Developments in Quantitative Finance Programme
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Dynamic Correlation Intensity Modelling for Portfolio Credit Risk

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Outline

- Assessing the impact of dependency at the portfolio level
- Incorporating dependency into intensity based models
- Dynamic correlation approach
- Dynamic correlation model with copula extension
- Examining levels of model implied default correlation
- Credit migration and default risk portfolio model
- Conclusions and further research

Introduction

- **Prior research:** focussed on the modelling of individual defaults but in order to assess the risk of a given portfolio it is crucial to examine the portfolio as a whole

- **Example**: supposing firms have perfect positive dependence then if one firm defaults they all default. Individual default likelihoods do not capture the true risk unless the firms are independent or perfectly dependent

- **Macroeconomic environment**: influences both the individual firm and industry or economy wide risk

- **Aim**: develop a portfolio model incorporating flexible dependence structure

Overview of Default Correlation

- **Default correlation**: is a measure of the tendency of one firm to default given that another firm has defaulted

- **Default correlation drivers**:

  - ***Common factors****:* such as interest rates and changes in economic growth affecting all firms

  - ***Direct firm links****:* such as if one firm is a creditor of another

Motivation: Impact of Default Dependency

- **Portfolio level**: a portfolio of investment grade debt can be extremely risky if the securities are highly dependent

- **Basket credit derivatives**: are securities whose payoff depends on the default behaviour of all the underlying bonds

  - This market is rapidly developing and a reliable theoretically based model which incorporates default dependence is essential to price these instruments

Intensity Based Approaches

- **Individual default intensity process**: is specified exogenously. The time of default is set equal to the first jump of a Cox process with given intensity

- **Conditional independence model**: intensity is specified through a linear combination of macroeconomic factors and default dependency incorporated through common factors

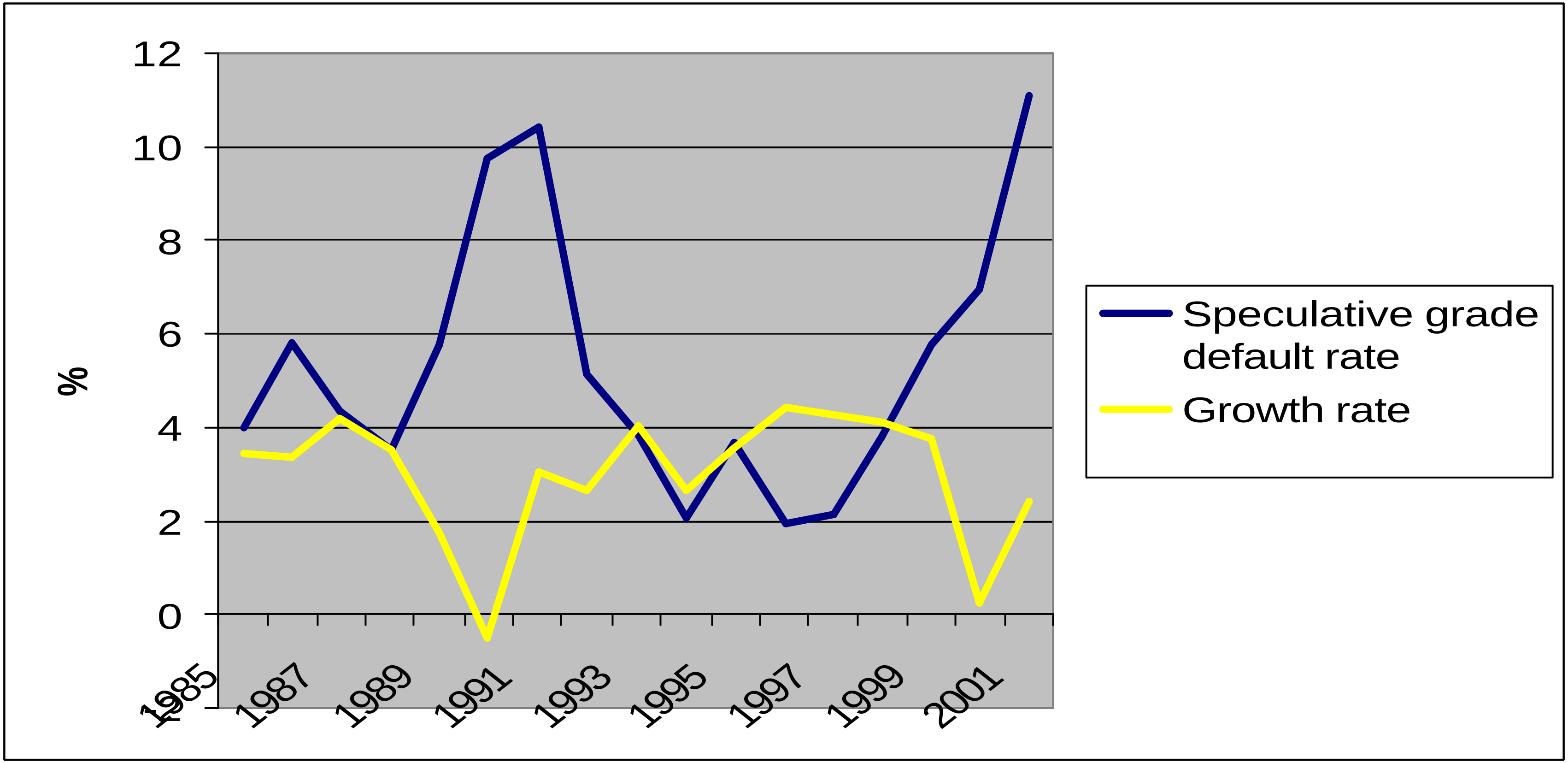
- **Regime shifting model**: intensity is defined by a linear combination of common factors but shifting between high and low states can occur. Dependency generated by common factors and depends on the regime

Empirical Features of Default Correlation

- **Empirical analysis**: Das *et al*. (2002) conducted an analysis of over 7000 U.S. firms over the period 1987-2000
- **Recession periods**: default probabilities and default correlation increase
- **Growth periods**: default probabilities and default correlation decrease
- **Conclusion**: default probabilities and default correlation are time varying in a way that is related to the macroeconomic environment

Empirical Evidence

- **Data**: annual U.S. growth rate, source: Bureau of Economic Analysis, U.S. Department of Commerce, and annual U.S. default rates for speculative grade, source: Hamilton *et al*. (2002a), from 1985 - 2001



Implication for Credit Risk

- **Implication:** credit risk is greater in periods of decline and lesser in periods of prosperity
- **Conditional independence model**: inadequately captures the time variation
- **Regime shifting model**: time variation is captured however
  - A regime shifting mechanism must be specified
  - Parameters for each regime must be calibrated and is non trivial
  - Regime shifts cause jumps in the intensity function

Aim of Dynamic Correlation Approach

- **Time variation:** in default intensities and in correlations between default intensities is incorporated

- **Macroeconomic environment**: influences the level of risk for individual firms and dependency between firms

- **Impact**: increased risk in periods of economic downturn and decreased risk during periods of economic growth is incorporated

Model Specification

- **Firm Default Intensity process**: is defined as

$$d\,l_t = \kappa_1(\zeta\, \mathbf{w}_t - l_t)dt + \sigma_1\sqrt{l_t}\,d\mathbf{Z}_{1t}$$

where ? is a scaling parameter and the individual firm Brownian motions are correlated with correlation matrix given below

- **Dynamic correlation**: matrix is defined depending on the stochastic weighting function

$$S_t = \boldsymbol{w}_t\Sigma_{HI} + (1-\boldsymbol{w}_t)\Sigma_{LO}$$

so that $\mathbf{S}_t$ is a mixture of the correlation matrix S_{HI} for *recession* periods and S_{LO} for *growth* periods

Model Specification...

- **Macroeconomic process**: GDP growth rate modelled using an Ornstein-Uhlenbeck process

$$dD_t = k_2(q_2 - D_t)dt + s_2 dZ_{2t}$$

- **Weighting function**: transforms the growth rate process to lie locally in [0,1] and is negatively correlated with the growth rate process

$$w_t = aD_t + b$$

where a:=-(ß-d)$^{-1}$ and b:=ß(ß-d)$^{-1}$ where ß and d are *upper* and *lower* bounds for the *historical growth rate*

Calibration Technique

- **Macroeconomic process**: calibrated using a maximum likelihood estimation procedure
- **Intensity process**: calibrated using derived yield spread dynamics in terms of intensities and the Kalman filter (Kalman, 1960)
  - ***Observable****: yield spread* is the difference between the yield on corporate and Treasury bonds of comparable maturities
  - ***Unobservable****: credit spread* is the part of the yield spread due to credit risk of corporate bonds. Yield spreads contain non credit information such as tax, liquidity and call and conversion features

Calibration Technique...

- **Dynamic correlation**: using optimal parameters the covariance of the unobserved variables -- which is an output of the filter -- is used as follows

  - Principal component analysis is applied to the corresponding correlation matrices. As the first component accounts for most of the variance in the data it is used as a proxy for the original variables

  - The maximum and minimum values of the first component are found and then the original correlation matrices that correspond to these values are determined

Growth Rate and Yield Spread Data

- **Growth rate data**: quarterly U.S. growth rate from the second quarter 1947 until the first quarter 2003, source: Bureau of Economic Analysis, U.S. Department of Commerce

- **Yield spread data**: zero coupon yield spread data with daily frequency covering the period April 1996 until April 2001, source: Bloomberg. For AA, A and BBB ratings of 4, 5, 7, 10, 15, 20, 25 and 30 year maturities -- 24 series in all

Dynamic Correlation Model with Copula Extension

- **Intensity models**: dependency arises through the intensity processes λ for which we may compute the *survival probabilities* from 0 to t as

$$P(t) = E[\exp(-\int_0^t \lambda(u)du)]$$

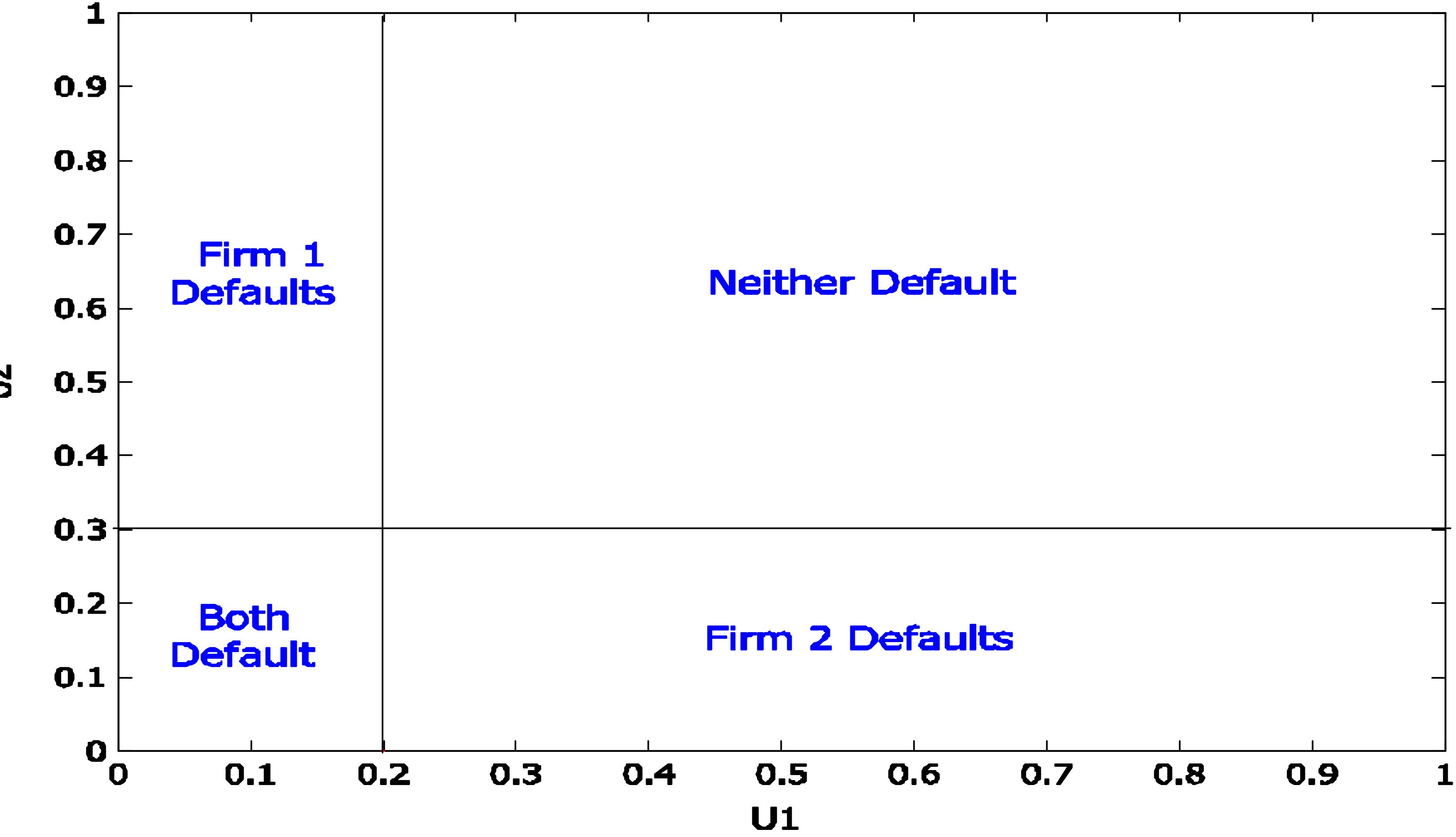
- **Default occurs**: e.g. for firm one if $P_1(t) = \mathbf{U}_1$ and for firm two if $P_2(t) = \mathbf{U}_2$, where P_1 and P_2 are default probabilities and $\mathbf{U}_1 \sim U(0,1)$ and $\mathbf{U}_2 \sim U(0,1)$ are *trigger variables*

- **Trigger variables**: are assumed to be independent in intensity models Schönbucher & Schubert (2001), Schönbucher (2003)

- **Motivation**: join the trigger variables using a copula function to allow a richer variety of dependence structures

Default and Survival Scenarios

❖Two firms with default probabilities of P_1=0.2 and P_2=0.3 respectively



Motivation: Copula Dependence

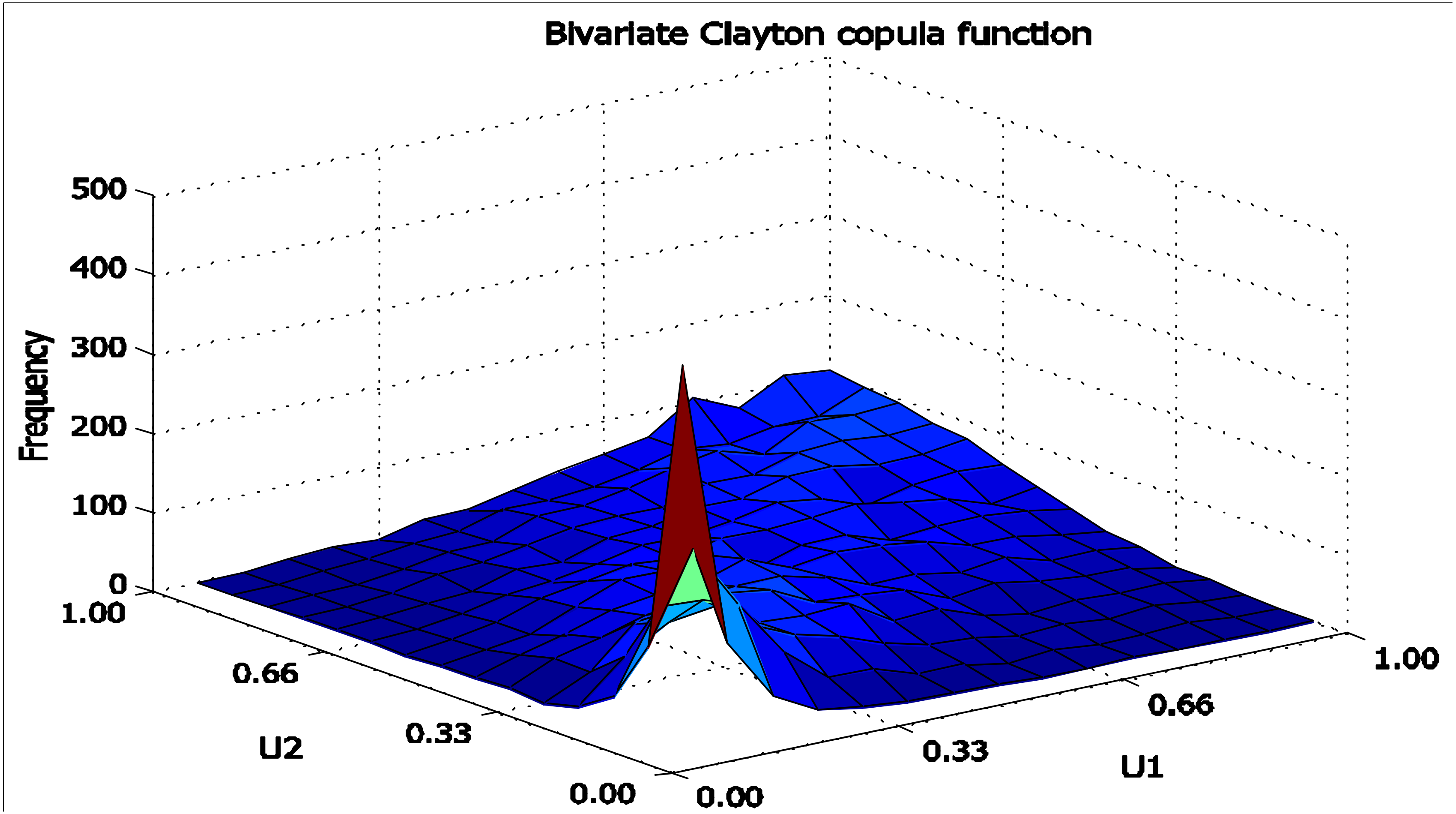
- **Joint default probabilities**: that arises when the trigger variables are independent are lower than those viewed historically

- **Clayton copula**: possesses lower-tail dependence and joining the trigger variables using this results in higher joint default probabilities

- **Tail dependence**: parameter is chosen such that the mean squared error between the model implied and the historical joint default probabilities is minimized

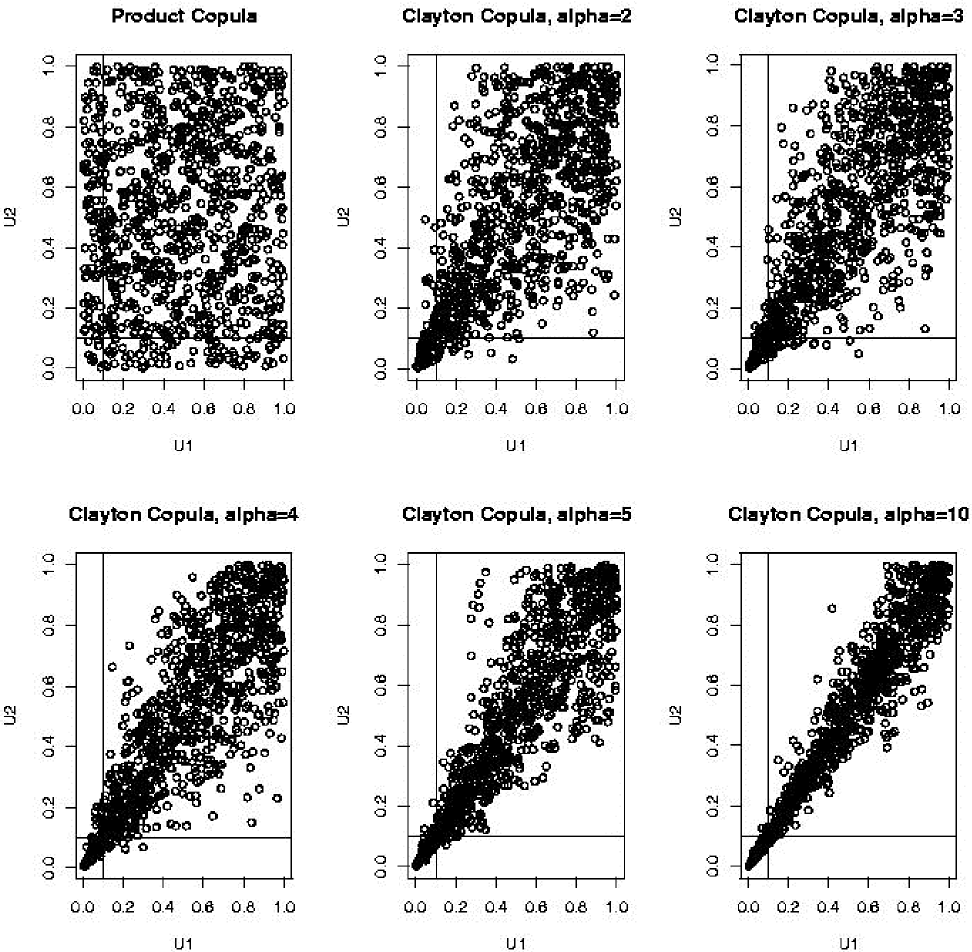
Bivariate Clayton Copula Function

- **Bivariate Clayton copula**: with dependence parameter e=2



Samples from Various Copula Functions

❖ Samples from independence and Clayton copulas



Examining Default Correlation

- **Empirical evidence**: Lucas (1995) examined historical 2-firm *default correlation* in various credit classes using Moody's default data from 1970-1993 as

$$r_{i,j} = \frac{P_{i,j} - P_i P_j}{\sqrt{P_i(1-P_i)P_j(1-P_j)}}$$

P_i, P_j are individual default probabilities and $P_{i,j}$ is joint default probability

- **Three factor model**: Duffee (2002)

- **Two factor model**: Dynamic correlation model

- **Two factor model**: Dynamic correlation model with Clayton extension

Historical Default Correlations

- **Empirical evidence**: Lucas' (1995) study provides historical default correlations for AA, A and BBB ratings for rolling five year periods ending 1975 to 1993

	Five Years		
	AA	**A**	**BBB**
AA	0.0%	1.0%	1.0%
A	1.0%	**1.0%**	1.0%
BBB	1.0%	1.0%	0.0%

Duffee's Model

- **Default intensity**: specified as follows

$$l_t = a + l^*_t + b_1(\mathbf{s}_{1t} - \bar{s}_{1t}) + b_2(\mathbf{s}_{2t} - \bar{s}_{2t})$$

- **Firm specific**: individual intensity function

$$d\,l^*_t = k\,(q - l^*_t)dt + s\sqrt{l^*_t}\,d\mathbf{Z}_t$$

- **Common factors**: short rate factors s_{1t} and s_{2t}

$$d\mathbf{s}_{i,t} = k_i(q_i - \mathbf{s}_{i,t})dt + s_i\sqrt{\mathbf{s}_{i,t}}\,d\mathbf{Z}_{i,t},\ \ i = 1,2$$

- **Default correlation:** determined by the $ß_1$ and $ß_2$ parameters

Comparison of 5-Year Default Correlations

Dynamic Correlation with Clayton Extension			
	AA	**A**	**BBB**
AA	0.00850%	1.07119%	1.27556%
A	1.07119%	0.02599%	1.43447%
BBB	1.27556%	1.43447%	0.02219%
Dynamic Correlation			
AA	-0.07072%	0.27838%	0.40493%
A	0.27838%	-0.04761%	0.31360%
BBB	0.40493%	0.31360%	-0.07202%
Duffee			
AA	0.00063%	0.00001%	-0.00025%
A	0.00001%	0.01365%	0.00031%
BBB	-0.00025%	0.00031%	0.01961%

Portfolio Credit Risk

- **Portfolio**: of 20 AA, 25 A and 55 BBB firms, with recovery rates of 0.25, bonds' principal value is $1 million. **P & L in millions** from 100,000 simulations

- **Intensity dependence structures differ**: otherwise models are identical

	Dynamic Correlation	Duffee
Mean	3.30	3.30
Median	3.35	3.34
Std. Dev.	0.93	0.93
1% Quantile	0.95	0.97
0.1% Quantile	0.10	0.15
99% VaR	2.36	2.33
99.9% VaR	3.20	3.15

Portfolio Credit Risk

- **Portfolio**: of 2 AA, 4 A and 22 BBB firms, with recovery rates of 0.25, bonds' principal value is $1 million. **P & L in millions** from 100,000 simulations
- **Trigger variable dependence structures differ**: otherwise models are identical

	Dynamic Correlation	Clayton Default Risk
Mean	2.95	2.95
Median	3.03	3.03
Std. Dev.	0.61	0.61
1% Quantile	1.14	1.13
0.1% Quantile	0.18	0.12
99% VaR	1.80	1.82
99.9% VaR	2.77	2.83

Empirical Features of Rating Transitions

- **Empirical analysis**: Bangia *et al*. (2000) conducted an analysis of 7,328 U.S. firms over the period 1981-1998
- **Recession periods**: likelihood of downgrading or defaulting increases and vice versa in growth periods
- **Growth periods**: likelihood of upgrading or staying in rating increases and vice versa in recession periods
- **Conclusion**: default and credit migration processes are influenced by the macroeconomic environment

Empirical Evidence

- **Quarterly transition matrices**: for growth and recession periods from Bangia *et al.* (2000) for AA, A and BBB to default rating D

	Growth Period					Recession Period			
	AA	**A**	**BBB**	**D**		**AA**	**A**	**BBB**	**D**
AA	98.08%	1.61%	0.12%	0.00%	**AA**	96.89%	2.79%	0.05%	0.00%
A	0.53%	98.06%	1.21%	0.00%	**A**	0.88%	96.44%	2.59%	0.00%
BBB	0.07%	1.47%	96.94%	0.02%	**BBB**	0.04%	1.11%	96.31%	0.11%

Aim of Credit Migration and Default Risk Approach

- **Credit risk**: encompasses all credit linked events including upgrades, downgrades and the default event itself

- **Aim**: to allow both the default risk and credit migration processes to be influenced by the macroeconomic environment

- **Impact**: increased risk in periods of economic downturn and decreased risk during periods of economic growth is incorporated

Model Specification

- **Upgrading process**: is defined as follows where ?$_u$ is a scaling parameter

$$d l_t^{up} = \kappa_u(\zeta_u \overline{\mathbf{w}}_t - l_t^{up})dt + \sigma_u \sqrt{l_t^{up}} d\mathbf{Z}_t$$

- **Staying in rating process**: is defined as follows where ?$_s$ is a scaling parameter

$$d l_t^{stay} = \kappa_s(\zeta_s \overline{\mathbf{w}}_t - l_t^{stay})dt + \sigma_s \sqrt{l_t^{stay}} d\mathbf{Z}_t$$

- **Downgrading process**: is defined as follows where ?$_d$ is a scaling parameter

$$d l_t^{down} = \kappa_d(\zeta_d \mathbf{w}_t - l_t^{down})dt + \sigma_d \sqrt{l_t^{down}} d\mathbf{Z}_t$$

Model Specification…

- **Macroeconomic process**: GDP growth rate modelled using an Ornstein-Uhlenbeck process as before

$$dD_t = k_2(q_2 - D_t)dt + s_2 dZ_{2t}$$

- **Weighting function**: transforms the growth rate process to lie in [0,1] and is *positively* correlated with the growth rate process

$$w_t = A D_t + B$$

where ? : = (ß-d)$^{-1}$ and ? : = -d (ß-d)$^{-1}$ where ß and d are *upper* and *lower* bounds for the *historical growth rate*

Model Specification...

- **Default process**: defined as follows

$$l_t^{default} = 1 - l_t^{up} - l_t^{stay} - l_t^{down}$$

- **Transition matrix**: given realizations of all processes each ? t is treated as a discrete time Markov chain with rating changes occurring at each ? t

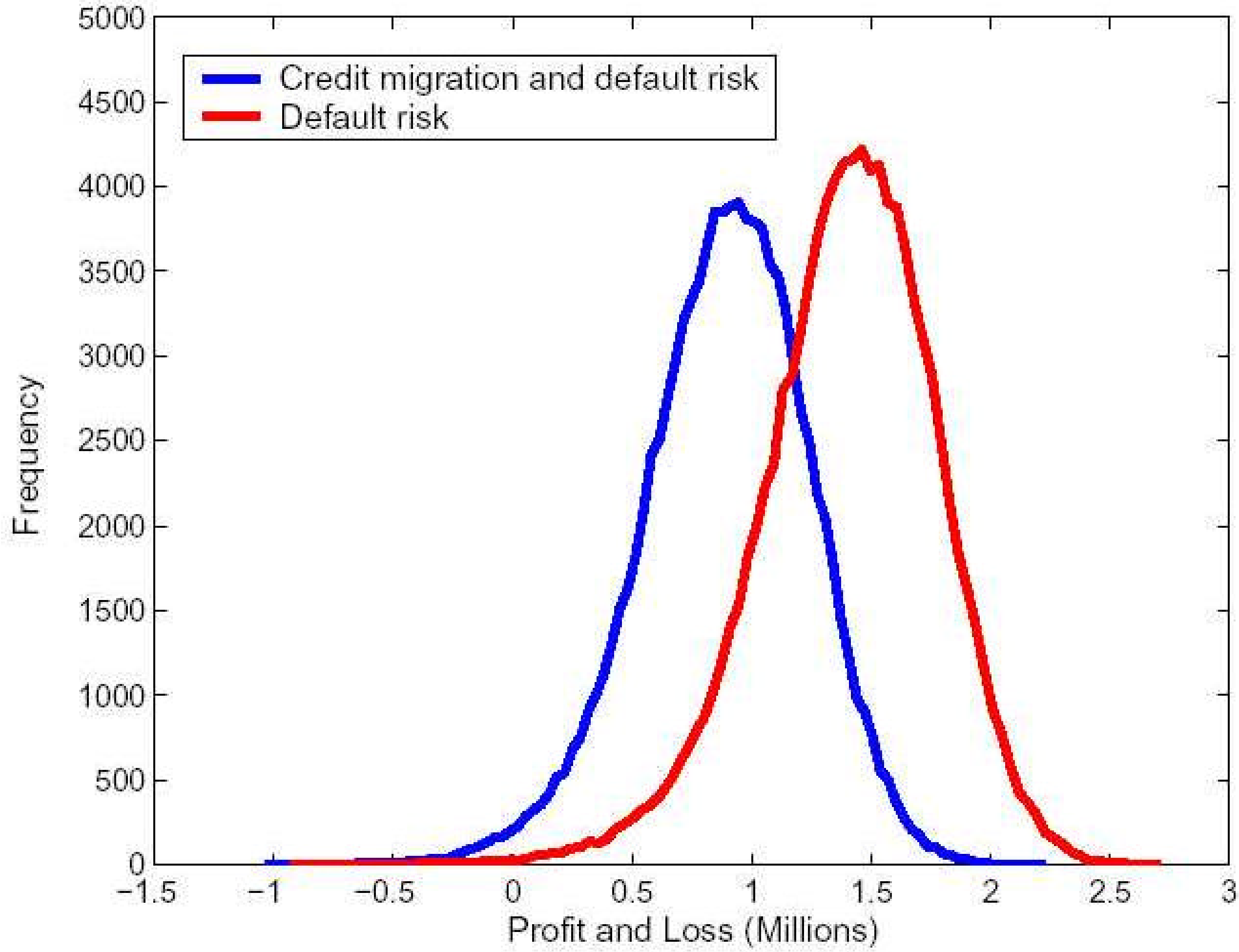
$$\lambda_{\Delta t} = \begin{pmatrix} \lambda_{\Delta t}^{AA,AA} & \lambda_{\Delta t}^{AA,A} & \lambda_{\Delta t}^{AA,BBB} & \lambda_{\Delta t}^{AA,D} \\ \lambda_{\Delta t}^{A,AA} & \lambda_{\Delta t}^{A,A} & \lambda_{\Delta t}^{A,BBB} & \lambda_{\Delta t}^{A,D} \\ \lambda_{\Delta t}^{BBB,AA} & \lambda_{\Delta t}^{BBB,A} & \lambda_{\Delta t}^{BBB,BBB} & \lambda_{\Delta t}^{BBB,D} \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

Portfolio Credit Migration and Default Risk

- **Portfolio**: of 20 AA, 20 A and 20 BBB firms, with recovery rates of 0.25, bonds' principal value is $1 million. **P & L in millions** from 100,000 simulations

	Default Risk	Credit Migration and Default Risk
Mean	1.40	0.89
Median	1.42	0.91
Std. Dev.	0.37	0.35
1% Quantile	0.38	-0.03
0.1% Quantile	-0.16	-0.42
99% VaR	1.02	0.92
99.9% VaR	1.56	1.31

Profit and Loss Distribution



Conclusions and Further Research

- **Incorporating dependency**: is crucial in determining the level of risk at portfolio level
- **Macroeconomic environment**: influences both the level of risk for individual firms and the level of dependence between firms
- **Dynamic correlation**: and stochastic mean reversion allows more flexibility and macroeconomic fluctuations can be incorporated
- **Copula dependence**: lower tail dependent copulas allow for greater extreme co-movement and result in higher levels of dependency
- **Default and credit migration risk**: processes incorporate fluctuations in the macroeconomic climate

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