HALL MAGNETIC RECONNECTION:
2D AND 3D RESULTS

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Hall magnetic reconnection (dependence on)
  • Initial width of current layer
  • Guide field

3D results
  • ‘Reconnection wave’
  • Magnetic ‘flux ropes’

Results based on NRL 3D Hall MHD code VooDoo
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  - Initial width of current layer
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3D results
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Results based on NRL 3D Hall MHD code VooDoo
- Ohm’s law (electrons frozen into magnetic field)
  \[ \mathbf{E} + \mathbf{V}_e \times \mathbf{B}/c = 0 \]

- Current definition (assumes quasineutrality)
  \[ \mathbf{J} = n_e (\mathbf{V}_i - \mathbf{V}_e) \Rightarrow \mathbf{V}_e = \mathbf{V}_i - \frac{1}{n_e} \mathbf{J} \]

- Electric field is written as
  \[ \mathbf{E} = -\frac{1}{c} \mathbf{V}_i \times \mathbf{B} + \frac{1}{n_e c} \mathbf{J} \times \mathbf{B} \]

- Physically, the Hall term decouples ion and electron motion on ion inertial length scales: \( L \lesssim c/\omega_{pi} \)
Equilibrium configuration: \( B_y(x) = B_0 \tanh(x/L) \)

Four initial widths: \( L = 1, 5, 10, \text{ and } 20 \)
   All other parameters the same

Boundary conditions: zero-gradient \( (\partial/\partial x = \partial/\partial y = 0) \)
   Steady state achieved
INITIAL PERTURBATION

Dependence on Current Layer Width

L20 animation
INITIAL/FINAL STATES

Dependence on Current Layer Width
Reconnected flux $\Phi$ is

$$\Phi(t) = \int_0^\infty B_x(0, y, t) \, dy = \int_0^{L_y/2} B_x(0, y, t) \, dy + \int_{L_y/2}^\infty B_x(0, y, t) \, dy$$

- First term is time independent in steady state.

- Final term is approximated as

$$\int_{L_y/2}^\infty B_x(0, y, t) \, dy \simeq \int_{t_A}^t B_x(0, L_y/2)V_y(0, L_y/2) \, dt = B_x(0, L_y/2)V_y(0, L_y/2)(t - t_A)$$

Reconnected flux rate $\partial \Phi / \partial t$ is

$$\frac{\partial \Phi(t)}{\partial t} \simeq B_x(0, L_y/2)V_y(0, L_y/2)$$
RECONNECTION RATE/ENERGIZATION TIME
Dependence on Current Layer Width
HALL MAGNETIC RECONNECTION

Dependence on Guide Field

Reconnection Rate/Kinetic Energy

Inflow/Outflow Velocities

![Graphs showing the relationship between reconnection rate, kinetic energy, inflow and outflow velocities with increasing guide field ratio.](image-url)
HALL MAGNETIC RECONNECTION

Dependence on Guide Field

- Reduction of inflow and outflow velocities (and hence reconnection rate) associated with additional $\mathbf{J} \times \mathbf{B}$ force from guide field.

\[
\frac{\partial \rho \mathbf{V}}{\partial t} = \mathbf{J} \times \mathbf{B}
\]

\[
\frac{\partial \rho V_x}{\partial t} = J_y B_z - J_z B_y \\
\hspace{1cm} \text{GF inflow}
\]

\[
\frac{\partial \rho V_y}{\partial t} = -J_x B_z + J_z B_x \\
\hspace{1cm} \text{GF outflow}
\]

- Guide field contribution opposes inflow and outflow velocities.
STRUCTURE OF ‘QUADRUPOLE FIELD’

Dependence on Guide Field

$B_{gf}/B_0 = 0$

$B_{gf}/B_0 = 5$
3D HALL RECONNECTION

Asymmetric propagation

‘Reconnection wave’

\[ \omega = k_z \frac{c}{4\pi en} \frac{\partial B_0}{\partial y} \]
‘Flux rope’ formation
Asymmetric propagation persists
SUMMARY

Hall Magnetic Reconnection

- Final state of reversed field plasma system nearly independent of initial current layer width
  - Magnetic reconnection rate
  - Steady state values of Hall electric field and velocity
  - Energization time is dependent on initial width of plasma
  - Results independent of initial perturbation and grid size
  - Preliminary result: Reconnection rate independent of system size (similar to Shay et al. finding)

- Guide field
  - Reduces reconnection rate and energization (but not much)
  - Quadrupole field disappears

- Localized perturbation in 3D leads to
  - Asymmetric propagation of ‘reconnection wave’
  - ‘Flux rope’ formation with a guide field
HALL MHD EQUATIONS

- **Magnetic field evolution**
  \[
  \frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E} = \nabla \times [\mathbf{V} + \mathbf{V}_B \times \mathbf{B}] \quad \text{where} \quad \mathbf{V}_B = -\mathbf{J}/ne
  \]

- **Continuity**
  \[
  \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{V} = 0
  \]

- **Momentum**
  \[
  \frac{\partial \rho \mathbf{V}}{\partial t} + \nabla \cdot \left[ \rho \mathbf{V} \mathbf{V} + (P + B^2/8\pi) \mathbf{I} - \mathbf{B} \mathbf{B}/4\pi \right] = 0
  \]

- **Pressure**
  \[
  \frac{\partial P}{\partial t} + \nabla \cdot P \mathbf{V} = -\left(\gamma - 1\right)P \nabla \cdot \mathbf{V}
  \]

- **Energy**
  \[
  \frac{\partial \epsilon}{\partial t} + \nabla \cdot \left[ \mathbf{V} (\epsilon + P + B^2/8\pi) - \mathbf{B}/4\pi (\mathbf{V} \cdot \mathbf{B}) \right]
  \]
  \[
  \nabla \cdot \left[ \mathbf{V}_B (B^2/8\pi) - \mathbf{B}/4\pi (\mathbf{V}_B \cdot \mathbf{B}) \right] = 0
  \]

where \( \epsilon = \rho V^2/2 + 3P/2 + B^2/8\pi \)
Cell definition: Staggered mesh
Nonuniform Cartesian grid
Finite volume method
High order spatial interpolation (8th order)
Adams-Bashforth time stepping (2nd order)
Hydro flux calculation (distribution function method)
Flux limiter (partial donor cell method)
Electric field
  - Ideal MHD (distribution function method)
  - Hall MHD (upwind scheme, subcycling, smoothing)
Courant condition
Normalizations (upstream values of $B$ and $n$):

$L \sim c/\omega_{pi0}; \ t \sim \Omega_{i0}t; \ V \sim V_{A0}; \ E \sim V_{A0}B_0$

Equilibrium configuration: $B_y(x) = B_0 \tanh(x/L)$

Density determined by pressure balance

Grid size: $L_x \simeq 70; \ L_y = 84$

Mesh: $90 \times 160$

Nonuniform grid in $x$ direction: $\gtrsim 32$ grid points in current layer

Initial magnetic field perturbation ($\delta B_x$ and $\delta B_y$)

Boundary conditions: zero-gradient ($\partial/\partial x = \partial/\partial y = 0$)

Steady state achieved