# Quantum transport in networks of weakly disordered metallic wires

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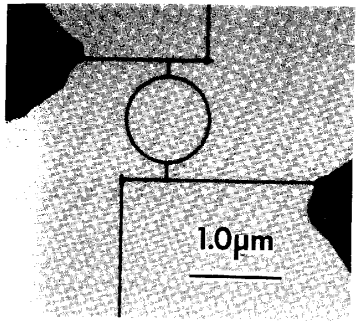
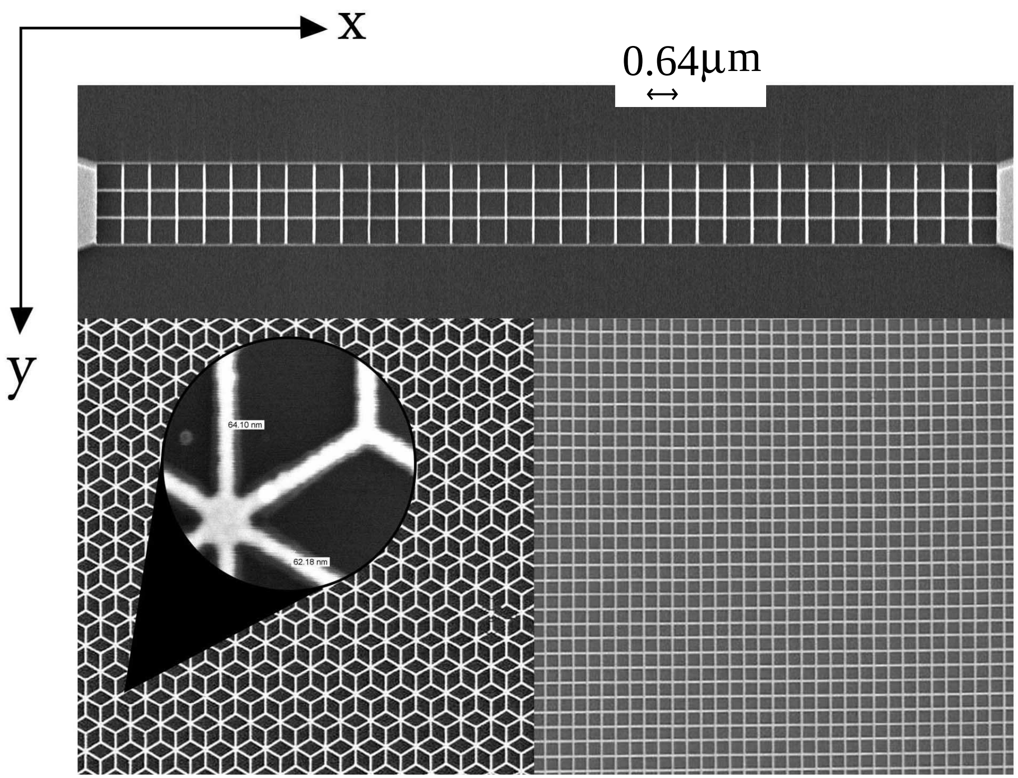
• Washburn & Webb (1985) : gold rings



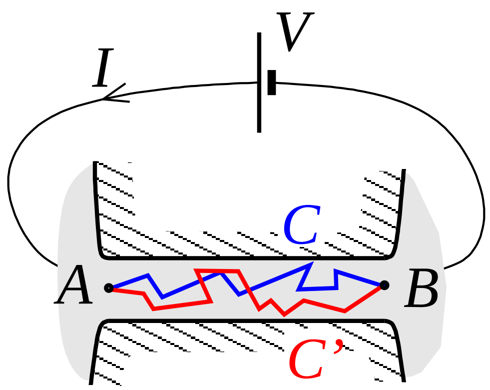

• Bäuerle, Saminadayar, Schopfer *et al* (2005) :

silver networks



# Contents

# Conductance of a weakly disordered metal




$$G = \frac{I}{V} \sim \overbrace{\left|\sum_{\mathcal{C}} \mathcal{A}_{\mathcal{C}}(A \to B)\right|^2}^{\text{Proba}(A\to B)}$$

$$= \sum_{\mathcal{C}} |\mathcal{A}_{\mathcal{C}}|^2 + \sum_{\mathcal{C}\neq\mathcal{C}'} \mathcal{A}_{\mathcal{C}}\, \mathcal{A}^*_{\mathcal{C}'}$$

Weak disorder $\boxed{k_F \ell_e \gg 1}$

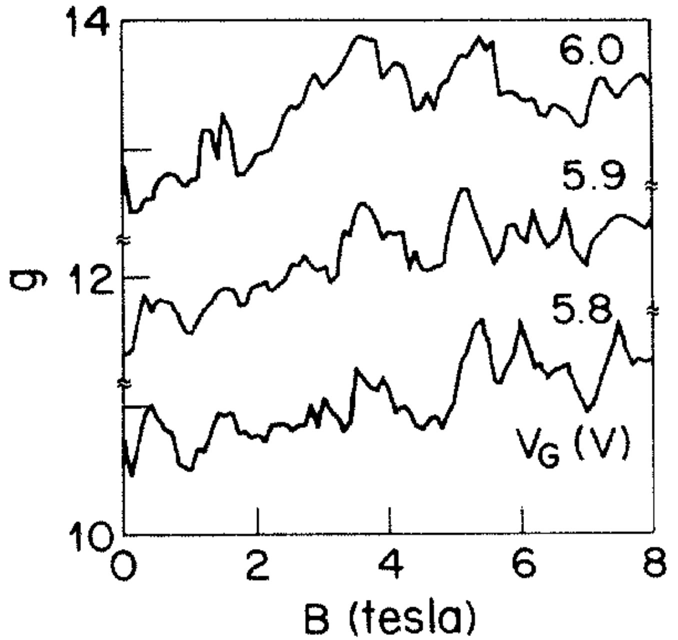
amplitude at Fermi energy $E_F = \frac{k_F^2}{2m_e}$ has a large random phase

$$\mathcal{A}_{\mathcal{C}} \propto \exp\left(\mathrm{i}k_F \ell_{\mathcal{C}} + \mathrm{i}e\int_{\mathcal{C}} \mathrm{d}\vec{r}\cdot\vec{A}\right)$$

# Averaging conductance of a mesoscopic wire

Skocpol *et al* PRL (1986)

Bäuerle *et al* PRL (2005)

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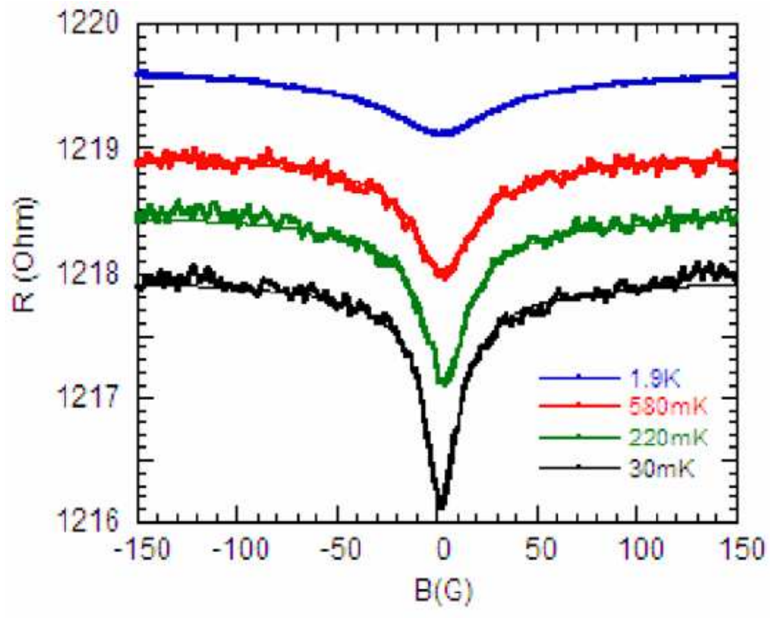


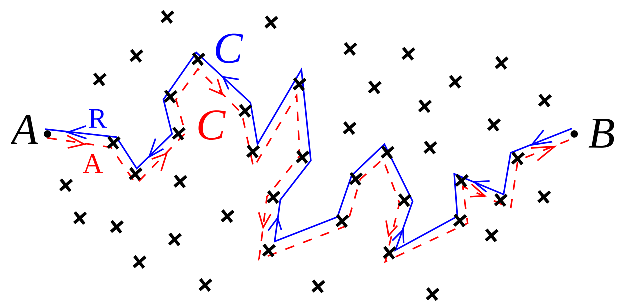

Averaging

**U**niversal **C**onductance **F**luctuations

**W**eak **L**ocalization

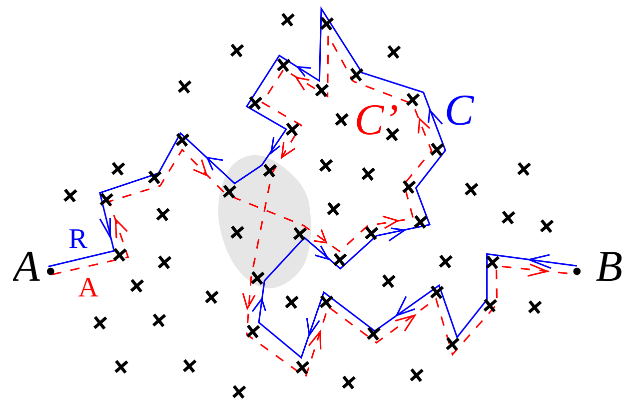
Classical transport : $\boxed{\mathcal{C} = \mathcal{C}'}$

$$G_{\text{Drude}} = \sigma_0 \frac{\text{section}}{\text{length}} \sim \langle \sum_{\mathcal{C}} |\mathcal{A}_{\mathcal{C}}|^2 \rangle$$



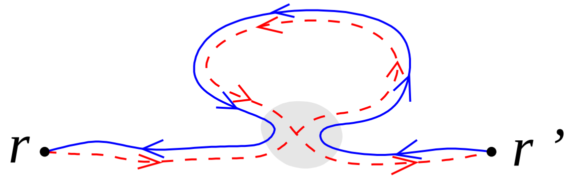

Quantum interferences : $\boxed{\mathcal{C} \neq \mathcal{C}'}$

Weak localization correction $\langle \Delta G \rangle$



## Weak localization correction

- Quantum crossing $\Rightarrow$ a small correction



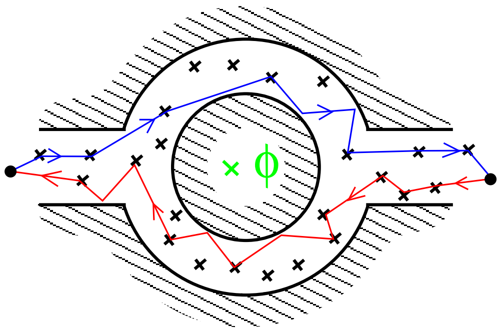

- Increase of backscattering : WL for the wire is $\langle \Delta G \rangle < 0$
- COHERENT contribution : loops smaller than $L_\varphi$ contribute

$$\boxed{\text{Experimental probe of phase coherence}}$$

- Magnetic field sensitivity : $\langle \Delta G(\mathcal{B}) \rangle$

# AB / AAS oscillations

- Aharonov-Bohm (AB) oscillations

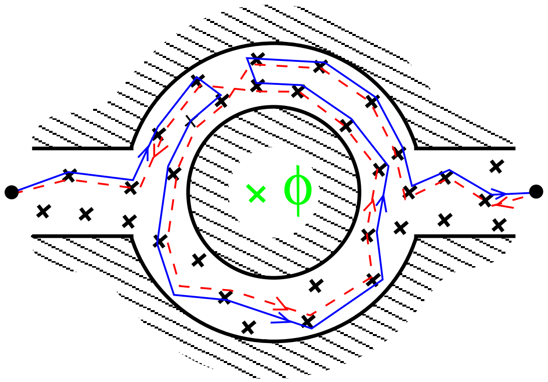



$$\Longrightarrow \mathcal{A}_{\mathcal{C}} \mathcal{A}^*_{\mathcal{C}'} \propto \mathrm{e}^{\mathrm{i}e\phi/\hbar} \times \mathrm{e}^{\mathrm{i}k_F(\ell_{\mathcal{C}} - \ell_{\mathcal{C}'})}$$

$G(\phi)$ : $h/e$ AB oscillations

$\Downarrow$ disorder averaging

- Al'tshuler-Aronov-Spivak (AAS) oscillations



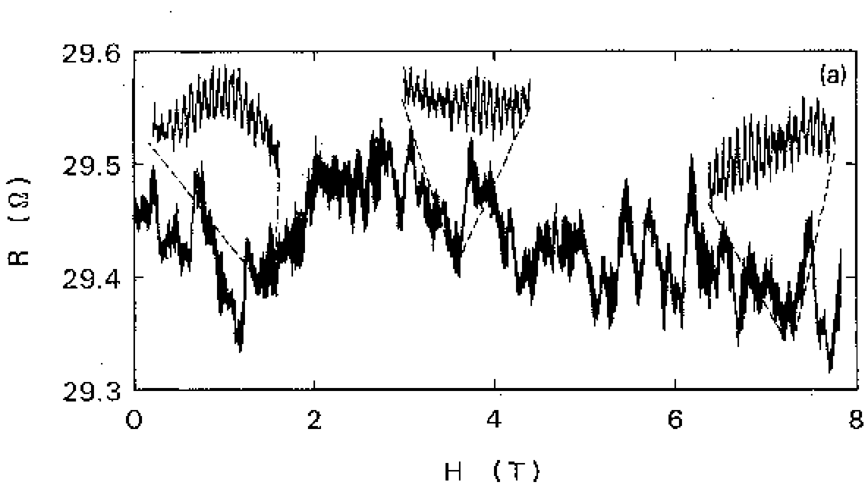

$$\Longrightarrow \mathcal{A}_{\mathcal{C}} \mathcal{A}^*_{\mathcal{C}'} \propto \mathrm{e}^{2\mathrm{i}e\phi/\hbar}$$

$\boxed{\langle \Delta G(\phi) \rangle : h/2e \text{ AAS oscillations}}$

# AB / AAS oscillations (2)

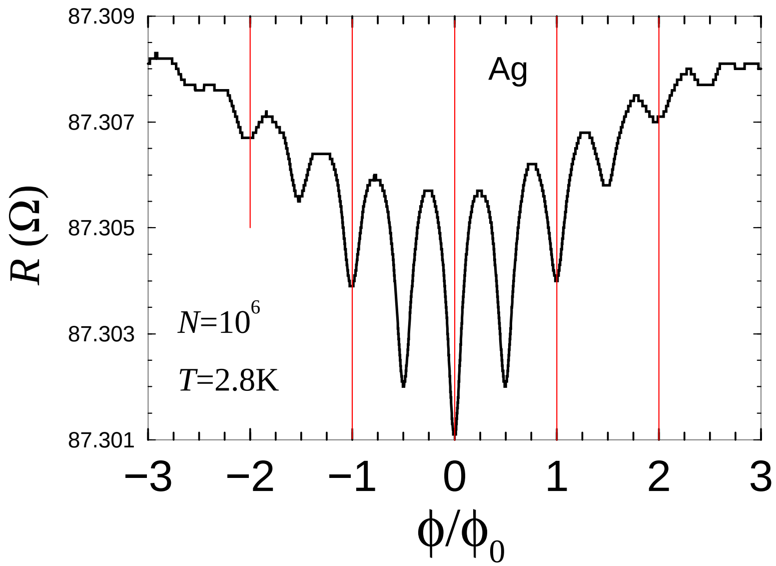
A mesoscopic ring (Au)

(Washburn & Webb, 1986)




A large square network (Ag)

(Schopfer *et al*, 2005)



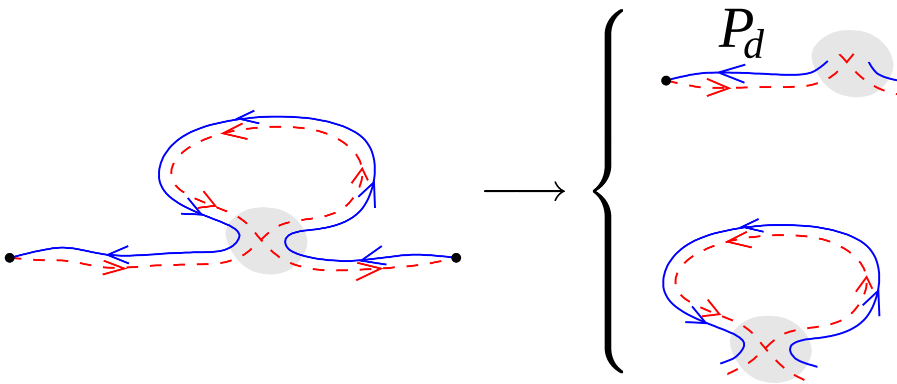

**UCF/AB**

**WL/AAS**

# Diffusion in graphs, spectral determinant

# & nonlocality of quantum transport

# Weak localization : two kinds of nonlocality




Classical

$\sim P_c(x,x)$ Quantum

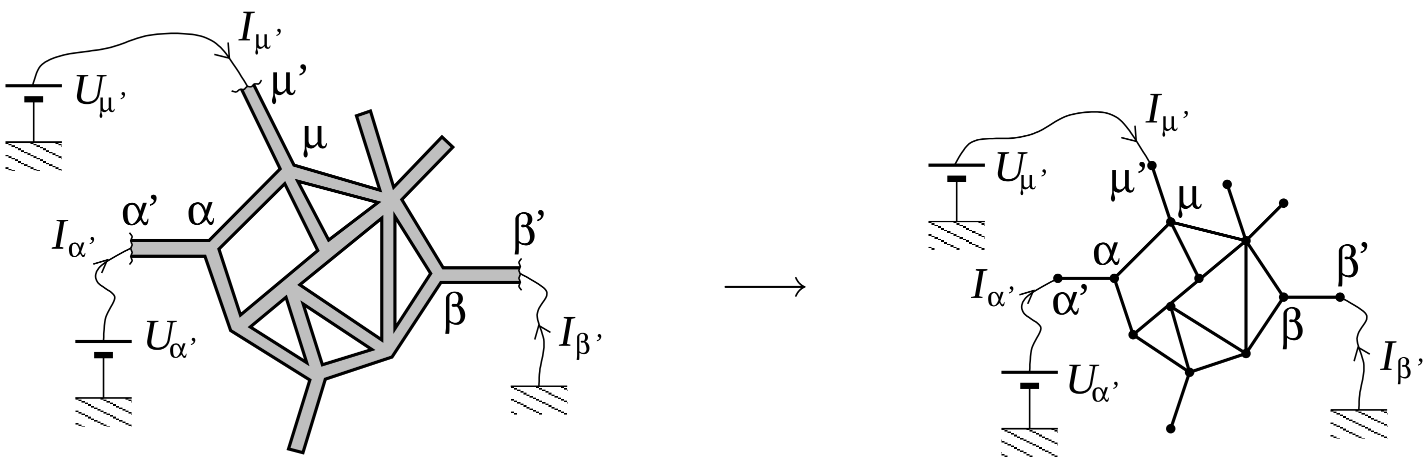
Network :

$$g_{\text{classical}} \propto \frac{1}{\mathcal{L}} \qquad \longrightarrow \qquad \langle \Delta g \rangle = -\frac{2}{\mathcal{L}^2} \sum_{\text{wire } i} \frac{\partial \mathcal{L}}{\partial l_i} \int_{\text{wire } i} \mathrm{d}x \, P_c(x,x)$$

CT & G. Montambaux, PRL**92** (2004).

$$P_c(x,x) = \langle\, x \,|\, \frac{1}{1/L_\varphi^2 - \Delta} \,|\, x \,\rangle$$

Diffusion is effectively 1d along wires



## Diffusion on a graph $(\gamma - \Delta) P_c(x, x') = \delta(x - x')$

Quantum transport $\longrightarrow$ Properties of diffusion

$$\langle \Delta\sigma \rangle = -\frac{e^2}{\pi} \langle\, x \,|\frac{1}{\gamma - (\nabla_x - 2\mathrm{i}eA)^2}|\, x \,\rangle = -\frac{e^2}{\pi} \int_0^\infty \mathrm{d}t\, \mathcal{P}(x, x; t)\, \mathrm{e}^{-\gamma t}$$

with $\gamma = 1/L_\varphi^2$

where

$$\left(\partial_t - [\nabla_x - 2\mathrm{i}eA(x)]^2\right) \mathcal{P}(x, x'; t) = \delta(t)\delta(x - x')$$

# Diffusion & Spectral determinant

$$\int \mathrm{d}x\, P_c(x,x) = \mathrm{Tr}\{\frac{1}{\gamma-\Delta}\} = \frac{\partial}{\partial\gamma}\mathrm{Tr}\{\ln(\gamma-\Delta)\} = \frac{\partial}{\partial\gamma}\ln\det(\gamma-\Delta)$$

Spectral determinant

$$\boxed{S(\gamma) \stackrel{\mathrm{def}}{=} \det(\gamma-\Delta)}$$

For a regular graph :

$$\langle\Delta\sigma\rangle = -\frac{e^2}{\pi}\frac{1}{\mathrm{Vol}}\frac{\partial}{\partial\gamma}\ln S(\gamma)$$

# Spectral determinant of a graph

M. Pascaud & G. Montambaux, PRL **82** (1999)

$$S(\gamma) = \prod_{(\alpha\beta)} \frac{\sinh \sqrt{\gamma} l_{\alpha\beta}}{\sqrt{\gamma}} \det \mathcal{M}$$

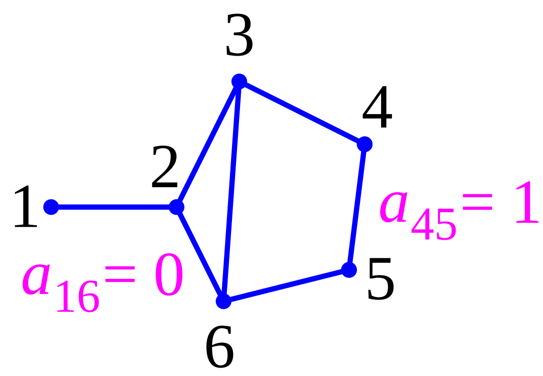
where

$$\mathcal{M}_{\alpha\beta} = \delta_{\alpha\beta} \left( \lambda_\alpha + \sqrt{\gamma} \sum_\mu a_{\alpha\mu} \coth(\sqrt{\gamma} l_{\alpha\mu}) \right) - a_{\alpha\beta} \frac{\sqrt{\gamma}\, \mathrm{e}^{-\mathrm{i}\theta_{\alpha\beta}}}{\sinh(\sqrt{\gamma} l_{\alpha\beta})}$$

$a_{\alpha\beta}$ : connectivity matrix

$a_{\alpha\beta} = 1$ if $(\alpha\beta)$ is a wire

$a_{\alpha\beta} = 0$ otherwise




Connection :

$\lambda_\alpha = 0$ for internal vertex & $\lambda_\alpha = \infty$ absorbing vertex (reservoir)

# Spectral determinant & trace formula

- Trace formula for Laplace operator on a graph : J.-P. Roth (1983)

$$Z(t) = \mathrm{Tr}\left\{ \mathrm{e}^{t\Delta} \right\} = \frac{\mathrm{Vol}}{2\sqrt{\pi t}} + \frac{V-B}{2} + \frac{1}{2\sqrt{\pi t}} \sum_{\mathcal{C}} \alpha(\mathcal{C}) \ell(\tilde{\mathcal{C}}) \mathrm{e}^{-\frac{\ell(\mathcal{C})^2}{4t}}$$

- Relation with spectral determinant : $\frac{\partial}{\partial\gamma} \ln S(\gamma) = \int_0^\infty \mathrm{d}t\, Z(t)\, \mathrm{e}^{-\gamma t}$

$$S(\gamma) = \mathrm{e}^{\sqrt{\gamma}\mathrm{Vol}} \gamma^{\frac{V-B}{2}} \prod_{\tilde{\mathcal{C}}} \left( 1 - \alpha(\tilde{\mathcal{C}}) \mathrm{e}^{-\sqrt{\gamma}\ell(\tilde{\mathcal{C}}) + \mathrm{i}\theta(\tilde{\mathcal{C}})} \right)$$

Akkermans, Comtet, Desbois, Montambaux & CT, Ann.Phys.(2000)

- $L$-functions on graphs : Chekhov, Russ. Math. Surv. (1999)
- Quantum chaos : Kottos & Smilansky PRL (1997), ...

- For Schrödinger operator $S(\gamma) = \det(\gamma - \Delta + V(x))$

  J. Desbois, J.Phys.A (2000)

- Spectral determinant for generalized boundary conditions

  J. Desbois, Eur. J. Phys. B (2001)

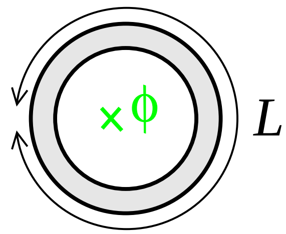
  - Review

    Comtet, Desbois & CT, J.Phys.A**38** (2005)

## AAS oscillations in an isolated ring

Harmonics of magnetoconductance (MC) :

$$\langle \Delta\sigma(\theta) \rangle = \sum_n \langle \Delta\sigma_n \rangle \, \mathrm{e}^{\mathrm{i}n\theta} \quad \text{with } \theta = 4\pi \frac{\phi}{\phi_0}$$



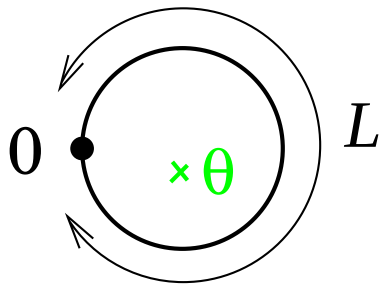

$$\boxed{\langle \Delta\sigma_n \rangle = -\frac{e^2}{\pi} \int_0^\infty \mathrm{d}t \quad \underbrace{\mathcal{P}_n(x,x;t)}_{\text{Proba to wind } n \text{ times}} \quad \mathrm{e}^{-t/\tau_\varphi}}$$

with $\gamma = 1/\tau_\varphi = 1/L_\varphi^2$

$$= -\frac{e^2}{\pi} \int_0^\infty \mathrm{d}t \, \frac{1}{\sqrt{4\pi t}} \, \mathrm{e}^{-\frac{(nL)^2}{4t}} \, \mathrm{e}^{-t/\tau_\varphi} = \boxed{-\frac{e^2}{h} L_\varphi \, \exp -|n|L/L_\varphi}$$

# Isolated ring (2) : spectral determinant

$$\mathcal{M}_{00} = \sqrt{\gamma}\left(2\coth\sqrt{\gamma}L - \frac{2\cos\theta}{\sinh\sqrt{\gamma}L}\right)$$




$$S(\gamma) = 2(\cosh\sqrt{\gamma}L - \cos\theta)$$

$$\langle\Delta\sigma(\theta)\rangle \propto -\frac{\sinh\sqrt{\gamma}L}{\cosh\sqrt{\gamma}L - \cos\theta} \longrightarrow \langle\Delta\sigma_n\rangle \propto -\mathrm{e}^{-|n|\sqrt{\gamma}L}$$

## Spectral determinant as a generating function

- Time $t$ is conjugated to spectral parameter $\gamma$
- Winding number $n$ is conjugated to magnetic flux $\theta$
- Starting point $x_0$ is conjugated to a parameter $\lambda_0$

$\delta$-potential : $$S^{(\lambda_0)}(\gamma) = \det[\gamma - \Delta + \lambda_0\,\delta(x - x_0)]$$

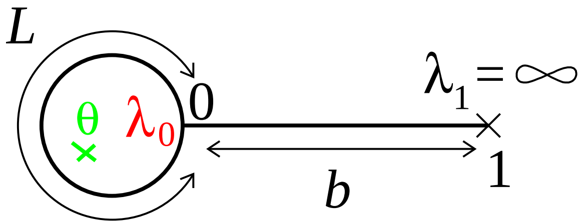
$$P_c(x_0, x_0) = \frac{\partial}{\partial\lambda_0} \ln S^{(\lambda_0)}(\gamma)\Big|_{\lambda_0=0}$$

- Study of occupation time and local time

Desbois, J.Phys.A (2002)

Comtet, Desbois & Majumdar, J.Phys.A (2002)

# Slow winding in a connected ring




$$\boxed{\mathcal{P}_n(0,0;t) = \,?}$$

$$\sqrt{\gamma}\, S^{(\lambda_0)}(\gamma) = \underbrace{\underbrace{\cosh b \sinh L}_{\text{arm}} + \underbrace{2 \sinh b (\cosh L - \cos\theta)}_{\text{ring}}}_{\text{same structure as for isolated ring :}} + \lambda_0 \frac{\sinh b \sinh L}{\sqrt{\gamma}}$$

$$2 \sinh \sqrt{\gamma} b \,\left[\cosh \sqrt{\gamma} L_{\text{eff}}(\gamma) - \cos\theta\right]$$

$$\boxed{\cosh \sqrt{\gamma} L_{\text{eff}}(\gamma) = \cosh \sqrt{\gamma} L + \frac{1}{2} \sinh \sqrt{\gamma} L \, \coth \sqrt{\gamma} b}$$

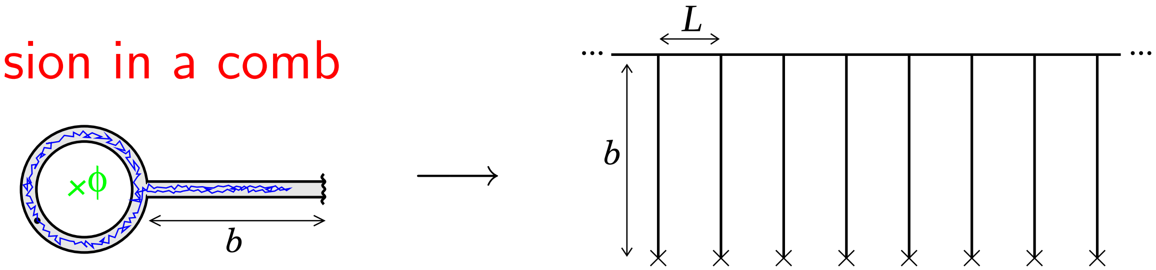
$$\int_0^\infty \mathrm{d}t\, \mathcal{P}_n(0,0;t)\, \mathrm{e}^{-\gamma t} = \frac{1}{2\sqrt{\gamma}} \frac{\sinh\sqrt{\gamma}L}{\sinh\sqrt{\gamma}L_{\mathrm{eff}}(\gamma)}\, \mathrm{e}^{-|n|\sqrt{\gamma}L_{\mathrm{eff}}(\gamma)}$$

- Scaling of winding with time : $\boxed{n_t \sim \sqrt{t}/L_{\mathrm{eff}}(1/t)}$

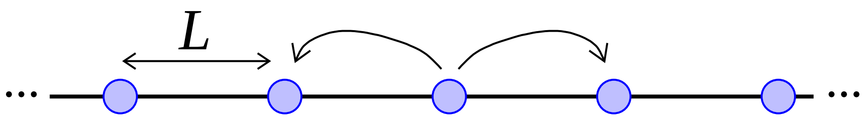
- Effective perimeter $L_{\mathrm{eff}}(\gamma)$ for $b \to \infty$ :

$$L_{\mathrm{eff}} \simeq \begin{cases} L & \text{for } \sqrt{\gamma}L \gg 1 \quad \longrightarrow \quad n_t \sim t^{1/2}/L \\ \sqrt{L}\gamma^{-1/4} & \text{for } \sqrt{\gamma}L \ll 1 \quad \longrightarrow \quad n_t \sim t^{1/4}/\sqrt{L} \end{cases}$$

## Interpretation : diffusion in a comb




trapping by the arms :



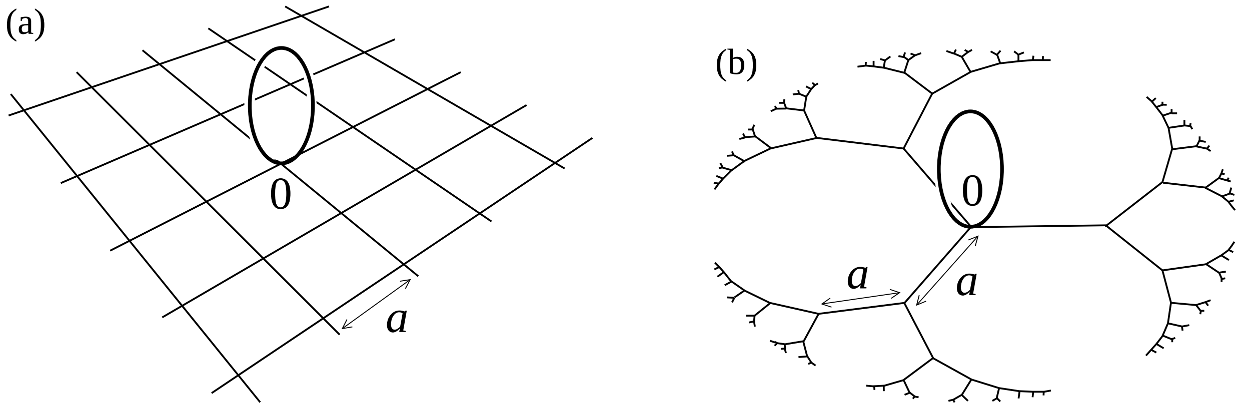

Trapping time distribution :

first return probability in 1d : $Q(t) \sim 1/t^{3/2}$ $\quad \longrightarrow n_t \sim t^{1/4}$

$$\boxed{\mathcal{P}_n(x,x;t) \simeq \frac{\sqrt{L}}{2\,t^{3/4}}\;\psi\!\left(\frac{n\sqrt{L}}{t^{1/4}}\right)}$$

$$\psi(0) = \frac{\Gamma(3/4)}{\pi\sqrt{2}} \quad \text{and} \quad \psi(\xi) \underset{\xi\gg1}{\simeq} \frac{4}{\sqrt{6\pi}}(\xi/4)^{1/3}\mathrm{e}^{-3(\xi/4)^{4/3}}$$

## Generalization : winding in a ring connected to a network

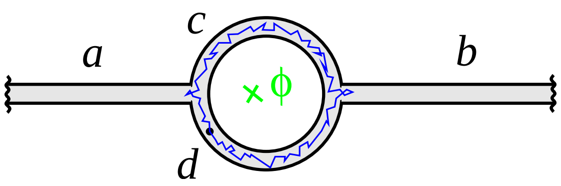


# Nonlocality of quantum transport – connected ring

Consequence for quantum transport :

- For $L_\varphi \ll L$ :

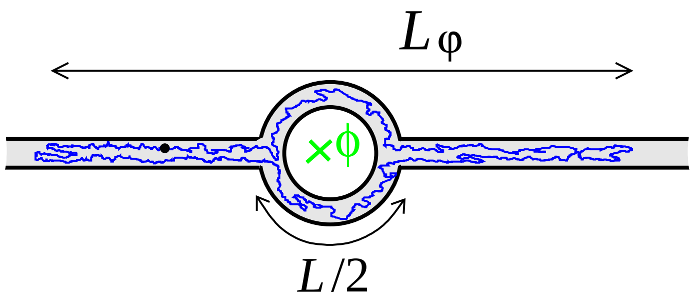
$$\langle \Delta g_n \rangle \sim -L_\varphi \left(\frac{2}{3}\right)^{2|n|} \mathrm{e}^{-|n|L/L_\varphi}$$




- The arms strongly manifest for $\boxed{L_\varphi \gtrsim L}$

$$\boxed{\langle \Delta g_n \rangle \sim -L_\varphi^{3/2}\, \mathrm{e}^{-|n|\sqrt{2L/L_\varphi}}}$$

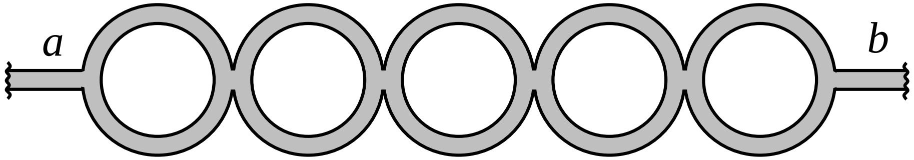
CT & G. Montambaux, J.Phys.A**38** (2005)

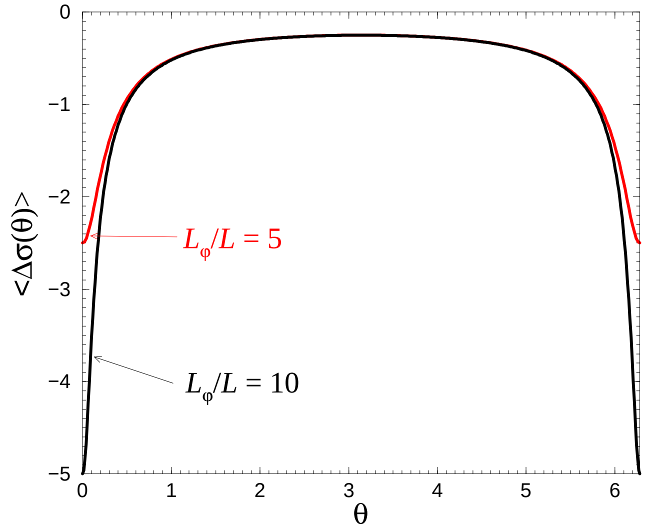



Winding is slow : $n_t \propto t^{1/4}$

(isolated ring : $n_t \propto t^{1/2}$)

# Nonlocality (2) : Chain of symmetric rings




$$\langle \Delta\tilde{\sigma}(\theta)\rangle_{\mathrm{osc}} = -\frac{L_\varphi}{2}\,\frac{\sinh(L/2L_\varphi)}{\sqrt{\cosh^2(L/2L_\varphi)-\cos^2(\theta/2)}}$$

$$\langle \Delta\tilde{\sigma}_n\rangle \simeq -L\int_0 \frac{\mathrm{d}\theta}{2\pi}\,\frac{\mathrm{e}^{-\mathrm{i}n\theta}}{\sqrt{(\frac{L}{L_\varphi})^2+\theta^2}} \simeq -\frac{L}{2\pi}\int_{L/L_\varphi}^{1/n}\frac{\mathrm{d}\theta}{\theta}$$

$$\langle \Delta\tilde{\sigma}_n\rangle \simeq -\frac{L}{2\pi}\ln(L_\varphi/nL) + \mathrm{cste}$$

# Chain of rings (2)

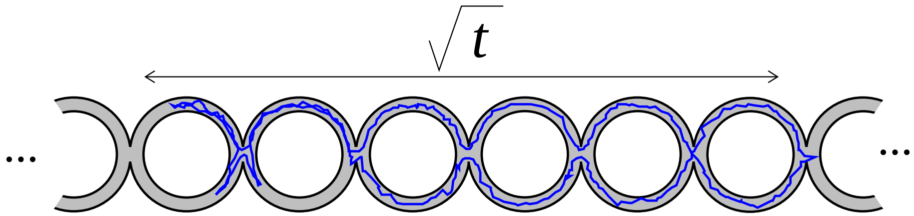
$$\begin{aligned} \langle \Delta\tilde{\sigma}_n \rangle &\simeq -\frac{L}{2\pi}\left[\ln(2L_\varphi/|n|L) + b_{|n|}\right] && \text{for } |n| \ll L_\varphi/L \\ &\simeq -\frac{L}{4}\,\frac{\mathrm{e}^{-|n|\,L/L_\varphi}}{\sqrt{\pi|n|\,L/2L_\varphi}} && \text{for } |n| \gg L_\varphi/L \end{aligned}$$

$$\Big\downarrow \ \text{Inverse Laplace}$$

$$\boxed{\mathcal{P}_n(x,x;t) \simeq \frac{L}{8\pi t}\mathrm{e}^{-(nL)^2/4t}} \ \text{for } t \gg L^2$$

$$\left(\mathcal{P}_n(x,x;t) \sim \tfrac{1}{\sqrt{4\pi t}}\mathrm{e}^{-(nL)^2/4t} \ \text{for } t \ll L^2\right)$$

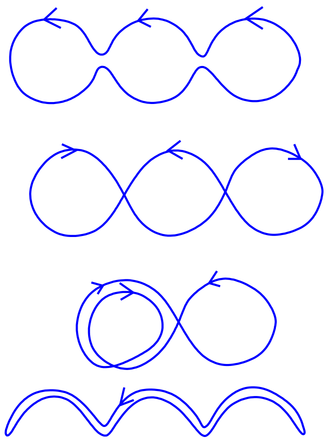
## Interpretation for $\mathcal{P}_n(x,x;t) \simeq \frac{L}{8\pi t}\mathrm{e}^{-(nL)^2/4t}$




$n = 3$

$n = -1$

$n = 0$



equiprobable

$\downarrow$

neglect counting

$\Downarrow$

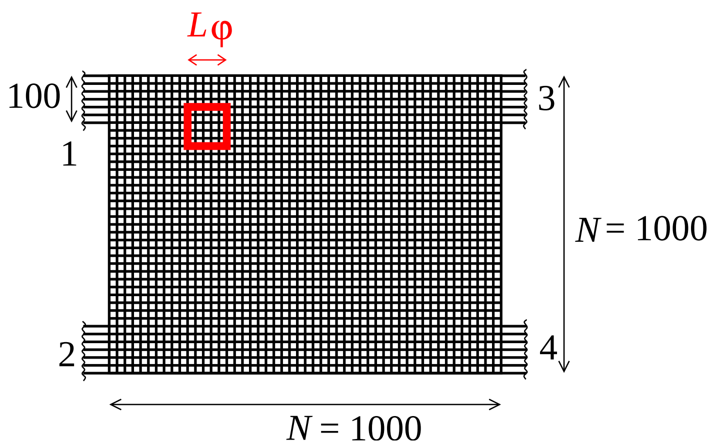
$$\sum_n \mathcal{P}_n(x,x;t) = \frac{1}{2\sqrt{4\pi t}} \quad \Rightarrow \quad \mathcal{P}_n(x,x;t) \sim \frac{L}{t} \text{ for } |n| \lesssim \frac{\sqrt{t}}{L}$$

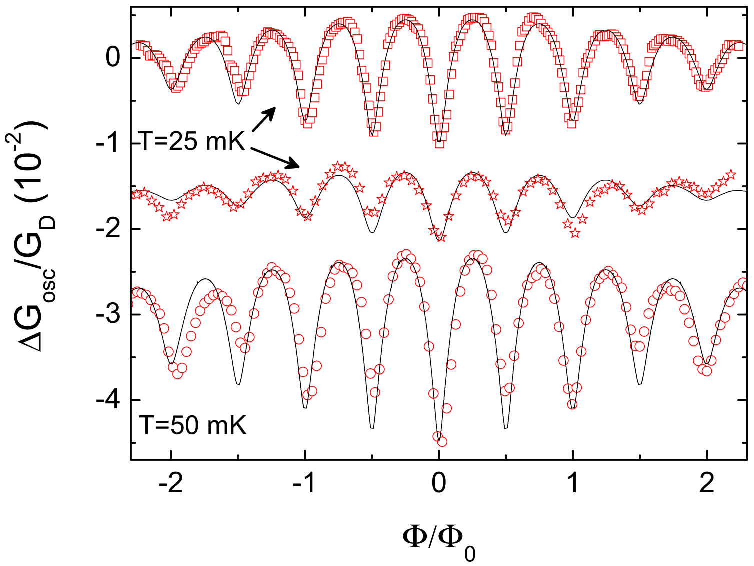
# Nonlocality in square network

→ Analysis with spectral determinant

→ Experiment : M. Ferrier *et al* PRL (2004).

$10^6$ cells etched in 2DEG
with $a = 1\ \mu$m



# Effects of electron-electron interaction

# in networks : decoherence & AA correction

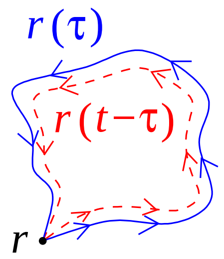
## Exponential relaxation of phase coherence

$$\langle \Delta\sigma \rangle \sim - \sum_{\text{loops } \mathcal{C}_t} \mathrm{e}^{-t/\tau_\varphi}$$

## Decoherence due to e-e interaction : AAK's model

Al'tshuler, Aronov & Khmel'nitzkiĭ, J.Phys.C**15** (1982).

$$\langle \Delta\sigma \rangle \sim - \sum_{\text{loops } \mathcal{C}_t} \langle \mathrm{e}^{\mathrm{i}\Phi[\mathcal{C}_t]} \rangle_V$$




$$\Phi[r(\tau)] = \int_0^t \mathrm{d}\tau \, [V(r(\tau), \tau) - V(r(\tau), t - \tau)]$$

$V(r, t)$ is the fluctuating electrostatic potential

## Decoherence due to electron-electron interaction (2)

$$\langle \Delta\sigma_n \rangle = -\frac{e^2}{\pi}\int_0^\infty \mathrm{d}t\, \mathcal{P}_n(x,x;t)\ \underbrace{\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}_n}}_{\text{replaces } \mathrm{e}^{-t/\tau_\varphi}}$$

Relaxation of phase coherence :

$$\boxed{\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}_n} = \left\langle \mathrm{e}^{-\frac{2}{L_N^3}\int_0^t \mathrm{d}\tau\, W(x(\tau),x(t-\tau))} \right\rangle_{\mathcal{C}_n}}$$

$$W(x,x') = \tfrac{1}{2}[P_d(x,x) + P_d(x',x')] - P_d(x,x') \text{ with } -\Delta P_d = \delta$$

Nyquist length : $L_N = \left(\frac{\nu_0 S D^2}{T}\right)^{1/3} = \left(\frac{\alpha_d}{\pi} N_c \ell_e L_T^2\right)^{1/3}$ with $L_T = \sqrt{D/T}$

$\Rightarrow$ decoherence depends on • network $(P_d)$

• trajectories $(\mathcal{C}_n)$

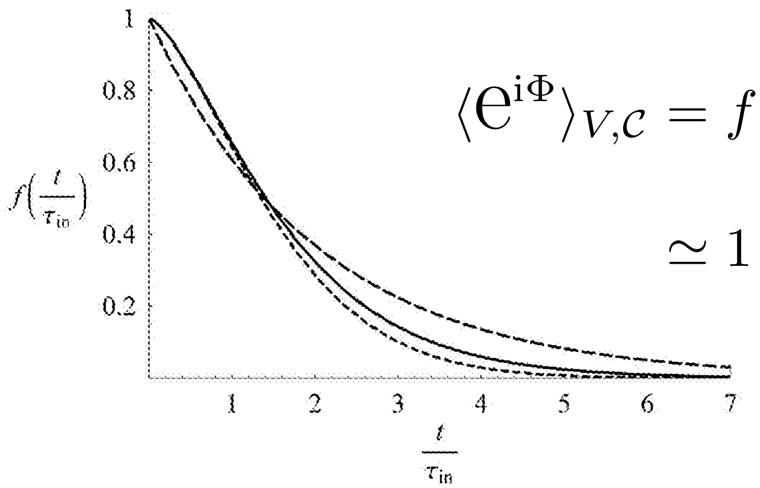
# Decoherence in the infinite wire (AAK, 1982)

$$\boxed{W_{\rm wire}(x,x') = \frac{1}{2}|x-x'|} \longrightarrow \langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}} = \left\langle \mathrm{e}^{-\frac{1}{L_N^3}\int_0^t \mathrm{d}\tau\,|x(\tau)-x(t-\tau)|}\right\rangle_{\mathcal{C}}$$

$$\langle\Delta\sigma\rangle = -\frac{e^2}{\pi}\int_0^\infty \frac{\mathrm{d}t}{\sqrt{4\pi t}}\,\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}} \times \mathrm{e}^{-t/\tau_\varphi} = \frac{e^2}{h}\,L_N\,\frac{\mathrm{Ai}(L_N^2/L_\varphi^2)}{\mathrm{Ai}'(L_N^2/L_\varphi^2)}$$

Relaxation of phase coherence : time scale $\tau_N = L_N^2$

Montambaux & Akkermans, PRL**95** (2005).




$$\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}} = f(t/\tau_N) \quad \text{with} \quad f(x) = \sqrt{\pi x}\sum_{m=1}^{\infty}\frac{1}{|u_m|}\mathrm{e}^{-|u_m|x}$$

$$\simeq 1 - \frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_N}\right)^{3/2} \qquad \text{for } t \ll \tau_N$$

$$\mathrm{Ai}'(u_m) = 0$$

# Decoherence in the isolated ring

$$\boxed{W_{\mathrm{ring}}(x,x') = \frac{1}{2}|x-x'|\left(1-\frac{|x-x'|}{L}\right)}$$

$$\langle\Delta\sigma_n(L_\varphi, L_N)\rangle = \overbrace{\frac{e^2}{h}\,L_N\,\frac{\mathrm{Ai}(L_N^2/L_\varphi^2)}{\mathrm{Ai}'(L_N^2/L_\varphi^2)}}^{\mathrm{AAK}}\,\mathrm{e}^{-|n|\ell_{\mathrm{eff}}} \text{ for } L \gg L_\varphi,\ L_N$$

CT & G. Montambaux, PRB**72** (2005).

- Combination of $L_\varphi$ & $L_N$ : $\ell_{\mathrm{eff}} = \left(\frac{L}{L_N}\right)^{3/2} \times \eta\left(\frac{L_c^2}{L_\varphi^2}\right)$ with $L_c = \frac{L_N^{3/2}}{L^{1/2}}$

$$\left(\ \frac{1}{\tau_\varphi} \longrightarrow \cancel{\frac{1}{\tau_\varphi} + \frac{1}{\tau_N}}\ \right)$$

- $L_\varphi = \infty$ : $\langle\Delta\sigma_n\rangle \propto \mathrm{e}^{-\frac{\pi}{8}|n|\left(\frac{L}{L_N}\right)^{3/2}} \sim \boxed{\mathrm{e}^{-nL^{3/2}T^{1/2}}}$ $\quad(L \gg L_N)$

Ludwig & Mirlin, PRB**69** (2004)

## Relaxation of phase coherence in the ring (1)

- Diffusion of the phase : Johnson-Nyquist

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\Phi = V$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\Phi^2\rangle_V = \int \mathrm{d}t\langle V(t)V(0)\rangle_V = 2e^2 T R_t \sim e^2 T \frac{r(t)}{\sigma_0 S} = \frac{r(t)}{\sqrt{D}\,\tau_N^{3/2}}$$

<u>Small time $t \ll \tau_D$</u> $\Rightarrow r(t) \sim \sqrt{Dt}$ $\langle\Phi^2\rangle_V \sim (\tfrac{t}{\tau_N})^{3/2}$

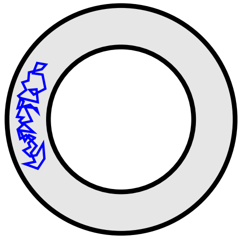
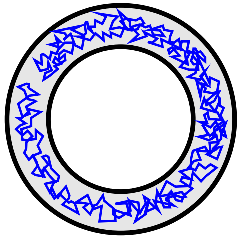
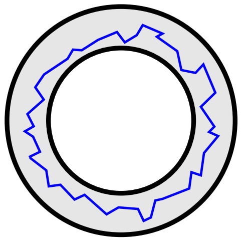
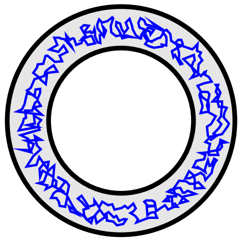
<u>Long time $t \gg \tau_D$</u> $\Rightarrow r(t) \sim L$ $\langle\Phi^2\rangle_V \sim \frac{\sqrt{\tau_D}}{\tau_N^{3/2}}\,t = \frac{t}{\tau_c}$

- For $\boxed{\tau_N \gg \tau_D}$ : $\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}_n} = \langle \mathrm{e}^{-\frac{1}{2}\langle\Phi^2\rangle_V}\rangle_{\mathcal{C}_n} \simeq \mathrm{e}^{-\frac{1}{2}\langle\Phi^2\rangle_{V,\mathcal{C}_n}}$

## Relaxation of phase coherence in the ring (2)

new time scale $\tau_c = \tau_N^{3/2}/\tau_D^{1/2} \propto T^{-1}$

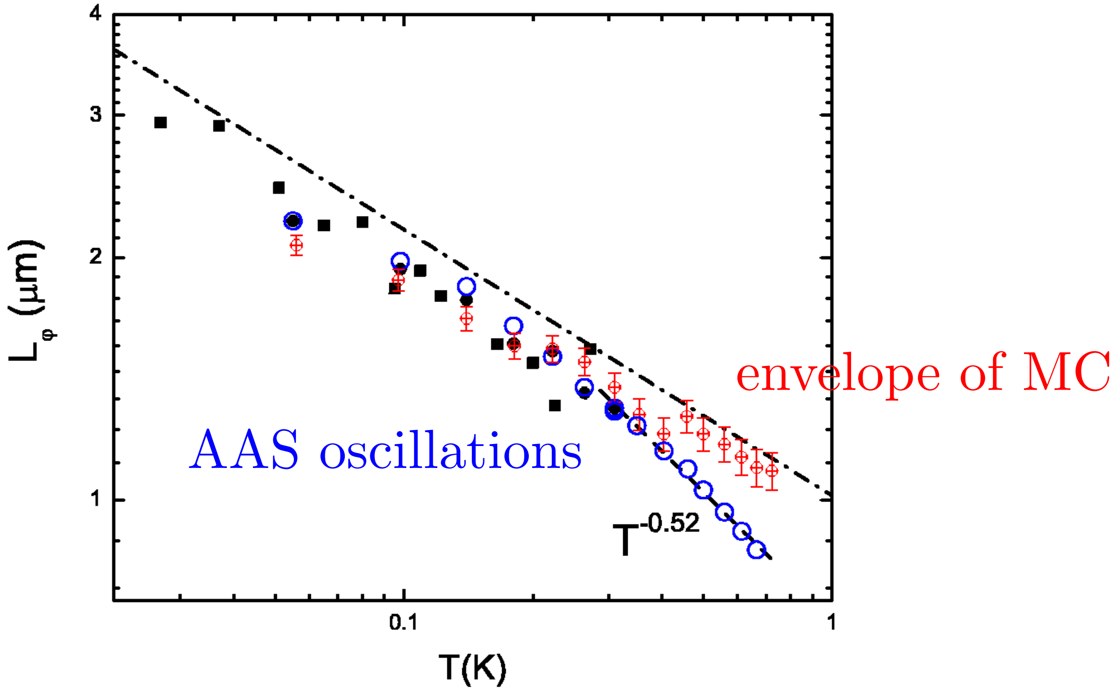
$$\boxed{\langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}_n} = \langle \mathrm{e}^{-\frac{1}{2}\langle \Phi^2 \rangle_V} \rangle_{\mathcal{C}_n} \simeq \mathrm{e}^{-\frac{1}{2}\langle \Phi^2 \rangle_{V,\mathcal{C}_n}} \ :}$$

| Harmonic | $t \ll \tau_D$ | | $t \gg \tau_D$ | |
|---|---|---|---|---|
| $n=0$ |  | $\mathrm{e}^{-\frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_N}\right)^{3/2}}$ (AAK) |  | $\mathrm{e}^{-\frac{1}{6}\frac{t}{\tau_c}}$ |
| $n \neq 0$ |  | $\mathrm{e}^{-\frac{1}{6}\frac{t}{\tau_c}}$ |  | $\mathrm{e}^{-\frac{1}{6}\frac{t}{\tau_c}}$ |

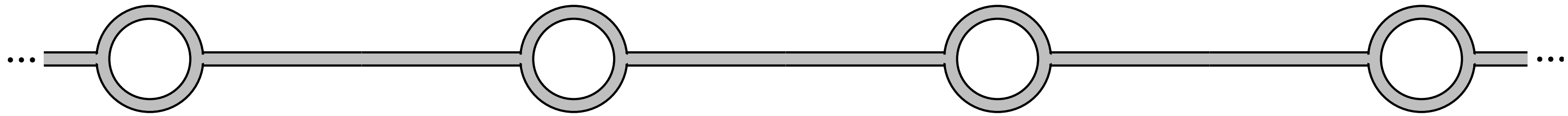
## Decoherence due to e-e interaction in ring : experiment

M. Ferrier, H. Bouchiat *et al*, to be published (2007).

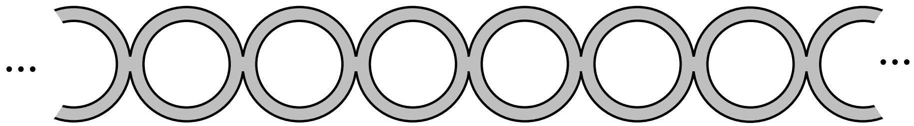
square network etched in 2DEG at high temperature



## Chains of rings



and



- $L_N \ll L$ : decoherence involves time scale $\tau_c = \tau_N^{3/2}/\tau_D^{1/2} \propto T^{-1}$

$$\langle \Delta g_n \rangle \sim \mathrm{e}^{-|n|L/L_c} = \mathrm{e}^{-|n|(L/L_N)^{3/2}}$$

- $L_N \gg L$ : $W(x,x') \sim W_{\mathrm{wire}}(x,x')$

decoherence involves time scale $\tau_N \propto T^{-2/3}$

$$\langle \Delta g_n \rangle \sim \mathrm{e}^{-|n|\sqrt{L/L_N}} \text{ and } \mathrm{e}^{-|n|L/L_N}$$

## Another interaction effect : AA correction

an electron is scattered by
- disordered potential
- electrostatic potential

Al'tshuler-Aronov correction to conductivity :

$$\langle \Delta\sigma \rangle_{\mathrm{ee}} = -\lambda_\sigma \frac{e^2}{\pi} \int_0^\infty \mathrm{d}t \left( \frac{\pi T t}{\sinh \pi T t} \right)^2 \mathcal{P}_d(t) \quad \text{with} \quad \mathcal{P}_d(t) = \frac{\mathrm{Tr}\left\{ \mathrm{e}^{t\Delta} \right\}}{\mathrm{Vol}}$$

$$\langle \Delta\sigma \rangle_{\mathrm{ee}} = -\lambda_\sigma \frac{e^2}{\pi \mathrm{Vol}} \sum_{n=1}^{\infty} \frac{1}{n} \left[ \gamma^2 \frac{\partial^3}{\partial \gamma^3} \ln S(\gamma) \right]_{\gamma = \frac{2n\pi}{L_T^2}}$$

CT & G. Montambaux, (2007).

→ Application to a large square network

# Conclusion

weakly disordered metal $\longrightarrow$ diffusive regime

quantum transport is also related to properties of diffusion

## Diffusion in a graph

- central object : $S(\gamma)$ (Pascaud & Montambaux, 1999)

  Akkermans, Comtet, Desbois, Montambaux & CT, Ann.Phys.**284** (2000)

  Desbois, J.Phys.A**33** (2000) ; Eur.J.Phys.B**24** (2001)

  Comtet, Desbois & CT, J.Phys.A**38** (2005)

- WL/AAS oscillations, nonlocality of quantum transport

  CT & Montambaux, Phys.Rev.Lett.**92** (2004)

  Ferrier *et al*, Phys.Rev.Lett.**93** (2004)

  CT & Montambaux, J.Phys.A**38** (2005)

- AB and AAS in large square networks

  Schopfer *et al*, Phys.Rev.Lett.**98** (2006)

## Electron-electron interaction in networks

- Fluctuations of $V$ & decoherence depend on geometry

  Ludwig & Mirlin, Phys.Rev.B**69** (2004)

  CT & Montambaux, Phys.Rev.B**72** (2005)

- Al'tshuler-Aronov correction

  CT & Montambaux, (2007)