Driven Magnetic Reconnection at the Dayside Terrestrial Magnetopause

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Two Special Magnetospheric Magnetic Field Topologies

The Sweet–Parker Time Scale Problem

\[ U_{in} \propto S^{-1/2} = \left( \frac{\eta c^2}{4\pi V_A \lambda} \right)^{1/2} \]

\[ U_{out} \propto V_A = B^{up} / \sqrt{4\pi \rho} \]


\[ E_R = U_{in} B^{up} \propto S^{-1/2} \]
Alfvénic Reconnection in the Context of Resistive MHD

- **Localized Plasma Resistivity:** The Petschek slow shock configuration becomes relevant.

  Biskamp et al., Localization, the key to fast magnetic reconnection, Phys. Plasmas, 8, 4279, 2001.

- **Magnetic Flux Pileup:** The outflow bottleneck is removed.


Two-dimensional, Incompressible Resistive MHD

\[ [f,g] \equiv \frac{\partial f}{\partial x} \frac{\partial g}{\partial z} - \frac{\partial f}{\partial z} \frac{\partial g}{\partial x} \]

\[ B_p = \nabla \psi \times \nabla y \quad U_p = \nabla \phi \times \nabla y \]

\[ \omega = -\nabla^2 \phi \quad J_y = -\nabla^2 \psi \]

\[ \frac{\partial \omega}{\partial t} + [\omega, \phi] = [J_y, \psi] \]

\[ \frac{\partial \psi}{\partial t} + [\psi, \phi] = \frac{1}{S} \nabla^2 \psi \]


\[ U_x = U_0 x \]
\[ U_z = -U_0 z \]

Flux pileup allows fast reconnection in thin current sheets.
Three-dimensional Magnetic Field Annihilation

Sonnerup, B. U. O. and E. R. Priest, Resistive MHD stagnation point flows at a thin current sheet

\[
\begin{align*}
U_x &= -U_0 x \\
U_y &= U_0 (1 - \kappa) y \\
U_z &= U_0 \kappa z \\
B_x &= 0 \\
B_y &= 0 \\
B_z &= f(x)
\end{align*}
\]

$\kappa$ determines the anisotropy of the bulk velocity field.
Limiting Anisotropic Stagnation Point Flows

\[ \kappa = 0 \quad \text{and} \quad \kappa = 1 \]
Magnetic Flux Pileup

Rescale the spatial coordinate $x$ as follows:

$$\xi = (SU_0)^{1/2} x$$

Then, one can show (Sonnerup and Priest, 1975) that $f(\xi)$ satisfies the following ODE:

$$\frac{d^2 f}{d\xi^2} + \xi \frac{df}{d\xi} + \kappa f = 0$$

Anderson and Priest (JGR, 98, 1993) and Craig et al. (Ap. J., 485, 383, 1997) showed that $f(\xi)$ can be expressed in terms of Kummer's function:

$$B_z(\xi) = f(\xi) = A\xi M\left(\frac{2 - \kappa}{2}, \frac{3}{2}, -\frac{\xi^2}{2}\right)$$
How does Magnetic Pileup Scale with Plasma Resistivity and Flow Anisotropy?
Flux Pileup Saturation

Biskamp, PRL, 44, 1069, 1983.


\[ E_y^{\text{max}} \propto S^{-1/2} \]

The Sweet-Parker scaling reappears!
Pre-saturation and Post-saturation Reconnection Rates

Pre-saturation reconnection electric field (solid lines):

\[ E_y(0) \propto S^{(\kappa - 1)/2} \]

Post-saturation reconnection electric field (dashed lines):

\[ E_y(0) \propto S^{-1/2} \]
Reconnection at the Dayside Magnetopause

see J. Raeder (JGR, 104, 17357, 1999) for a description of the General Geospace Circulation Model (GGCM)

12/31/1999 Time = 01:59:00 \( y = 0.00R_E \)

Steady solar wind conditions, southward IMF, and constant plasma resistivity:

\( S = \{500, 1000, 2000, 5000, 10000\} \)
A Closer Look at the Subsolar Magnetopause

\[ S = 10000 \]
Magnetic Pileup and Associated Plasma Depletion

\[ S = 10000 \]
Comparison with the 3D Annihilation Resistive MHD Solutions

\[ U_x \frac{\partial B_z}{\partial x} - C(x) B_z - \frac{1}{S} \frac{\partial^2 B_z}{\partial x^2} = 0 \]

\[ C(x) \equiv \left[ \frac{\partial U_z}{\partial z} - \nabla \cdot U \right]_{y=0,z=0} \]
Flux Pileup Scalings Observed in the GGCM Simulations

Green Squares: GGCM simulations
Red Lines: Sonnerup and Priest (1975)
Generalized Ohm’s Law

Assuming quasineutrality, and neglecting terms proportional to \( m_e/m_i \), one can derive the following expression for the electric field.

\[
E = -\frac{1}{c}U \times B + \frac{1}{n_{ec}} J \times B - \frac{1}{ne} \nabla \cdot P_{CM}^e
\]

\[
- \frac{m_e}{ne^2} \frac{\partial J}{\partial t} - \frac{m_e}{ne^2} \nabla \cdot (UJ + JU)
\]

\[
+ \sum_\alpha \int d\mathbf{v} q_\alpha n_\alpha v \left( \frac{\partial f_\alpha}{\partial t} \right)_e
\]

Ideal MHD term (Frozen Flux Theorem)
Hall term (Electron Frozen Flux Theorem) [ION INERTIAL SCALE]
Electron Pressure Anisotropy (thaws magnetic flux) [ION INERTIAL SCALE]
Electron Inertia (thaws magnetic flux) [ELECTRON INERTIAL SCALE]
Particle Scattering (thaws magnetic flux - e.g., resistivity)
Hall Electric Fields Near an X-type Neutral Line

\[ E = -\frac{1}{c} U \times B + \frac{1}{\text{nec}} J \times B + \frac{1}{S} J \]
Generation of Toroidal Magnetic Field in Two-fluid Reconnection

Teresawa, T., GRL, 10, 475, 1983
Mandt et al., GRL, 21, 73, 1994.

\[ \frac{\partial B_y}{\partial t} \approx B \cdot \nabla U_{ey} \]
Hall MHD Theory of Magnetic Field Annihilation

\[ \xi \equiv (SU_0)^{1/2} x \quad \zeta \equiv (SU_0)^{1/2} z \]

\[ \frac{\partial^2 B_y}{\partial \xi^2} + \frac{\partial^2 B_y}{\partial \zeta^2} - \xi \frac{\partial B_y}{\partial \xi} + \zeta \frac{\partial B_y}{\partial \zeta} \]

\[ B_y = C \xi \zeta \]

\[ \frac{dB_x}{d\zeta} + \alpha \zeta B_x = E_y \left( \frac{S}{U_0} \right)^{1/2} \]

\[ \alpha \equiv 1 + S \delta_i C \]

\[ B_x = E_y \left( \frac{S}{U_0} \right)^{1/2} \exp \left( -\frac{1}{2} \alpha \xi^2 \right) \int_0^\xi du \exp \left( \frac{1}{2} \alpha u^2 \right) \]

Estimate of Maximum Pileup Reconnection Rates

Define $\ell$ to be the location of the local maximum of the magnitude of the poloidal magnetic field component $B_x$:

$$\ell = \frac{\sqrt{2} \chi}{(SU_0)^{1/2} (1 + S\delta_i C)^{1/2}} \quad \frac{1}{2\chi} = D_+(\chi)$$

With $p_{min} = \beta - (B_{x\text{max}})^2$, it follows from $p_{min} > 0$ that:

$$E_y < E_{y\text{max}} \approx 1.31 (\beta U_0)^{1/2} \left( \frac{1 + S\delta_i C}{S} \right)^{1/2}$$
Effects of Hall Electric Fields on Magnetic Flux Pileup

Pileup saturates at a level which is independent of $S$
Flux Pileup Saturation in Hall MHD Simulations of Magnetic Island Coalescence
Magnetospheric Magnetic Field Topology under Northward IMF Conditions
Null-null Loop Topologies

135 degrees clock angle

315 degrees clock angle
Magnetic Field Topology at the Dayside Magnetopause under Northward IMF Conditions
Distribution of Current Density on the Magnetic Separatrix Surfaces

Boundary between Solar Wind and Open Field Lines

Boundary between Open and Closed Field Lines
Conclusions

- Magnetic flux pileup appears to be a ubiquitous feature of driven reconnection in resistive Hall MHD when the plasma resistivity is constant.

- Hall electric fields provide a possible solution of the flux pileup saturation problem which occurs in resistive MHD models of driven magnetic reconnection.

In a driven reconnection scenario where the spatial scale of the stagnation point flow is comparable to the ion inertial scale, fast reconnection is possible in thin current sheets without the need for spatially localized resistivity or a Petschek slow shock configuration.