How inefficient are simple asset-allocation strategies?

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Motivation

- **Ancient wisdom**
  - Rabbi Issac bar Aha (Talmud, 4th Century): Equal allocation
    A third in land, a third in merchandise, a third in cash.

- **Advances since then:**
  - Markowitz (1952);
  - Tobin (1958);
  - Sharpe (1964) and Lintner (1965);
  - Samuelson (1969) and Merton (1969); and,
  - Merton (1971).

Two challenges post-foundational work

1. **Implementation** of strategies from optimal portfolio models.
   - Implementing optimal policies requires **estimation** of parameters.
   - Traditionally, estimate using **classical** statistics: MLE, OLS, GMM.
   - But portfolio weights using these estimated **behave very poorly**.
     - **Extreme** weights;
     - The weights **fluctuate** a lot over time;
     - Portfolio has very **poor performance** out of sample.

2. **Explicit characterization** of dynamic portfolio policies.

Improving implementation of optimal models

1. **Bayesian** estimators that incorporate a **prior**.
   - A non-informative **diffuse prior**

     Barry (1974), and Bawa, Brown, and Klein (1979),
   - **Empirical** Bayes-Stein shrinkage estimators.

     Jobson and Korkie (1980), Jorion (1985, 1986), Frost and Savarino (1986)
   - "**Data-and-model**" approach.

     Pástor (2000), and Pástor and Stambaugh (2000)

2. **Subjective views** combined with priors from model.

   Black and Litterman (1990, 1992).

3. **Impose portfolio constraints**

   Frost and Savarino (1988), Chopra (1993), Jagannathan and Ma (2003).

Characterization of dynamic portfolio strategies

- **Recent work - analytical models of dynamic choice** (partial list)
  Brennan and Xia (2000, 2002) Campbell and Viceira (1999, 2001), Campbell, Chan, and Viceira (2003), Campbell, Cocco, Gomes, and Viceira (2001), Chacko and Viceira (2004), Kim and Omberg (1996), Liu (2001), Skiadas and Schroder (1999), Wachter (2002), and Xia (2001).

- **Recent work - numerical** (very partial list)
  Balduzzi and Lynch (1999), Brennan, Schwartz, and Lagnado (1997), Lynch (2001), and Lynch and Balduzzi (2000).

- **Estimation of return processes in dynamic models**
  Avramov (2004), Barberis (2000), Cremers (2002), Johannes, Polson, and Stroud (2002), Kandel and Stambaugh (1996).

Our objective

- **Evaluate the inefficiency** from using **simple portfolio rules**.
  - No estimation of parameters, and
  - No optimization.

Examples of simple portfolio rules:

- Hold only a single asset ("**all your eggs in one basket**");
  For instance, holding the market portfolio.
- The $1/N$ **naive diversification** rule. We study two versions of this:
  1. With rebalancing in order to maintain $1/N$ allocation over time;
  2. Without rebalancing ("buy-and-hold").

- **Identify** circumstances under which optimal portfolios perform well.

Why focus on the $1/N$ asset allocation rule

- **Easy to implement** - no estimation or optimization required.
- **Investors use** such simple portfolio rules, in practice.
  - See Benartzi and Thaler (2001) and Liang and Weisbenner (2002).
  - Investors exhibit inertia in investment and rebalancing decisions;
    - ⋆ Employees often accept the default allocation made by employers (Madrian and Shea (2000) and Choi, Laibson, Madrian, and Metrick (2001))
    - ⋆ Many employees never revise these initial allocations (Choi, Laibson, Madrian, and Metrick (2004)).
- MacKinlay and Pastor (2000) show when a risk factor is missing from an asset pricing model, then if one exploits this under the assumption that all assets have same expected return, one gets the $1/N$ allocation.

Interpretation of the $1/N$ portfolio policy

- Simple, but **not** simplistic:
  - Does have **some diversification**, though not "optimally" diversified
  - With rebalancing – **contrarian**;
  - Without rebalancing – **momentum**.

Bottom line

▶ What we do

- Compare performance of simple portfolio rules to that of optimal portfolio rules using **out-of-sample** Sharpe ratio, CEQ& Turnover.

▶ What we find

- The $1/N$ rule (with or without rebalancing) is **not** very inefficient.
  - ⋆ It often has a **higher Sharpe ratio** out-of-sample than portfolios from static and dynamic models of optimal asset allocation.
  - ⋆ It has a **lower turnover** than strategies from optimizing models.

▶ Intuition

- **Out-of-sample**, gains from optimal diversification not big enough to offset the loss arising from estimation error.

Methodology

1. **Choose** a window $(M < T)$ over which to estimate the parameters.
2. **Estimate** the parameters for each model being considered.
3. **Solve** model for optimal portfolio weights using estimated parameters.
4. **Measure** the return from holding these portfolio weights over the next period, that is, **out-of-sample**.
5. **Repeat** this "rolling-window" procedure until the end of the data set.
6. **Calculate** quantities to report

Quantities that we calculate

1. **Sharpe ratio** for the time series of in-sample and out-of sample returns.
   (a) For a **single asset** (typically, the market portfolio)
   (b) For portfolio of **just risky assets** (excluding riskfree asset)
   (c) For the **entire** portfolio (including riskfree asset—not reported)
2. **Certainty equivalent** value (CEQ).
3. **P-values** for difference in Sharpe-ratio and CEQ vs. simple strategies
4. **Turnover** for the portfolio (definition on next slide).
5. **Summary statistics** about the path of individual weights over time.

Turnover

- **Notation:**
  - $w_j(t)$ = the portfolio weight in asset j chosen at time t
  - $w'_j(t)$ = the portfolio weight before rebalancing at time $t+1$
  - $w_j(t+1)$ = the desired portfolio weight at time $t+1$
- **Definition:**

$$\text{Turnover} = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N}\left(\left|w'_j(t) - w_j(t+1)\right|\right)$$

- **Example:** For the $1/N$ strategy
  - $w_j(t) = w_j(t+1) = 1/N$
  - But $w'_j(t)$ is different, because changes in asset prices have caused a change in the relative weights in the portfolio.

Models of static optimal portfolio choice considered

1. **Single** asset (typically, the market)

2. **Simple strategy** of $1/N$ portfolio (without and with rebalancing)

3. **Classical Mean-Variance** using MLE (without & with constraints).

4. **Minimum Variance Portfolio** (without & with constraints)

5. **Empirical-Bayes Portfolio** (without & with constraints)

6. **Three-fund** strategy of Kan and Zhou (2005)

7. **Bayesian "Data-and-Model" Portfolios**

8. **Dynamic** model with stochastic bond returns – Campbell and Viceira

9. **Dynamic** model with stochastic stock returns – Campbell and Viceira

Data sets considered: Ten

- We consider more than a single data set because:
  1. We did not want our results to be **limited to a particular data set**; this was specially important given the nature of our findings.
  2. We wished to **use same data set as used in the original paper** proposing a particular asset-allocation model.
  3. We thought that it would be useful to experiment with simulated data **to understand better the results** we find from empirical data.

List of data sets considered

In addition to the 3-month US T-bill, the data sets include:

1. Twenty years of monthly returns on ten sector portfolios;
2. Seventy-five years of monthly returns on ten industry portfolios;
3. Thirty years of monthly returns on nine international equity indexes;
4. Seventy years of monthly returns on US market, HML and SMB;
5. Seventy-five years of monthly returns on 20 portfolios of firms sorted by size and B/M, HML, SMB, & US market, under a single-factor model;
6. Same data as above, but assuming a three-factor model;
7. Same data as above, but assuming a four-factor model;
8. Stochastic interest rates: Same data as in Campbell and Viceira.
9. Time-varying expected returns: Same data as in Campbell and Viceira.
10. Simulated data

Results

Summary of results

- **In-sample**, by construction, Sharpe ratio **highest** for mean-var strategy.
- **Out-of-sample**, Sharpe ratio is usually **lowest** for mean-var strategy.
- **Out-of-sample**, min-var-const does well (Jagannathan and Ma, 2003).
  - But, P-value for difference in Sharpe ratio & CEQ not significant
  - Turnover is 2-6 times higher than $1/N$;
  - Several assets have zero investment leading to unbalanced portfolio.
- **Dynamic allocation strategies** also do not out-perform $1/N$.
- In general, unconstrained policies (Bayes-Stein, 3-Fund, Data-Model) perform worse than constrained strategies and $1/N$.

Table 1: Ten S&P sector portfolios

Statistic \ Strategy	Single asset	$1/N$ (no rebal)	$1/N$ ($rebal$)	Mean-var	Mean-var constr.	Min-var	Min-var constr.	Bayes Stein	Bayes constr.	3-fund	DataModel $\omega = 0.50$
Panel A: Statistics about in-sample performance											
Mean	0.00629	0.00779	**0.00779**	0.05995	0.01410	0.00278	0.00646	0.02411	0.00938	0.01980	0.01837
Variance	0.00190	0.00172	**0.00172**	0.02427	0.00406	0.00113	0.00140	0.00983	0.00333	0.00667	0.00647
Sharpe Ratio	0.14436	0.18762	**0.18762**	**0.38481**	0.22136	0.08292	0.17258	0.24320	0.16242	0.24246	0.22836
Panel B: Statistics about out-of-sample performance											
Mean	0.00629	0.00786	**0.00779**	0.00653	0.00733	0.00304	0.00315	0.00454	0.00780	0.00327	0.00592
Variance	0.00190	0.00208	**0.00172**	0.00677	0.00675	0.00138	0.00142	0.00314	0.00527	0.00229	0.00327
Sharpe Ratio	0.14436	0.17245	**0.18762**	**0.07942**	0.08924	0.08200	0.08344	0.08107	**0.10752**	0.06829	0.10362
pVal.-(rebal.)	0.09067	0.29577	**0.50000**	0.11912	0.09219	0.04578	0.01298	0.09122	0.13804	0.05113	0.11898
pVal.-(no rebal.)	0.14583	0.50000	**0.29577**	0.12786	0.05336	0.09573	0.06908	0.10131	0.09964	0.05910	0.12990
CEQ	0.00534	0.00682	**0.00693**	0.00315	0.00396	0.00235	0.00244	0.00297	0.00517	0.00212	0.00429
pVal.-(rebal.)	0.20244	0.47548	**0.50000**	0.15915	0.16533	0.03799	0.02497	0.08886	0.27042	0.03852	0.20952
pVal.-(no rebal.)	0.24599	0.50000	**0.48048**	0.15137	0.10245	0.06941	0.06095	0.09544	0.22560	0.04900	0.21648
Turnover	0.00000	0.00000	**0.03051**	**1.18940**	0.13813	0.19964	0.07526	0.68375	**0.11093**	0.60460	0.62193
Panel C: Statistics about portfolio weights											

Table 2: Ten industry portfolios

Strategy Statistic	Single asset	$1/N$ (no rebal)	$1/N$ ($rebal$)	Mean-var	Mean-var constr.	Min-var	Min-var constr.	Bayes	Bayes constr.	3-fund	DataModel $\omega = 0.50$
Panel A: Statistics about in-sample performance											
Mean	0.00532	0.00595	**0.00595**	0.01225	0.00722	0.00494	0.00555	0.00635	0.00591	0.00499	0.00686
Variance	0.00219	0.00193	**0.00193**	0.00332	0.00204	0.00134	0.00145	0.00173	0.00196	0.00135	0.00225
Sharpe Ratio	0.11377	0.13531	**0.13531**	**0.21239**	0.15995	0.13490	0.14599	0.15269	0.13357	0.13606	0.14463
Panel B: Statistics about out-of-sample performance											
Mean	0.00532	0.00561	**0.00595**	-0.04805	0.00421	0.00589	0.00536	-0.02781	0.00478	-0.02050	0.06387
Variance	0.00219	0.00189	**0.00193**	1.74820	0.00386	0.00144	0.00141	0.75968	0.00341	0.58878	11.52589
Sharpe Ratio	0.11377	0.12895	**0.13531**	**-0.03634**	0.06780	**0.15536**	0.14253	-0.03190	0.08193	-0.02672	0.01881
pVal.-(rebal)	0.00920	0.07918	**0.50000**	0.01150	0.02864	0.29838	0.40615	0.01305	0.05794	0.01542	0.05883
pVal.-(no rebal)	0.03783	0.50000	**0.07918**	0.01427	0.03456	0.24106	0.32810	0.01613	0.07028	0.01893	0.06961
CEQ	0.00423	0.00466	**0.00498**	-0.92215	0.00228	**0.00517**	0.00465	-0.40765	0.00308	-0.31489	-5.69908
pVal.-(rebal)	0.24601	0.39145	**0.50000**	0.00000	0.03302	0.45583	0.41638	0.00000	0.09199	0.00000	0.00000
pVal.-(no rebal)	0.34142	0.50000	**0.38817**	0.00000	0.04349	0.38045	0.49689	0.00000	0.12082	0.00000	0.00000
Turnover	0.00000	0.00000	**0.02162**	13132.02	0.15501	**0.46800**	0.05570	218.16550	0.15616	214.96018	756.47789

Table 3: Nine international equity indexes

Strategy / Statistic	Single asset	$1/N$ (no rebal)	$1/N$ $(rebal)$	Mean-var	Mean-var constr.	Min-var	Min-var constr.	Bayes	Bayes constr.	3-fund	DataModel $\omega = 0.50$
Panel A: Statistics about in-sample performance											
Mean	0.00525	0.00555	**0.00555**	0.01105	0.00679	0.00654	0.00624	0.00693	0.00661	0.00574	0.00651
Variance	0.00179	0.00189	**0.00189**	0.00279	0.00181	0.00165	0.00167	0.00176	0.00181	0.00207	0.00190
Sharpe Ratio	0.12392	0.12767	**0.12767**	**0.20902**	0.15952	0.16083	0.15270	0.16529	0.15549	0.12635	0.14936
Panel B: Statistics about out-of-sample performance											
Mean	0.00525	0.00498	**0.00555**	-0.03519	0.00496	0.00633	0.00623	-0.01171	0.00453	0.00486	0.01058
Variance	0.00179	0.00195	**0.00189**	0.23951	0.00342	0.00181	0.00172	0.04907	0.00285	0.00722	0.01702
Sharpe Ratio	0.12392	0.11286	**0.12767**	**-0.07191**	0.08480	0.14896	0.15014	-0.05284	0.08476	0.05724	0.08111
pVal.-(rebal.)	0.43233	0.05442	**0.50000**	0.00973	0.17411	0.20912	0.15833	0.01256	0.14731	0.12904	0.25897
pVal.-(no rebal.)	0.29735	0.50000	**0.05442**	0.01520	0.24587	0.09942	0.07061	0.01993	0.21755	0.18834	0.32892
CEQ	0.00435	0.00401	**0.00461**	**-0.15495**	0.00325	0.00543	0.00537	-0.03624	0.00310	0.00125	0.00207
pVal.-(rebal.)	0.43430	0.33052	**0.50000**	0.00000	0.23193	0.30362	0.31318	0.00000	0.19342	0.10596	0.00000
pVal.-(no rebal.)	0.41331	0.50000	**0.33742**	0.00000	0.33071	0.19696	0.20417	0.00000	0.29034	0.15712	0.00000
Turnover	0.00000	0.00000	**0.02931**	**124.14**	0.21196	0.21400	0.06647	51.59943	0.17886	24.01118	40.89281

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Table 3 (cont.): Nine international equity indexes

Strategy Statistic	Single asset	$1/N$ (no rebal)	$1/N$ ($rebal$)	Mean-var	Mean-var constr.	Min-var	Min-var constr.	Bayes Stein	Bayes constr.	3-fund	DataModel $\omega = 0.50$
Panel C: Statistics about portfolio weights											
Canada											
Min	0.000	0.099	**0.111**	-100.443	0.000	-0.152	0.000	-61.238	0.000	-50.784	-67.241
Max	0.000	0.125	**0.111**	123.586	0.415	0.289	0.302	57.876	0.444	19.670	29.587
Avg	0.000	0.111	**0.111**	-0.068	0.006	0.059	0.088	0.057	0.005	0.255	-0.015
StdDev	0.000	0.004	**0.000**	15.567	0.043	0.137	0.110	7.662	0.039	4.037	5.720
France											
Min	0.000	0.090	**0.111**	-14.640	0.000	-0.146	0.000	-8.954	0.000	-7.438	-9.801
Max	0.000	0.124	**0.111**	18.686	1.000	0.126	0.052	8.719	0.842	2.924	7.157
Avg	0.000	0.111	**0.111**	0.239	0.139	-0.062	0.002	0.069	0.099	-0.056	0.135
StdDev	0.000	0.004	**0.000**	2.320	0.293	0.057	0.008	1.147	0.208	0.602	0.871
Germany											
Min	0.000	0.095	**0.111**	-72.513	0.000	0.024	0.000	-33.643	0.000	-22.326	-29.718
Max	0.000	0.125	**0.111**	83.761	0.244	0.349	0.277	39.391	0.272	13.592	15.709
Avg	0.000	0.111	**0.111**	0.246	0.003	0.149	0.120	0.216	0.011	0.248	0.179
StdDev	0.000	0.004	**0.000**	9.767	0.023	0.086	0.077	4.636	0.044	2.012	2.683
Italy											
Min	0.000	0.091	**0.111**	-17.089	0.000	0.015	0.008	-8.010	0.000	-2.800	-4.316
Max	0.000	0.133	**0.111**	35.657	0.574	0.136	0.111	21.800	0.369	18.106	23.871
Avg	0.000	0.111	**0.111**	0.172	0.034	0.075	0.062	0.121	0.044	0.086	0.089
StdDev	0.000	0.006	**0.000**	2.818	0.098	0.028	0.023	1.576	0.085	1.169	1.561

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Table 3 (cont.): Nine international equity indexes

Strategy Statistic	Single asset	$1/N$ (no rebal)	$1/N$ ($rebal$)	Mean-var	Mean-var constr.	Min-var	Min-var constr.	Bayes Stein	Bayes constr.	3-fund	DataModel $\omega = 0.50$
Panel C: Statistics about portfolio weights											
Japan											
Min	0.000	0.090	**0.111**	**-247.675**	0.000	-0.194	0.009	-115.370	0.000	-72.607	**-96.187**
Max	0.000	0.128	**0.111**	321.523	1.000	0.364	0.244	150.699	1.000	51.376	80.426
Avg	0.000	0.111	**0.111**	2.215	0.467	0.078	0.129	1.144	0.402	0.598	1.229
StdDev	0.000	0.006	**0.000**	37.378	0.426	0.126	0.071	17.726	0.376	7.491	10.508
Switzerland											
Min	0.000	0.102	**0.111**	-52.351	0.000	-0.146	0.000	-31.958	0.000	-26.521	-35.046
Max	0.000	0.123	**0.111**	39.380	1.000	0.220	0.166	18.380	1.000	7.115	12.417
Avg	0.000	0.111	**0.111**	0.197	0.205	0.047	0.051	0.137	0.165	0.131	0.141
StdDev	0.000	0.004	**0.000**	5.861	0.350	0.089	0.045	3.062	0.259	1.963	2.779
UK											
Min	0.000	0.098	**0.111**	-106.734	0.000	-0.276	0.000	-49.807	0.000	-22.854	-30.190
Max	0.000	0.122	**0.111**	136.233	0.806	0.435	0.252	63.764	1.000	21.628	35.068
Avg	0.000	0.111	**0.111**	0.959	0.061	-0.042	0.025	0.449	0.076	0.167	0.580
StdDev	0.000	0.004	**0.000**	15.652	0.150	0.159	0.064	7.329	0.132	2.799	3.872
US											
Min	0.000	0.102	**0.111**	-599.852	0.000	-0.148	0.378	-279.532	0.000	-136.736	-181.236
Max	0.000	0.122	**0.111**	765.660	1.000	1.052	0.622	358.756	1.000	122.167	189.040
Avg	0.000	0.111	**0.111**	4.063	0.084	0.437	0.505	2.320	0.159	1.575	2.629
StdDev	0.000	0.004	**0.000**	85.998	0.232	0.250	0.058	40.381	0.263	15.794	22.141
World											
Min	1.000	0.105	**0.111**	**-1481.952**	0.000	-1.407	0.000	-693.868	0.000	-235.645	**-364.088**
Max	1.000	0.118	**0.111**	**1168.282**	0.000	1.244	0.244	544.931	0.000	322.161	**426.549**
Avg	1.000	0.111	**0.111**	-7.023	0.000	0.259	0.016	-3.514	0.000	-2.004	-3.968
StdDev	0.000	0.002	**0.000**	169.285	0.000	0.576	0.048	80.064	0.000	33.228	46.685

Table 4: Market, HML and SMB portfolios

Table 5: Market, HML, SMB, and 20 size- and B/M-sorted portfolios

	Single asset	$1/N$ (no rebal)	$1/N$ ($rebal$)	Mean var	Mean-var constr.	Min-var	Min-var constr.	Bayes	Bayes constr.	3-fund	DataModel $\omega = 0.50$
Panel A: Statistics about in-sample performance											
Sharpe	0.11377	0.16744	**0.16744**	**0.50882**	0.21090	0.03317	0.26176	0.40478	0.17001	0.40478	0.27233
Panel B: Statistics about out-of-sample performance											
Sharpe	0.11377	0.18656	**0.16744**	**-0.00467**	0.18906	-0.02181	**0.25574**	-0.00641	0.19577	-0.00730	0.04042
pVal	0.00974	0.00003	**0.50000**	0.00945	0.22637	0.00595	0.04149	0.00886	0.19244	0.00857	0.03755
CEQ	0.00423	0.00800	**0.00705**	-0.55112	0.00745	-0.00023	**0.00398**	-0.36234	0.00727	-0.34149	-0.20462
pVal	0.03427	0.23069	**0.50000**	0.00000	0.40656	0.00234	0.09641	0.00000	0.45045	0.00000	0.00000
Turnover	0.00000	0.00000	**0.01850**	63.35506	0.30349	0.13283	**0.03235**	49.28308	0.25722	47.41400	151.83048

Table 6: MKT, HML, SMB, MOM, 20 size-& B/M-sorted portfolios

Table 7: MKT & 10-year bond with stochastic interest rates

Table 8: Dynamic model: Time-varying expected returns ($\gamma = 3$)

Statistic	Single asset	$1/N$ (no rebal)	$1/N$ ($rebal$)	Mean	Mean-var constr.	Min var	Min-var constr.	Bayes var	Bayes constr.	3-fund	Dynamic (re-est)	Dynamic (one-est)
Panel A: Statistics about in-sample performance												
Sharpe	0.30517	0.35383	**0.35383**	0.33910	0.33910	0.38078	0.31903	0.33235	0.31060	0.36425	**0.39357**	**0.39357**
Panel B: Statistics about out-of-sample performance												
Sharpe	0.30517	**0.39866**	**0.35383**	0.35169	0.20207	0.30686	0.30686	0.33634	0.19936	0.14312	**0.12151**	**0.36337**
pVal	0.05869	0.00006	**0.50000**	0.13240	0.00051	0.38137	0.38137	0.14002	0.03676	0.07369	0.00391	0.01285
CEQ	**0.00349**	0.00324	**0.00311**	0.00276	0.00274	**0.00318**	0.00318	0.00272	0.00254	0.00237	**0.00191**	**0.00201**
Turnover	0.00000	0.00000	**0.02677**	5.40181	0.12014	0.00500	0.00500	4.28032	0.07281	1.21319	**1.27328**	**34.84525**
Panel C: Statistics about portfolio weights												
5yrBond												
Min	0.000	0.452	**0.500**	-6.956	0.000	0.959	0.959	-3.141	0.000	-7.694	**-18.776**	**-37.199**
Max	0.000	0.577	**0.500**	48.577	0.720	0.980	0.980	45.709	0.950	9.865	**5.900**	**278.641**
Avg	0.000	0.497	**0.500**	1.438	0.395	0.970	0.970	1.706	0.755	1.197	0.071	8.804
StdDev	0.000	0.019	**0.000**	7.579	0.251	0.006	0.006	6.596	0.302	1.869	2.912	36.199
Market												
Min	1.000	0.423	**0.500**	-47.577	0.280	0.020	0.020	-44.709	0.050	-8.865	**-4.900**	**-277.641**
Max	1.000	0.548	**0.500**	7.956	1.000	0.041	0.041	4.141	1.000	8.694	19.776	38.199
Avg	1.000	0.503	**0.500**	-0.438	0.605	0.030	0.030	-0.706	0.245	-0.197	0.929	-7.804
StdDev	0.000	0.019	**0.000**	7.579	0.251	0.006	0.006	6.596	0.302	1.869	2.912	36.199

Insights based on simulated data

Table 9: Simulated data: $M = 10$ **years,** $T = 600$ **years**

Table 10: Simulated data: $M = 100$ years, $T = 600$ years

Statistic	Single asset	$1/N$ (no rebal)	$1/N$ ($rebal$)	Mean-Var	Mean-var constr.	Min-Var	Min-var constr.	Bayes	Bayes constr.	3-fund	Data&model $\omega = 0.50$	Mean true
Panel A: Statistics about in-sample performance												
Sharpe	0.14079	0.13473	**0.13473**	**0.14145**	0.12792	0.12611	0.13010	0.13621	0.12578	0.13619	0.14092	0.14073
Panel B: Statistics about out-of-sample performance												
Sharpe	**0.14079**	0.12586	**0.13479**	**0.12965**	0.12331	0.12694	0.13062	0.13251	0.12813	0.13053	0.13802	**0.14079**
pVal	**0.12045**	0.04048	**0.50000**	0.21793	0.02374	0.09650	0.13861	0.34486	0.06247	0.23839	0.26917	0.12045
CEQ	**0.00549**	0.00561	**0.00513**	**0.00524**	0.00549	0.00436	0.00461	0.00493	0.00519	0.00485	0.00540	0.00549
pVal	0.14122	0.04563	**0.50000**	0.37170	0.11505	0.02093	0.06474	0.29151	0.42268	0.21734	0.26935	0.14122
Turnover	0.00000	0.00000	**0.02485**	0.05852	0.03144	0.03805	0.02093	0.04128	0.04614	0.04624	0.02829	0.00000

Table 11: Simulated data: $M = 10$ years and $N = 100$ assets

Statistic	Single asset	$1/N$ (no reb)	$1/N$ ($rebal$)	Mean var	Mean-var constr.	Min var	Min-var constr.	Bayes	Bayes constr.	3-fund	DataModel $\omega = 0.50$	Mean-var true
Panel A: Statistics about in-sample performance												
Sharpe	0.14431	0.14430	**0.14430**	**0.18374**	0.14620	0.05342	0.13009	0.13126	0.10604	0.13074	0.15531	**0.14328**
Panel B: Statistics about out-of-sample performance												
Sharpe	0.14431	0.13690	**0.14534**	**0.00281**	0.12914	0.03801	0.13112	-0.00026	0.13037	-0.01452	0.00157	**0.14431**
pVal	0.18651	0.01200	**0.50000**	0.00000	0.00636	0.00000	0.00457	0.00000	0.01165	0.00000	0.00000	0.18651
CEQ	0.00564	0.00687	**0.00562**	**-29.04618**	0.00494	0.00072	0.00345	-5.73108	0.00473	-8.95993	-68.54554	**0.00564**
pVal	0.46081	0.00000	**0.50000**	0.00000	0.02420	0.00000	0.00000	0.00000	0.00627	0.00000	0.00000	0.46081
Turnover	0.00000	0.00000	**0.03527**	**367.59855**	0.23188	2.01365	0.08993	147.18731	0.24611	88.64578	319.07492	**0.00000**

Robustness checks

- Results reported for 10 datasets: 10+ strategies (many others nested).
- In all, we had more than 500 tables – reported only 10 – tried to pick the setting that would be least favorable to the $1/N$ strategy
- Considered risk aversion $= \{\mathbf{1}, 2, \mathbf{3}, 4, 5, 10, 20\}$.
- Reported results for $M = 120$; also considered $M = \{40, 60\}$.
- Reported results for holding interval of **one period** (month or quarter); also considered holding period of one year.
- Reported results for the portfolio of **only-risky-assets**; also considered total-portfolio (including riskless asset.)
- In the simulations, considered $M = \{5, 10, 20, 30, 50, 100\}$ years; and $N = \{4, 10, 25, 50, 100\}$ assets.

Conclusions

▶ Experiment considered

Compared performance of simple "$1/N$" allocation rule (with and without rebalancing) to allocation rules from optimizing models.

- Considered **static and dynamic** models of optimal portfolio selection
  - ⋆ Minimum variance portfolios;
  - ⋆ Mean variance portfolios;
  - ⋆ Bayes-Stein shrinkage portfolios;
  - ⋆ Pastor-Stambaugh "data-and-model" portfolios.
  - ⋆ Dynamic models with stochastic stock and bond returns.
- Considered both **unconstrained and constrained** strategies.
- Considered several **different data sets**.

Takeaways

- **Main finding**
  - The $1/N$ strategy not very inefficient.
    - ⋆ It often has a higher Sharpe ratio than for portfolios from optimal static and dynamic models.
    - ⋆ Estimation error erodes gains from optimal diversification (relative to naive diversification)

- **Main message**
  - Focus future effort on **estimation** rather than optimization
  - 1/N strategy is a good **benchmark** to evaluate out-of-sample new strategies for optimal asset allocation.

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