} \theta_C &\approx 360^\circ/27.397 \\ &\approx 13.14^\circ \end{aligned} \tag{6}$$

which is within 1% of the measured value $\theta_C \approx 13.02^\circ$.

It may possibly be relevant that the anomalistic month, that is the interval between successive occurrences of lunar perigee, is about 27.63 sidereal days, and $360^\circ/27.63 \approx 13.03^\circ$. But consideration of such minutiae is beyond the scope of this paper.

4. Pseudoscalar mesons

4.1. **Overview.** In this section I only consider the three light quarks. The masses of the quarks themselves are very uncertain. The simplest independent particles composed of quarks (i.e. hadrons) are the pseudoscalar mesons, that in the standard model are made out of a quark and an anti-quark, with certain quantum superpositions in certain cases. Hence there are 9 such mesons involving three quarks, identified in the standard model as two charged pairs, of pions and kaons, and five neutral particles: one pion, two kaons, the eta and eta-prime mesons.

Symmetry suggests that this collection of 9 particles should form a basis for a generally covariant bosonic representation. However, there is no plausible such representation. The closest matches are the 10-dimensional representation on the symmetric square of spacetime, and its dual. At this point we appear to have a choice between abandoning the principal of general covariance, or abandoning the standard model description of pseudoscalar mesons.

Given the remarkable success of the principle of general covariance in making accurate predictions of five fundamental parameters that the standard model cannot explain, it seems worthwhile to keep hold of this principle, in the hope that it can provide some further insight into the pseudoscalar mesons. If, for example, the new model provides more accurate mass formulae than the Gell-Mann–Okubo formula [14, 15], then the balance of probability shifts towards the new model.

To begin with, we need to understand how these two 10-dimensional representations break up on restriction to various subgroups. For $SL(2,\mathbb{C})$ they each split as 4+6. The 4-dimensional representation is identical to the restriction of one of the two 4-dimensional generally covariant representations, which were used in [3] to describe charged fermions. The 6-dimensional representation is equivalent to that used in the standard model for the photon. This analogy suggests there are 4 charged and 6 uncharged particles in this representation.

For $SO(3,1)$ both 10-dimensional representations split as $1+9$, which is reminiscent of Gell-Mann's splitting of adjoint $SU(3)$ as $1+8$. The closest match to $SU(3)$ inside $SL(4,\mathbb{R})$ is $SL(3,\mathbb{R})$, which however splits the representation quite differently, as $1+3+6$. An individual observer, or experiment, is probably best served by restricting to the rotation group $SO(3)$, where we see $1+1+3+5$. With respect to this symmetry group it is natural to associate the pions with the 3, and put the kaons in the 5. Then there is the option of putting the eta meson with the kaons in the 5, and putting the eta-prime in one of the scalars, for a close match with the standard model. But the principle of general covariance suggests that it may be better to put the eta and eta-prime in the two scalars, which leaves one spare dimension in the kaon representation.

4.2. **Kaons.** We have to decide between the principle of general covariance, which suggests that the kaons are best modelled by a 5-dimensional real representation, and the standard model, which uses instead two 2-dimensional complex representations. The crucial question then is, how many distinct kaons are actually observed in experiments, rather than proposed by theory?

In the standard model, the four kaons are denoted K^+, K^-, K_0 and $\bar{K_0}$. The neutral kaons are supposed to oscillate rapidly between the two states, which means that they cannot be experimentally distinguished. There are two further kaon states, K_1 and K_2, that in the standard model are described as quantum superpositions of K_0 and $\bar{K_0}$. The two states K_1 and K_2 are certainly different from K_0, and also can be clearly distinguished experimentally from each other, because their lifetimes differ by nearly three orders of magnitude, and the decay products include an odd number of pions in one case, and an even number of pions in the other. Therefore the number of kaon states that can be distinguished experimentally is five, as my model predicts, and not four, as the standard model asserts.

My model goes further, and predicts the mass ratio of the charged to neutral kaons. Since the kaons live in the 5-dimensional ('spin 2') representation of $SO(3)$, the mass ratio is a quadratic formula in simple trigonometric functions of the spatial angles. The kaons are implicated in CP-violation, and involve strange quarks, so the angle must be related to the Moon. There is essentially only one such spatial angle, the angle of 5.14° already highlighted above. The prediction is therefore

$$\begin{array}{rcl} m(K^+)/m(K^0) & \approx & \cos^2(5.14^\circ) \\ & \approx & .99197, \end{array} \tag{7}$$

compared to the experimental value, given by

$$\begin{array}{rcl} m(K^+) & = & 493.677(13) \\ m(K^0) & = & 497.648(22) \\ m(K^+)/m(K^0) & \approx & .99202. \end{array} \tag{8}$$

4.3. **The eta meson.** The η meson in the proposed model lies in one of the two scalar representations. One of these is essentially the square of the time representation, or its dual, while the other lies inside the symmetric square of the space representation, or its dual, along with the kaons. The close relationship between the eta meson and the kaons in the standard model suggests that the symmetric square $S^2(3) = 1+5$ is the appropriate representation for kaons plus the eta meson. If so, we should expect a quadratic formula in trigonometric ratios, to relate the η mass to the kaon mass(es). The only plausible conjecture with these properties is

$$\begin{aligned} m(K^0)/m(\eta) &\approx \cos 23.44^\circ \times \cos 5.14^\circ \\ &\approx .9175 \times .9960 \\ &\approx .9137 \end{aligned} \tag{9}$$

compared to an experimental value

$$\begin{aligned} m(K^0)/m(\eta) &\approx 497.65/547.51 \\ &\approx .90893. \end{aligned} \tag{10}$$

Here the discrepancy between conjectured and experimental values is around .5%. Perhaps the reason for this rather larger discrepancy than in some other cases is due to the same cause as in the standard model, that is, some 'mixing' between the different neutral particles.

4.4. **Pions.** The pions are allocated instead to the product of space and time (or the dual thereof), so the pion mass ratio must involve a frequency ratio. As with all the other pseudoscalar mesons, the Moon appears to be implicated, so that the appropriate frequency ratio is (essentially) the number of days in a month. The pions are bosons, so there is no extra factor of 2 as appeared in the proposed formula for the proton/neutron mass ratio.

Again we must consider what sort of month is most appropriate. The argument that the Cabibbo angle relating down and strange quarks should use a sidereal month does not necessarily apply, since quarks are a very different type of 'particle' from pions. There is an argument that pions are more closely related to the observer, while the quarks are more closely related to a more abstract 'inertial frame'. If so, then we should use a lunar month and a solar day, just as we used a solar year and a solar day in the case of the nucleons.

Using this particular value gives an estimate

$$\begin{aligned} m(\pi^+)/m(\pi^0) &\approx 1 + 1/29.53 \\ &\approx 1.03386. \end{aligned} \tag{11}$$

Experimental values give

$$\begin{aligned} m(\pi^+) &\approx 139.570 \\ m(\pi^0) &\approx 134.977 \\ m(\pi^+)/m(\pi^0) &\approx 1.03403. \end{aligned} \tag{12}$$

The discrepancy here is around .00017, or about 1/60 of a percent.

4.5. **More about neutral kaons.** A very simplified picture of the experimental properties of neutral kaons is that they are typically produced as K_0, and spontaneously transform into a mixture of K_1 and K_2. Then the K_1 mesons decay rapidly into two pions, leaving the K_2 mesons, whose decay rate is almost three orders of magnitude slower, and which decay either into three pions or into one pion and two leptons. The property of kaons for which they are famous, however, is that a small proportion of the K_2 mesons appear to change spontaneously into K_1 mesons [16], since they are observed to decay into two pions.

This property is the crucial evidence for CP-violation in the standard model. I have provided evidence above that CP-violation is related by a duality to the rotation of the Earth, on an axis tilted at 23.44° relative to the axis of revolution about the Sun. The rotation of the Earth picks out a preferred copy of $SO(2)$ inside $SO(3)$, and splits the 5-dimensional kaon representation as $1+2+2$. One of the 2-dimensional representations contains the charged kaons, and the other one contains K_1 and K_2. To understand how the K_2 mesons are converted into K_1 mesons, therefore, we have to understand how the geometry of $SO(2)$ is related to the experiment.

It would appear from this discussion that the rate of conversion depends on both the latitude of the experiment and its orientation. However, it is also possible that this choice of $SO(2)$ inside $SO(3)$ is hidden inside the unobservable part of the experiment, and that the observable part is independent of these parameters. In that case, the choice of $SO(2)$ macroscopically is made instead by the direction of travel of the kaon beam in the experiment. Then the rate of conversion depends only on the curvature of the Earth, not on rotation. This second possibility seems more likely, given that the kaon representation does not explicitly involve time at all, but only space.

In either case, the experimental results depend on a projection from a circle onto a straight line, and therefore on sines or cosines of particular angles in space. The original experiment was conducted with a kaon beam 57 feet in length, which at one end was a pure K_2 beam, and at the other had a small proportion of K_1 mesons, measured indirectly by counting the number of two-pion decays and the total number of decays detected.

The relevant angle is the angle subtended by a length of 57 feet at the centre of the Earth. The radius of the Earth varies from 3950 miles at the poles to 3963 miles at the equator, so the angle is approximately .00000273 radians. There is only one plausible conjecture for the proportion of K_1 mesons detected, and that is the sine of this angle. The K_1 decay rate is about 570 times as fast as the K_2 decay rate, so the expected proportion of K_1 decays predicted by the new model is approximately

$$570 \times 0.00000273 \quad \approx \quad .0016. \tag{13}$$

The actual proportion reported by the experimenters was $.002 \pm .0004$, so that the proposed new model correctly predicts what the experiment found, to 1σ.

5. Quarks

5.1. **Elementary particles.** In the standard model, there are at least 9 massive fundamental fermions (excluding the neutrinos for now), namely three leptons and six quarks. In the new model, there can be only four truly fundamental particles, but when we put them into a particular spacetime environment by tensoring with V, we see the 16-dimensional representation $V \otimes V'$, which allows us to distinguish up to 16 'different' particles.

If as before we regard the scalar component as an energy or mass component, the remaining 15 dimensions look very much like the 15 fundamental particles of a single generation in the standard model. That is, we can imagine three leptons: a neutrino, and left- and right-handed electrons; and 12 quarks: three colours of left- and right-handed up and down quarks. This is the basis on which Georgi and Glashow built their grand unified theory [17] based on $SU(5)$.

This standard catalogue of elementary particles is not easy to reconcile with the idea that symmetries are described by subgroups of $SL(4,\mathbb{R})$, nor with the suggestions from [3] concerning possible relationships between the gauge groups of the standard model and subgroups of $SL(4,\mathbb{R})$. At any rate, there is no subgroup of $SL(4,\mathbb{R})$ that can reproduce this pattern of symmetries without breaking the rotation symmetry of (internal) space. On the other hand, I have already used such symmetry-breaking as a potential explanation for the three generations of leptons, so a group such as the affine general linear group $AGL(2,\mathbb{R})$ might do the job, being the subgroup of $SL(3,\mathbb{R})$ that fixes a particular direction in space.

However that may be, the fact remains that there are at least 9 apparently independent masses in the standard model, while I claim there can be no more than four. I therefore need at least five universal mass equations to explain the standard model masses. To date I have only partially succeeded in this objective, having found three plausible equations to explain the relationship between lepton masses and the lightest four of the six quark masses, which will be described in Section 5.2. The other two quarks (bottom and top) are more challenging (see Section 5.4).

5.2. **Quarks and leptons.** The model implies that there is effectively only a 4-dimensional space of fermion masses and charge, identified with V'. If we assume, for example, that there is a basis for this space consisting of the four lightest quarks, up, down, strange and charm, then the lepton masses must be writable as simple linear combinations of these quark masses. There is a good deal of variation in quark masses quoted in the literature. For present purposes I take the following consensus values in MeV/c^2:

$$\begin{aligned} m(u) &= 2.3 \pm 0.7 \pm 0.5 \\ m(d) &= 4.8 \pm 0.5 \pm 0.3 \\ m(s) &= 95 \pm 5 \\ m(c) &= 1275 \pm 25. \end{aligned} \tag{14}$$

In order to match the charge as well as mass, the only possible equation for the electron is

$$m(e) + m(u) = m(d). \tag{15}$$

This equation is certainly consistent with experiment, though only just, and in view of the large error bars, it is not very discriminating. It therefore provides little, if any, additional support for the proposed new model.

The muon mass is close to the strange quark mass, and in order to equate the charge we need

$$m(\mu) = m(s) + 2m(d). \tag{16}$$

The right-hand side evaluates to around $105 \pm 5\text{MeV}/c^2$, in complete agreement with the measured value of the left-hand side

$$m(\mu) = 105.6583745(24). \tag{17}$$

Similarly, we can relate the tau mass to the charm quark mass, adding in 5 strange quarks in order to balance the charge.

$$m(\tau) \quad = \quad m(c) + 5m(s). \tag{18}$$

The right hand side now evaluates to

$$1275(25) + 475(25) \quad = \quad 1750(35)\text{MeV}/c^2. \tag{19}$$

This is consistent with the measured value of the tau mass

$$m(\tau) \quad = \quad 1776.86(12). \tag{20}$$

5.3. **Quark symmetries.** Adding up the three equations (15), (16) and (18), we obtain a more symmetrical equation:

$$m(e) + m(\mu) + m(\tau) + m(u) \quad = \quad m(c) + 6m(s) + 3m(d). \tag{21}$$

In [3] I suggested that the group $SL(3,\mathbb{R})$ could play a role similar to that of the strong gauge group $SU(3)$ in the standard model, in which case it must have something to do with the symmetries of quarks. This group splits the space $V \otimes V'$ into dimensions $1 + 1 + 3 + 3' + 8$, which does not look much like what we see in the above equation.

But if we look instead at the action on $V \otimes V$, or $V' \otimes V'$, then we see a splitting $3+3'+1+3+6$. This arises from the splitting of spacetime into space and time, that I shall express in the form $V = S \oplus T$, such that the antisymmetric and symmetric squares split as

$$\begin{aligned} \Lambda^2(S \oplus T) &= \Lambda^2(S) \oplus (S \otimes T) \\ S^2(S \oplus T) &= S^2(S) \oplus (S \otimes T) \oplus S^2(T). \end{aligned} \tag{22}$$

Now we see the multiplicities in the equation (21) reflected in the dimensions of the representations, where we expect to see them.

This suggests that when it comes to quarks we need to change focus from the representation $V' \otimes V$ to the representation $V' \otimes V'$. The equation (21) then includes 14 of the $6+10 = 16$ dimensions of $V' \otimes V'$, leaving two dimensions available for the bottom and top quarks, which do not participate in this particular mass equation. In order to obtain mass equations involving these two quarks, therefore, it would seem to be necessary to include other particles besides quarks and leptons.

5.4. **The heavy quarks.** The above discussion suggests that there are no equations that relate the heavy quark masses directly to the leptons and/or the light quarks. Equation (36) of [3], involving both the Higgs boson and the neutron, indeed suggests that some of the universal mass equations mix boson and fermion masses. Thus particles related by the weak force (such as the top and bottom quarks) may have masses related to the masses of the Z and W bosons. Similarly, particles related by the strong force (such as the down, strange and bottom quarks) may have masses related to the masses of pions (regarded as massive mediators of the strong force in the atomic nucleus).

The latter suggestion leads to the following possibilities for the bottom quark:

$$\begin{aligned} m(b) + m(s) + m(d) + m(\pi^+) + 2m(\pi^0) &= 5m(n), \\ m(b) + m(s) + m(d) + 2m(\pi^+) + m(\pi^-) &= 5m(n). \end{aligned} \tag{23}$$

Substituting in experimental values for the down and strange quarks, given in equation (14) above, and the pions:

$$\begin{aligned} m(\pi^{\pm}) &\approx 139.570 \\ m(\pi^0) &\approx 134.977 \end{aligned} \tag{24}$$

gives two competing suggestions for the mass of the bottom quark in MeV/c^2 as

$$\begin{aligned} m(b)_{p1} &= 4190 \pm 5, \\ m(b)_{p2} &= 4180 \pm 5, \end{aligned} \tag{25}$$

both of which are consistent with the current experimental value

$$m(b)_e = 4180 \pm 30. \tag{26}$$

The former suggestion leads to the rather tentative equation:

$$m(t-b) + m(c-s) + m(d-u) + m(\eta') = m(Z^0) + m(W^+). \tag{27}$$

The idea here is that the weak force relates the heavier quark to the lighter in each generation, which accounts for the Z and W bosons, and then the η' meson (which is symmetric in the up, down and strange quarks) is required in order to link the three generations symmetrically. Substituting in experimental values, including

$$m(\eta') \approx 957.78, \tag{28}$$

gives an estimate

$$m(t)_p = 173620 \pm 50 \tag{29}$$

compared to an experimental value

$$m(t)_e = 173210 \pm 510 \pm 710. \tag{30}$$

Another pair of plausible equations relate the charm quark to the strange quark:

$$\begin{aligned} m(c) - m(s) &= m(K^+) + m(\eta) + m(\pi^0), \\ m(c) - m(s) &= m(K^0) + m(\eta) + m(\pi^+). \end{aligned} \tag{31}$$

Let us take experimental values

$$\begin{aligned} m(K^+) &\approx 493.68, \\ m(K^0) &\approx 497.65, \\ m(\eta) &\approx 547.51. \end{aligned} \tag{32}$$

Then the two estimates are as follows:

$$\begin{aligned} m(c)_{p1} &= 1271 \pm 5, \\ m(c)_{p2} &= 1280 \pm 5, \end{aligned} \tag{33}$$

compared to an experimental value

$$m(c)_e = 1275 \pm 25. \tag{34}$$

6. Conclusion

6.1. **Summary.** In this paper I have used a previously proposed generally covariant model of quantum mechanics [3] to derive 14 accurate numerical predictions, of 14 unexplained parameters of fundamental physics. In addition, the model provides qualitative explanations for the chirality of the weak force, the existence of three generations of electrons and the existence of five rather than four experimentally distinguishable kaons. Adding this evidence to the two numerical mass equations obtained in [3] creates a weight of evidence in favour of the new model that cannot be ignored.

Sixteen equations reduce the number of fundamental parameters from around twenty in the standard model to around four. Three more are needed in order to relate the units of distance, time, spin and charge to each other. Distance and time are related by c, the speed of light. Spin and charge are related by h/e, the ratio of the Planck constant to the charge on the electron. The relationship of charge or spin to distance or time is described by a duality automorphism. The absolute constant obtained by the duality automorphism in [3] is the mass of the neutron.

6.2. **Measurement.** The underlying theme of this paper is that measurement of properties of fundamental particles is essentially a process of setting up an isomorphism between an intrinsic particle copy of $SL(4,\mathbb{R})$ and a macroscopic copy chosen by the observer. In practice, the whole group is never used, only some suitable subgroup, but this does not alter the basic principle. If we perform measurements on two particles, then we have two particle copies of the group, say P_1 and P_2, and two macroscopic copies, say M_1 and M_2. The measurements define isomorphisms $\phi_1 : P_1 \to M_1$ and $\phi_2 : P_2 \to M_2$. The observer who sets up the experiment can specify an isomorphism $\mu : M_1 \to M_2$. Now if we suppose that the two particles are entangled, then there is a canonical isomorphism $\pi : P_1 \to P_2$.

At this point the system is over-determined, because for consistency we require $\mu\phi_1 = \phi_2\pi$. Knowledge of any three of the isomorphisms determines the fourth. In particular, if we perform two compatible experiments on two entangled particles we always get the same answer. On the other hand, if we perform incompatible experiments, such as measuring the spin in perpendicular directions, then the result is not determined by quantum mechanics. This is essentially because the loop of four isomorphisms does not close properly, in the sense that the chosen subgroup of P_1 maps to two different subgroups of M_2 under the two maps $\mu\phi_1$ and $\phi_2\pi$.

The details are actually quite complicated, and I will not go into them here. But the general principle is that knowing three isomorphisms, such as (a) the experimental set-up, (b) the entanglement and (c) the result of one measurement, gives us the fourth isomorphism, and therefore information about (d) the result of the other measurement. The hidden variables [18] are hidden in plain sight.

The central issue here is that measurement of spin in the standard model uses a particle group $SU(2)$ and a macroscopic group $SO(3)$. It is impossible to set up the required isomorphism between these two groups, since they are not isomorphic. Therefore, the standard interpretation of quantum mechanics attempts to describe the measurement in terms of the 2-to-1 homomorphism from $SU(2)$ to $SO(3)$. But I suggest that this group homomorphism is physically meaningless.

Therefore a number of unphysical variables appear. The 'measurement problem' can be interpreted as the problem of assigning physical meaning to these unphysical variables. The solution is to use a macroscopic group $SU(2)$ for the measurement of electron spin, in place of $SO(3)$. This discussion essentially solves the measurement problem once and for all.

6.3. **Neutrino oscillations.** The simplest example to explain is probably neutrino oscillation [19, 20] for terrestrial neutrinos. The generation of a neutrino is defined by the weak interaction with an electron. The generation of the electron is defined by its mass, and therefore the symmetry-breaking between the three generations is defined by the symmetry-breaking of $SO(3)$ caused by the gravitational field.

We therefore set up isomorphisms between four copies of $SO(3)$. The two macroscopic copies are the rotation symmetry groups at the two ends of the experiment, related by the angle and direction of the rotation around the Earth. The two internal copies are equal, because they describe the same neutrino. The first measurement is a definitive measurement as an electron neutrino. Hence the second measurement is converted by the rotation around the Earth into a well-defined quantum superposition of the three generations. This is what has been consistently observed experimentally.

A similar argument applies to solar neutrinos, except that it is not possible to specify the isomorphism between the two macroscopic copies of $SO(3)$, which is essentially random. Hence solar neutrinos are detected in random generations, as experiment confirms [21].

The proposed model therefore provides the same mathematical explanation for three very different experimental results: (a) CP-violation of neutral kaons, (b) measurement of spins of entangled electrons and (c) neutrino oscillations.

Notice also that neutrino oscillation for both solar and terrestrial neutrinos is predicted by the new model independently of whether the neutrino has zero or non-zero mass. Since a non-zero mass has not been detected by any experiment, Occam's razor declares that neutrinos should be assumed to be massless. This allows them to travel through space at the same speed as light, as has been observed to high accuracy by astronomical observations. In subsequent work I shall consider a possible role for neutrinos in quantum gravity [22].

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