Quantum Error Correction and the Low Energy Problem in Quantum Gravity

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• The Low-Energy Problem in Quantum Gravity: find propagating modes in a background-independent quantum theory of gravity. Necessary, in order to make contact with physics.

• Quantum Information Theory: the low-energy problem is analogous to error-correction. The propagating modes are “noiseless subsystems”.

• Example: the Schwarzchild black hole in LQG

• Implications for background-independence and locality.

• The relation between error correction and the renormalization group?
• Noiseless subsystems in quantum gravity: work in progress with David Poulin (IQC/PI)

• The Schwarzchild black hole as a noiseless subsystem: O.Dreyer, FM and L.Smolin, hep-th/0409056

• Noiseless subsystems and the renormalization group: work in progress with O.Dreyer, L.Smolin and P.Zanardi
Quantum gravity must contain general relativity and quantum theory.

**GR:** Only events are physical. Any coordinates we may use to describe them have no physical meaning. Coordinate distances are not physical quantities. In general relativity the metric is a dynamical variable. There is no background space and time. **GR** is a background-independent theory.

Quantum gravity must be background-independent.
Spin Networks

Background independent quantum space

\[ i, j, k, .. \in SU(2) \]

\[ \rho : i \otimes j \otimes k \otimes l \rightarrow 1 \]
Important results about spin networks:

1. In the large spin limit, abstract (not embedded) graphs reproduce directions in Euclidean 3-space (Penrose)

2. They are the **basis states** for background independent spatial geometry (Loop Quantum Gravity) (Rovelli Smolin Ashtekar Lewandowski)

3. Area and volume operators have discrete spectra (Rovelli Smolin)
Area operator:

\[ \hat{A}(s) = l_{Pl}^2 \sqrt{i(i + 1)} \]

Discreteness:

\[ \hat{A}(s') = l_{Pl}^2 \left( \sqrt{i(i + 1)} + \sqrt{j(j + 1)} \right) \]
Local evolution of spin network graphs is by **local moves:**

There is a small number of generating moves. (cf quantum gates)
Evolution of spin networks generates a “spacetime network”, or Spin Foam.

A spin foam $\Gamma$ is a 2-dimensional complex with faces labeled by irreducible representations of the group and edges labeled by intertwiners. Spacelike cuts through spin foams are spin networks.
Background independent models of quantum gravity:

\[ A(S_{in}, S_{out}) = \sum_{\Gamma} \sum_{\text{labels on } \Gamma} \prod_{f \in \Gamma} \dim j_f \prod_{v \in \Gamma} A_v (\{j\}) \]

\[ = \sum_{\text{histories}} \sum_{\text{labels}} \prod_{\text{events}} A(\text{event}) \]

A choice of 2-complexes, group and amplitudes gives a model.
The low-energy problem

Planck scale events, causal structure, “spacetime”

Low energy ordinary spacetime
I need to be able to identify effective propagating degrees of freedom.

\[ A(S_{in}, S_{out}) = \sum_{\Gamma} \sum_{\text{labels on } \Gamma} \prod_{f \in \Gamma} \text{dim } j_f \prod_{v \in \Gamma} A_v(\{j\}) \]

Cf. condensed matter theory:

\[ H = -g \sum_{\langle ij \rangle} \sigma_i \cdot \sigma_j \]

| \[ k \rangle = \sum_k f(k) |k\rangle \]

How can we obtain similar results for a background independent theory?

Note: To recover QFT in the low energy limit, we need to coarse-grain \( \times 10^{20} \) and be left with a unitary theory.
The low-energy problem

Background-independent quantum gravity theory

LAB (particles on a fixed background)

This is the main open problem of background-independent approaches to quantum gravity (loop quantum gravity, spinfoams, etc).
The Idea:
What we see (QFT&GR) is what nature encoded. (Almost) noiseless with respect to the Planck scale fluctuations.

Propagating mode: A subsystem of $\mathcal{H}_S$ that is conserved under $\phi$, i.e., evolves unitarily: **Noiseless Subsystem**

When symmetries are present, protected subsystems (free particles) exist.

\[
\mathcal{H}^{\text{Total}} = \mathcal{H}^S \otimes \mathcal{H}^\mathcal{E}
\]

\[
H = H_S \otimes 1_{\mathcal{E}} + 1_S \otimes H_\mathcal{E} + H_I
\]

\[
\phi : \mathcal{H}^S \rightarrow \mathcal{H}^{S'}
\]
Noiseless Subsystems in Quantum Error Correction
Example:

\[ \mathcal{H}_S = C^2 \otimes C^2 \otimes C^2 \]

Symmetric under \( S_3 \)

\[ H^{\text{int}} = \sum_{\alpha} S_\alpha \otimes E_\alpha \]

\[ \mathcal{H}_S \simeq \mathcal{H}_{\frac{3}{2}} \oplus \mathcal{H}_{\frac{1}{2}} \oplus \mathcal{H}_{\frac{1}{2}} \]

\[ |\lambda, s_z\rangle \simeq |\lambda\rangle_{NS} \otimes |s_z\rangle \]

\[ \lambda = 0, 1 \]

\[ s_z = \pm \frac{1}{2} \]

\[ S_\alpha = \sum_{i=1}^{3} \frac{\sigma^i_\alpha}{2} \]

\[ \alpha = x, y, z \]

\( su(2) \) rotations

\[ \mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \simeq \mathcal{H}_{NS} \otimes \mathcal{H}_{\frac{1}{2}} \]

Noiseless subsystem

Protected

Noisy
On

\[ \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \simeq \mathcal{H}_{NS} \otimes \mathcal{H}_{1/2} \]

the evolution takes the form

\[ S_\alpha \simeq 1 \mathcal{H}_{NS} \otimes \sigma(\alpha) \]

That is, \( \mathcal{H}_{NS} \) behaves as a “free particle” (coherent degree of freedom).

Note: \( \mathcal{H}_{NS} \) is not a subspace of \( \mathcal{H}_S \).
Noiseless subsystems

- Interacting system \( \mathcal{H}_S \) and environment \( \mathcal{H}_E \).

- Interaction algebra \( \mathcal{A}^{\text{int}} \subseteq \mathcal{B} (\mathcal{H}_S^S) \) decomposes as

  \[
  \mathcal{A}^{\text{int}} \cong \bigoplus_j 1_{\mu_j} \otimes \mathcal{B} (C^{d_j})
  \]

- Corresponding interaction commutant \( \mathcal{A}^{\text{int}}' \subseteq \mathcal{B} (\mathcal{H}_S^S) \) is

  \[
  \mathcal{A}^{\text{int}}' \cong \bigoplus_j \mathcal{B} (C^{\mu_j}) \otimes 1_{d_j}
  \]

- Symmetries in \( \mathcal{A}^{\text{int}} \) \( \Leftrightarrow \) non-trivial \( \mathcal{A}^{\text{int}}' \)

- \( \mathcal{H}_S \) decomposes as

  \[
  \mathcal{H}_S \cong \bigoplus_j C^{\mu_j} \otimes C^{d_j}
  \]

Knill, Laflamme & Viola, Zanardi & Rasetti

\( \mathcal{H}_S = C^{2 \otimes 3} \)

\( su(2) \)

\( S_\alpha \cong 1_{\mathcal{H}_S} \otimes \sigma_\alpha \)

\( \mathcal{H}_{\frac{3}{2}} \oplus \mathcal{H}_{\mathcal{NS}} \otimes \mathcal{H}_{\frac{1}{2}} \)
Noiseless Subsystems in Quantum Gravity
• **Noiseless subsystems** are useful for describing the long-term behavior of the system because they are conserved.

• If we divide the quantum gravitational field into subsystems, those properties that are **conserved under interactions** between the subsystems will characterize the low-energy limit spacetime geometry.

• The **commutant** $\mathcal{A}^{\text{int}'}$ should include the symmetries of a classical spacetime (e.g. Poincare, deSitter).

Example....
Example of a NS:
The Schwarzschild black hole in Loop Quantum Gravity

O. Dreyer, FM and L. Smolin, hep-th/0409056
We can use the spin networks to count the states of a black hole.
\[ H_A^S = \text{invariant states of } U(1) \text{ Chern-Simons on punctured } S^2 \]

\[ H_{\text{total}} = \sum_{\{j_\alpha\}} H_A^S \otimes S^{E}_{\{j_\alpha\}} \]

Q: Which states in \( H_A^S \) correspond to the quantum analogue of a Schwarzschild black hole of area \( A \)?
Use symmetries in the system-environment interaction:

A classical SBH has $SO(3)$ symmetry.

If there is a microscopic analogue of a SBH, there must be a microscopic analogue, $G_q$, of $SO(3)$ in the dynamics, that becomes $SO(3)$ in the classical limit.

This corresponds to a non-trivial $A^{\text{int}'}$ that identifies the QSBH as a noiseless subsystem, i.e.,

$$G_q \subseteq A^{\text{int}'}$$

We do not know $B(\mathcal{H}_A^S)$ or $A^{\text{int}}$, but it is possible to restrict it sufficiently to find the noiseless subsystem corresponding to $G_q$. 
$G_q : \text{Microscopic } SO(3)$

- $G_q$ is a discrete group
- $A^{\text{int}}$ must act simultaneously and locally on $\mathcal{H}^S$ and $\mathcal{H}^E$
- A sufficient set of generators for $A^{\text{int}}$ is:
  - Add/remove punctures
  - Braid (symmetric)

\[
B = \sum_{\langle i,j \rangle} B_{ij}
\]

\[
\begin{array}{c}
\begin{array}{c}
m_1 \quad j_1
\end{array}
\quad B_{12}
\begin{array}{c}
m_2 \quad j_2
\end{array}
\end{array}
\Rightarrow e^{\frac{2\pi i (m_1 + m_2)}{k}}
\begin{array}{c}
m_1 \quad j_1
\end{array}
\begin{array}{c}
m_2 \quad j_2
\end{array}
\]

- $G_q : \mathcal{H}_A^S \longrightarrow \mathcal{H}_A^S \Rightarrow \text{braidings only}$
\[ m_1 \quad \xrightarrow{B_{12}} \quad e^{\frac{2\pi i(m_1 + m_2)}{k}} \quad \xrightarrow{P_{12}} \quad e^{\frac{2\pi i(m_1 + m_2)}{k}} \quad m_2 \]

\[ m_1 \quad \xrightarrow{P_{12}} \quad m_2 \quad \xrightarrow{B_{12}} \quad e^{\frac{2\pi i(m_1 + m_2)}{k}} \quad \xrightarrow{B_{12}} \quad e^{\frac{2\pi i(m_1 + m_2)}{k}} \quad m_1 \]

\[ \Rightarrow G_q \subset \prod_{j} P_{K_j} \quad K_j = \text{no. of punctures with same } j \]

- \( G_q \) must be transitive \( \Rightarrow K_j = \text{number of punctures} \)

All \( j \) are the same in a quantum Schwarzschild black hole.
The state space of the quantum Schwarzschild black hole is the noiseless subsystem corresponding to the $G_q$ symmetry of the dynamics.

\[
\mathcal{H}^{QSBH}_A = \mathcal{H}^{QBH}_j
\]

Quantum Schwarzschild Black Hole

\[
S_A^{QSBH} \approx S_{j_{\text{min}}}^{QBH} = \ln \dim \mathcal{H}^{QBH}_{j_{\text{min}}}
\]

Quasinormal modes: excited states of the NS.

Rotating black holes: no transitivity.
Lessons

- The NS is a dynamical construction. 
  NB: Symmetries at the kinematical level gives $\mathcal{H}_{m}^\text{QBH}$.

- For the low-energy behavior, we only need the interaction dynamics, and from that, we only need the symmetries.

- $A^\text{int}$ depends on the energy level/approximation we are interested in.

- Conversely, in model-building, we can use the low-energy symmetries to constrain the microscopic dynamics.
Propagating modes vs background independence

The NS method is background-independent in the sense that it does not rely on a particular graph/state and is defined via the dynamics.

**BUT:** The NS is a subsystem of a subsystem: $\mathcal{H}_{j}^{QBH}$ in $\mathcal{H}_{A}^{QSBH}$. It finds the first if you give the second.

There are very interesting situations in QG when the boundary is given.
The boundary does not need to be a physical one:

\[ C^2 \otimes C^2 \otimes C^2 \]

\[ \bullet \bullet \bullet \]

\[ C^2 \]

\[ \bullet \]

\[ \text{high energy modes} \]

\[ \text{environment} \]

\[ \text{boundary} \]

\[ \text{low energy modes} \]

Relate NS to renormalization group for quantum systems.
NS as quasiparticles.
In progress with O.Dreyer, L.Smolin and P.Zanardi.
NS & locality, or, Microscopic vs Macroscopic locality

One can try to assign geometric/local/causal properties to the subfoams/subgraphs. But they are not eigenstates of the dynamics. One has to take into account the sum over foams.

Note that in the $C^2 \otimes 3$ example, the preserved qubit is none of the originals.

NS method suggests: Assign locality/geometric/causal properties to the effective NSs.
Outlook:

Black hole case: Quasinormal modes as excitations in Rotating black holes BH in spin foams/other models
\[ G_q \rightarrow SO(3) \]

Propagating modes in quantum gravity with a boundary

Separate scales: environment \leftrightarrow small-scale fluctuations system \leftrightarrow coarse-grained fluctuations

NS theory: Approximate/emergent NS definition Poincare symmetry