

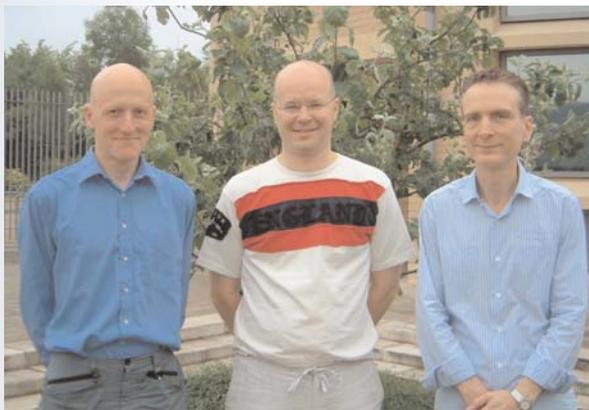
Developments in Quantitative Finance

24 January to 22 July 2005

Report from the Organisers:

D Duffie (Stanford), DG Hobson (Bath), LCG Rogers (Cambridge) and JA Scheinkman (Princeton)

S Wilkinson



Left to right: LCG Rogers, DG Hobson and JA Scheinkman

Scientific Background

Mathematical finance sprang to life in the early 1970s with the development of the now-legendary Black–Scholes–Merton option pricing formula. This formula was quickly extended by Ross, Harrison, Kreps, Pliska and others into a general theory for the pricing of options; these authors showed that the fair price of an option is the expected discounted payoff of the option, where the expectation is taken not with respect to original probability measure, but rather with respect to an equivalent martingale measure under which the discounted price process is a martingale.

Although the theory is based on stochastic models, and there are immediate applications of statistical inference and data analysis, part of the explanation for the rapid growth of financial mathematics is that the field provides fertile grounds for collaboration between researchers with different backgrounds: numerical analysis and computational methods are required to calculate specific answers; functional analysis and the general theory of stochastic process have combined to give exact conditions for the fundamental theorems; and convex analysis is used to investigate difficult

constrained investment problems via the dual functions.

In 1995 the Newton Institute hosted a programme entitled *Financial Mathematics*. After a gap of a decade, in which new problems of interest such as securitisation, credit derivatives, risk measures and model uncertainty have come to the fore, the subject was ripe for a further programme, which aimed to give the same impetus to the field as the earlier meeting.

Structure of the Programme

The six-month programme attracted an extremely large number of participants. There were 47 long-stay participants, including 17 from the UK, and a further 154 short-stay participants, including 64 from the UK. In addition there were almost as many further visitors, from both the finance industry and academia, who attended one of the industry days or workshops.

In addition to two large workshops (one in April held as a satellite meeting at the ICMS in Edinburgh, the second a larger event in July) the programme held a series of themed events, often of a week in duration. The purpose of these weeks was to gather in one place a group of researchers with interests in a particular field of financial mathematics, and for those people to engage with the longer-stay participants and other interested parties to hold a mini-workshop. Typically these events were relatively unstructured, which gave them a spontaneous feel and provided the perfect forum for free discussion and the presentation of the most recent but not necessarily most polished results. Organisation of these themes and the decisions over whom to invite were delegated to a distinguished figure in the relevant field.

Between the themed events the programme was quiet, but only in a relative sense. There were still

often 20 scientists in residence and 4 or 5 seminars per week, but talks ranged over a wider range of subjects and there was greater opportunity for prolonged discussions. All long- and short-stay participants were allowed the opportunity to give presentations or seminars on their latest research. In most cases these were organised to take place soon after arrival, so that the seminar could act as an introduction to the other participants.

The programme also combined with interested parties in the Centre for Mathematical Sciences, the Judge Business School and the Faculty of Economics at the University of Cambridge to hold a weekly Tuesday seminar in the late afternoon. Benoit Mandelbrot spoke on *Fractal and Multi-Fractal Finance*. The Tuesday seminar was a prelude to regular informal social events.

The Rothschild Professor, Stan Pliska, gave a seminar entitled *Portfolio Optimisation: The Quest for Useful Mathematics*.

Themed Events and Workshops

Continuous Time Processes based on Infinite Activity Innovations

Themed Week, 31 January–4 February 2005

Organisers: O Barndorff-Nielsen and N Shephard

It has long been recognised that whilst the classical Brownian models for asset prices postulate continuous price processes, in reality market prices exhibit jumps. One of the simplest classes of models that address this phenomenon is the class of Lévy models. This themed week was concerned with recent advances in the field and especially the problems of inference for price processes based on infinite activity Lévy models, and on models in which the underlying process has a mean-reverting volatility with innovations driven by a subordinator.

Credit

Themed Fortnight, 21 February–4 March 2005

Organiser: M Davis

Following the financial collapse of several major companies, and the downgrading of debt ratings for several others, credit and credit risk remain major problems of interest in both financial



M Kabanova

Anna Pavlova during her seminar

markets and academia. This was reflected both in the fact that this themed event on credit extended to two weeks (including an industry workshop: see page 42) and in that a further day of talks was organised by Bielecki, Jeanblanc and Rutkowski later in the programme on 29 June. The literature in credit is roughly divided between structural and reduced form models and the latest research on both these topics was presented. A major issue is how to model joint and correlated defaults, whether via a copula or by introducing a correlation into the stochastic processes that drive the individual securities. The continued growth of the credit market and the introduction of innovative products such as CDOs and CDO²s mean that new challenges involve modelling portfolios of credit-based securities with option-like payoffs. New approaches, including drawing a parallel between sequences of defaults and arrivals in queues, were presented.

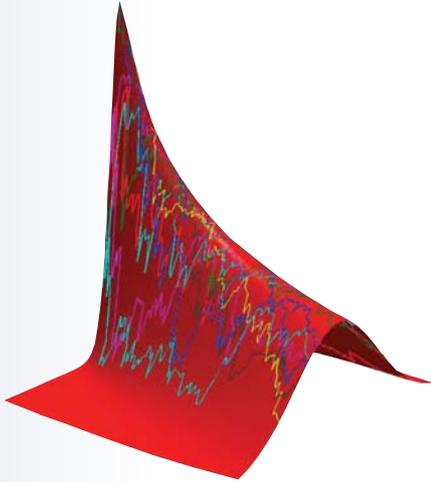
Risk Management

Themed Week, 7–11 March 2005

Organisers: P Artzner and P Embrechts

This themed week was concerned with risk measurement and management, including the problems of model uncertainty. The recently introduced concepts of coherent and convex risk measures played a prominent role in the week, along with good-deal bounds.

On Thursday 10 March, as part of this theme week, the programme held an LMS Spitalfields Day entitled *Risk Management of Hedge Funds*, which covered what they are, how to model them and their risks, and how they behave when they



The transition density of a drifting Brownian motion, superimposed with some typical sample paths

face extreme events. The day attracted a large number of PhD students and other visitors.

International Finance

Themed Days, 14–15 March 2005

Organiser: R Uppal

Most of the participants who were present specifically for this theme were from finance or economics departments. Nonetheless, the most striking difference between this workshop and other parts of the programme was in the type of question that was under consideration rather than in the mathematical technologies and tools that were utilised. Indeed, the approach was generally to attempt to solve problems in full equilibrium, and to use fundamental economic variables to derive the prices of goods and securities in many countries simultaneously, rather than to take the more traditional approach in mathematical finance which is to work in a partial equilibrium model in which prices are modelled exogenously. Many challenging problems on the existence of equilibria were posed for the mathematicians.

The Interface between Quantitative Finance and Insurance

Satellite Workshop at ICMS, 4–8 April 2005

Organiser: A Cairns, with C Kluppelberg, S Pitts and LCG Rogers

This workshop, supported by Watson Wyatt, considered research at the interface between insurance, pensions and quantitative finance. In addition to their insurance risks, insurance

companies face economic and financial risks, which have traditionally been the remit of more mainstream finance. One of the tasks facing the actuarial industry is to incorporate the advances in modelling and understanding which have been achieved in financial mathematics. In return, the securitisation of insurance risk throws up new challenges for the financial mathematician. By bringing together researchers from the relevant disciplines this workshop achieved significant progress in opening a dialogue on these issues.

Fundamentals

Themed Week, 25–29 April 2005

Organiser: W Schachermayer

This week was concerned with the fundamental properties of option prices and consumption/investment problems in a general financial market model. The main topics of interest were risk measures, the problems of optimal risk sharing and definition of risk measures in a dynamic context; and marginal utility based pricing, the link with minimal distance martingale measures and the sensitivity and convergence properties of such prices.

Computational Finance

Themed Week, 9–13 May 2005

Organisers: M Broadie and P Glasserman

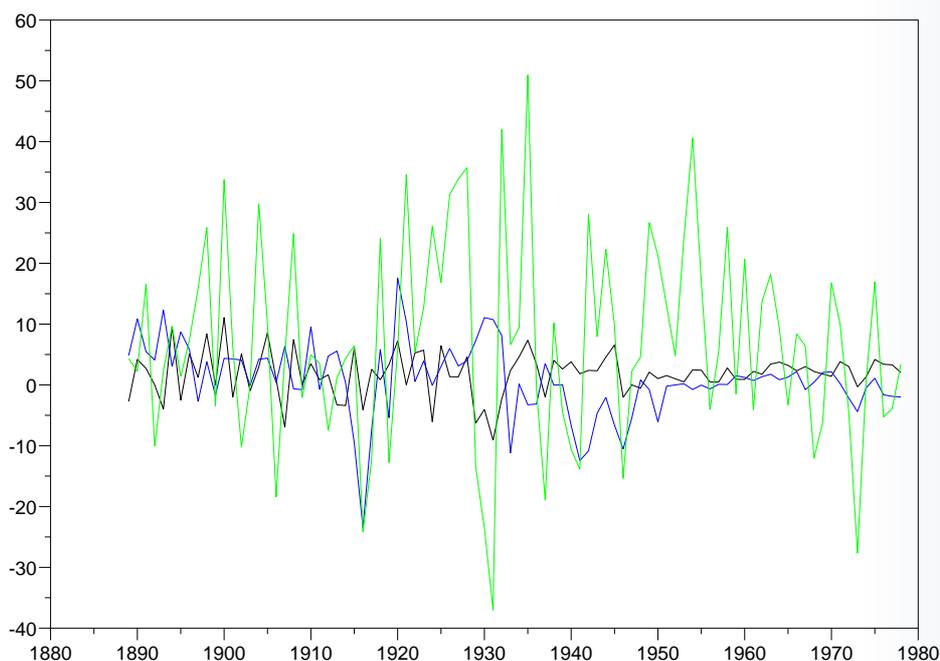
A long-standing problem in financial mathematics is to determine the optimal exercise boundary for an American put option, and this week reported on the latest advances in this style of problem, together with the problems associated with efficient evaluation of option prices under stochastic volatility models and bounds on the prices of volatility options and variance swaps.

Monte Carlo Methods

Themed Week, 16–20 May 2005

Organiser: N Touzi

Monte Carlo methods have undergone a recent renaissance in financial mathematics. This followed both from the discovery that the solution of a dual problem allows the American option pricing problem to be recast in such a way as to be amenable to Monte Carlo methods, and from the recent advances in Malliavan calculus which mean that Monte Carlo techniques can be used as a robust method to determine delta-hedging ratios.



The dataset used by Mehra & Prescott in their celebrated 1985 paper on what has come to be known as the equity premium puzzle, showing consumption growth (black), the return on riskless securities (blue) and the return on the S&P500 (green)

Beauty Contests in Finance

Themed Days, 23–24 May 2005

Organisers: N Kiyotaki and H Shin

Here the use of the phrase ‘Beauty Contests’ alludes to a quotation from Keynes, and refers to the fact that in deciding on optimal behaviour agents need to consider how their behaviour will influence the actions of others, and how the behaviour of others should influence their own actions. This can continue in an infinite cycle leading to challenging problems relating to the existence and form of equilibria, and more generally to problems involving agent interaction. These two themed days were held in conjunction with the Cambridge Endowment for Research in Finance.

Econometrics

Themed Week, 13–17 June 2005

Organisers: L Hansen and JA Scheinkman

The problems of statistical inference for financial time series can be extremely complicated not least because the data often show non-stationarity. This week reported on some of the latest advances in volatility modelling, the analysis of time series and high-frequency data, and how inference can be improved with the use of moment estimates

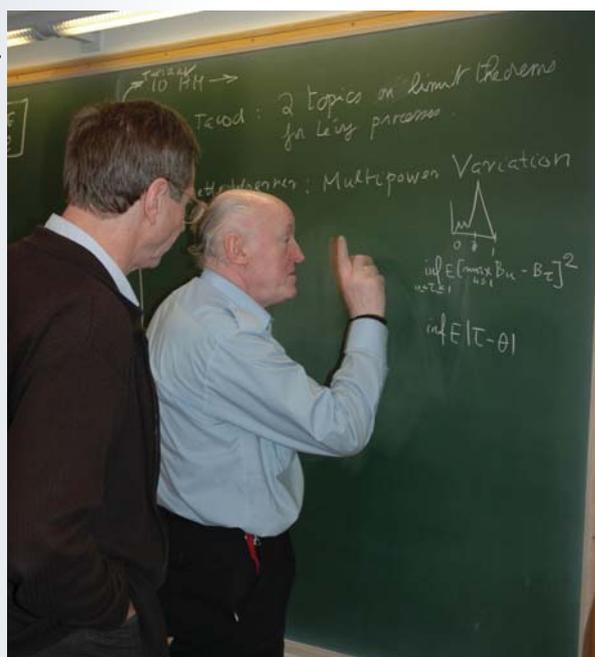
derived from continuous time models. The week also provided the opportunity for several of the participants to continue their collaborations on eigenvalues and principal components in volatility modelling.

Quantitative Finance: Developments, Applications and Problems

Workshop, 4–8 July 2005

Organiser: V Henderson, with DG Hobson, S Pliska and LCG Rogers

The objective of this conference, which was supported by Nomura International and the EC, was to bring together academics from various fields, including mathematics, economics and finance, as well as City professionals, to discuss the latest developments in the theory of mathematical finance and the application of this theory to current problems facing the industry; and to identify the substantive problems facing academic researchers and practitioners. With 25 speakers and over 100 delegates, the workshop was able to summarise many of the themes and developments of the programme as a whole. Major themes that were well represented included credit and credit derivatives, utility indifference pricing and martingale measures, optimal stopping and hedging, and



Albert Shiryaev at the board with a colleague during the 'Developments in Quantitative Finance' programme

volatility modelling. The final talk of the meeting was given by Steve Ross, mentioned above as one of the founders of Arbitrage Pricing Theory and mathematical finance, who spoke on behavioural finance and the implications and lessons to be learned from this topic by researchers in other branches of finance.

Industry Events

In addition to the above themed events, and with the support of BNP Paribas, we ran four industry events organised by M Musiela:

- *Credit*, 25–26 February 2005
- *Interest Rates*, 18 March 2005
- *Modelling Philosophy*, 22 April 2005
- *Volatility*, 13–14 May 2005

These 1- or 2-day events, with 5–8 speakers per day divided between industry and academia, had audiences of 60–100 delegates made up of long-stay participants on the programme, practitioners and other interested researchers including large numbers of PhD students. These meetings had a more practical focus than the other themed events. Speakers from the City raised important issues and

proposed some solutions, and academics reported some of their research in related areas.

The *Credit* and *Volatility* events were especially successful and well received by the participants.

Outcome and Achievements

Without exception the participants on the programme found that the Isaac Newton Institute provided an excellent environment for research with ideal facilities augmented by efficient and effective administrative support. The thanks of the programme organisers are also given to the theme organisers, each of whom gathered together an outstanding group of senior and younger researchers in order to work on topics in their field.

The chance to visit the Newton Institute provided many researchers with the opportunity both to extend existing collaborations and to develop new ones. Short-stay participants and visitors during themed weeks were able to meet co-workers in the same specialism. Long-stay participants were able to take a more holistic view, and those fortunate enough to be able to spend the entire programme at the Institute saw as full a spectrum of problems and approaches as could be realised in six months.

The main themes of the programme, and the main achievements in terms of publications, preprints and collaborations, only became apparent as the programme progressed. In the event the topics of greatest interest included option pricing and portfolio optimisation in incomplete markets, real options and endogenous exercise, full equilibrium models and dynamic risk measures. Credit risk modelling was another important theme, and the presence of many of the world's experts at the Institute, both in February and again in June, led to several fruitful interactions.

Finally, thanks are also due to BNP Paribas and Nomura International, who gave unconstrained financial support to the programme over and above that provided by the Institute's regular financial supporters. This extra support was used in part to augment the already distinguished list of speakers at the programme, and in part to support PhD students and other junior visitors.