Tree shapes

- induced by neutral models for speciation -

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The constant rate birth- and death processes

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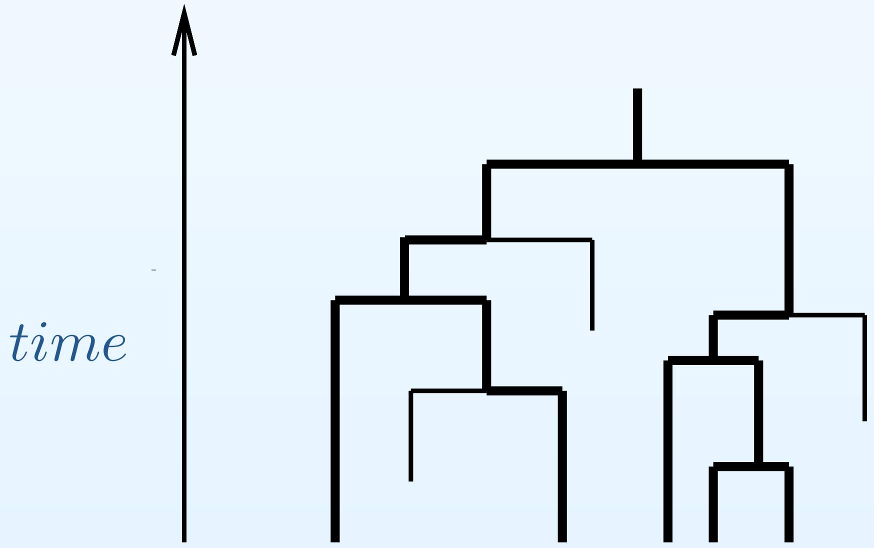

birth-death process

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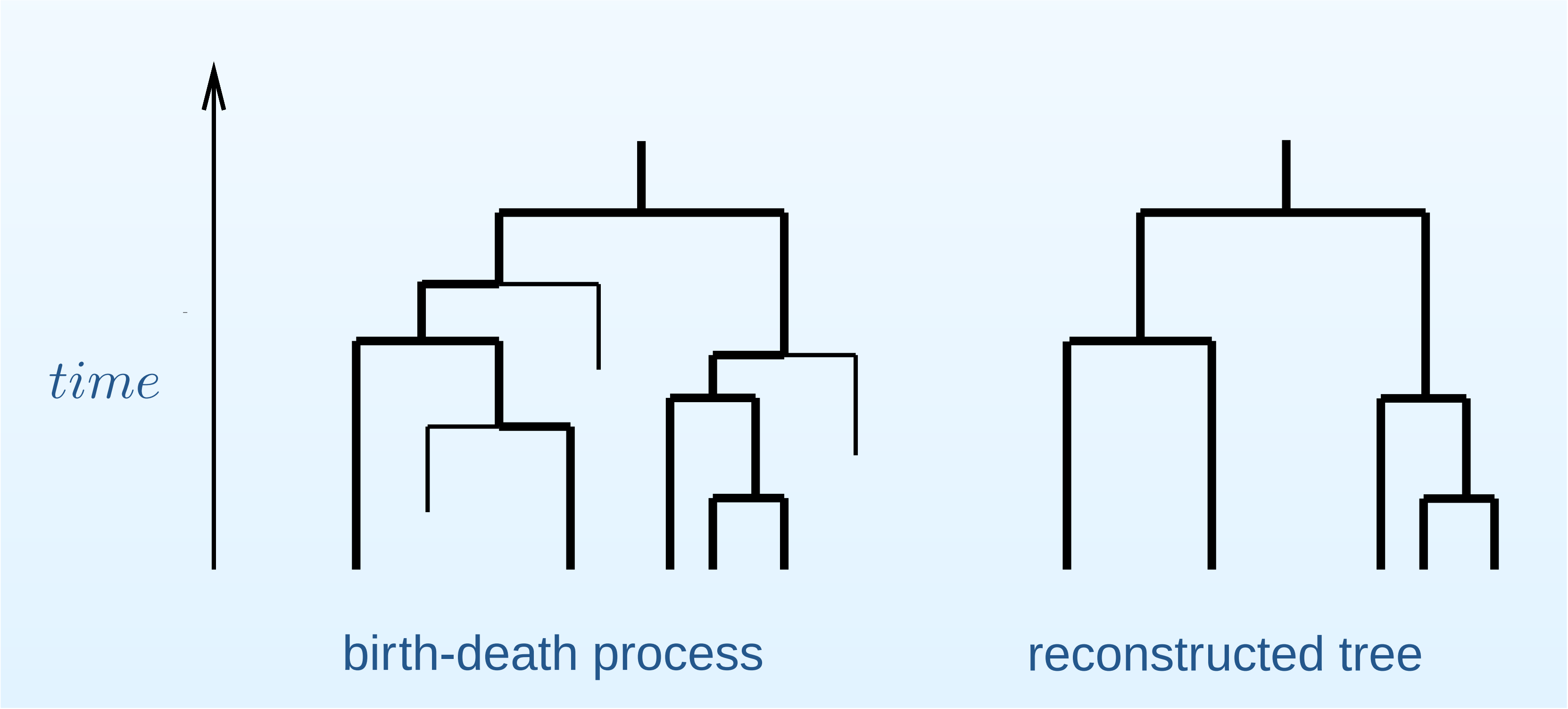
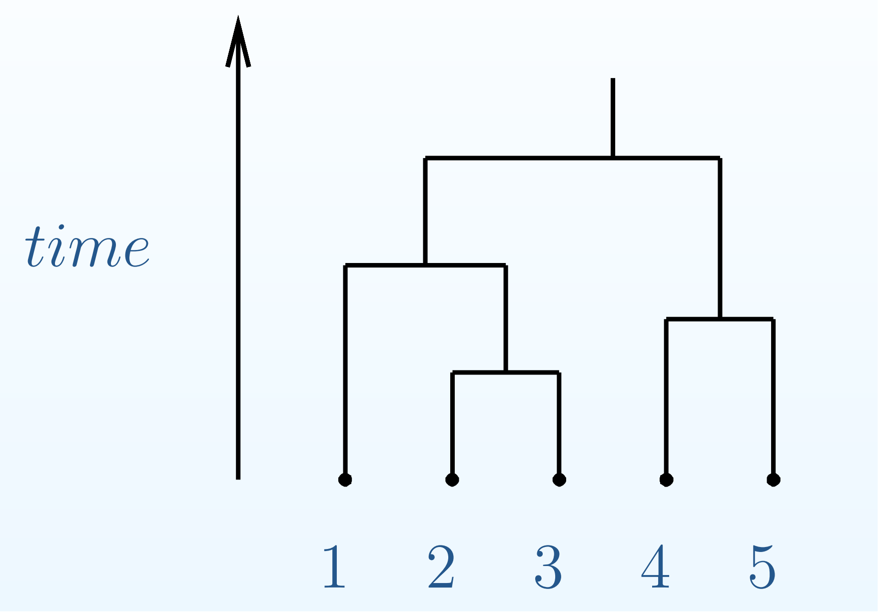


Reconstructed trees



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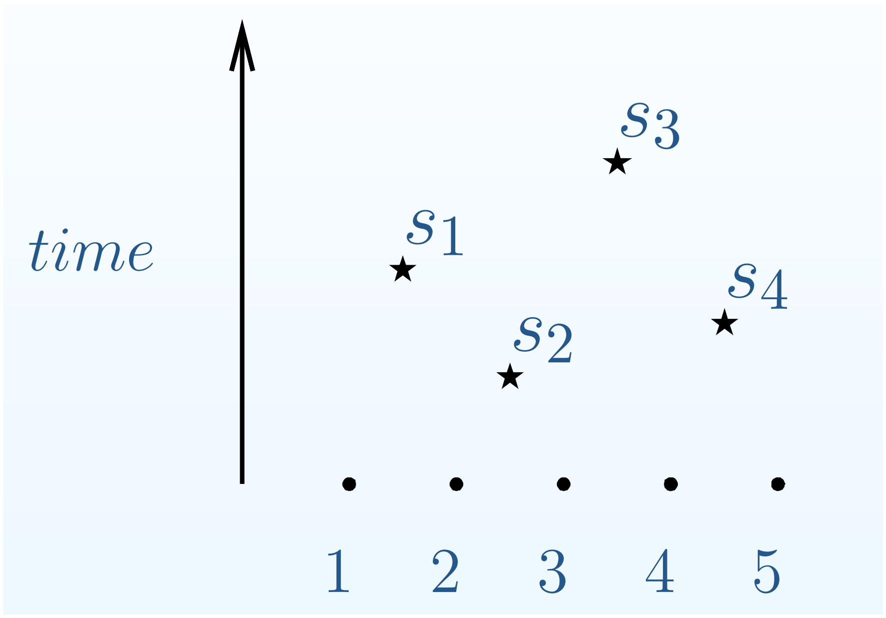
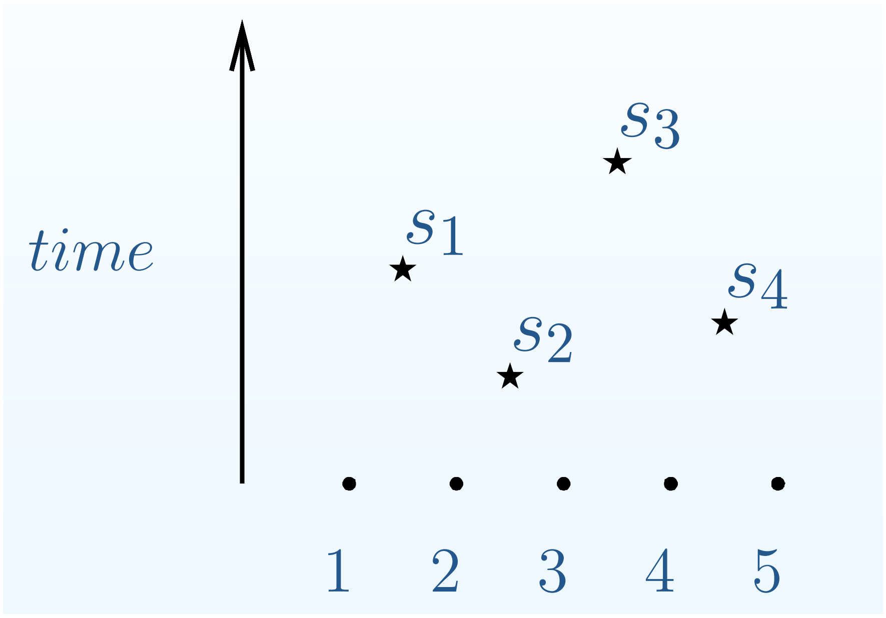


- Point Process representation.

Reconstructed trees



- Point Process representation.
- Speciation times s_i i.i.d.

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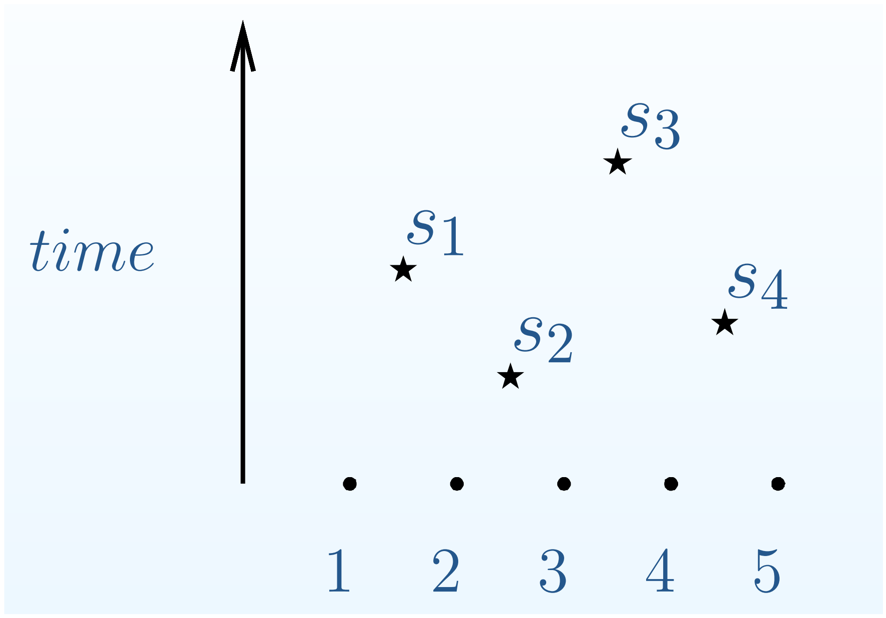

- Point Process representation.
- Speciation times s_i i.i.d.

$$f(s_i|\lambda, \mu, t_{or} = t, n) = (\lambda - \mu)^2 \frac{e^{-(\lambda-\mu)s_i}}{(\lambda - \mu e^{-(\lambda-\mu)s_i})^2} \frac{\lambda - \mu e^{-(\lambda-\mu)t}}{1 - e^{-(\lambda-\mu)t}}$$

Successive speciation events

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$$
\begin{aligned}
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&\quad e^{-(\lambda-\mu)(k+1)s}\frac{(1-e^{-(\lambda-\mu)s})^{n-k-1}}{(\lambda-\mu e^{-(\lambda-\mu)s})^{n+1}}
\end{aligned}
$$

in particular :

$$
\begin{aligned}
f^{Yule}_{\mathcal{A}_n^k}(s) &= (k+1)\binom{n}{k+1}\lambda\frac{(e^{\lambda s}-1)^{n-k-1}}{e^{\lambda s n}} \\
f^{CBP}_{\mathcal{A}_n^k}(s) &= (k+1)\binom{n}{k+1}\lambda^{n-k}\frac{s^{n-k-1}}{(1+\lambda s)^{n+1}}
\end{aligned}
$$

Expectation of successive speciation events

$$\mathbb{E}[\mathcal{A}_n^k] = (k+1)\binom{n}{k+1}(-1)^k \sum_{i=0}^{n-k-1} \binom{n-k-1}{i} \frac{(\lambda-\mu)^{k+i}}{k+i+1} \frac{1}{\mu^{k+i+1}} \cdots$$

$$\left[\log\left(\frac{\lambda}{\lambda-\mu}\right) - \sum_{j=1}^{k+i} \binom{k+i}{j} \frac{(-1)^j}{j} \left(1 - \left(\frac{\lambda}{\lambda-\mu}\right)^j\right)\right]$$

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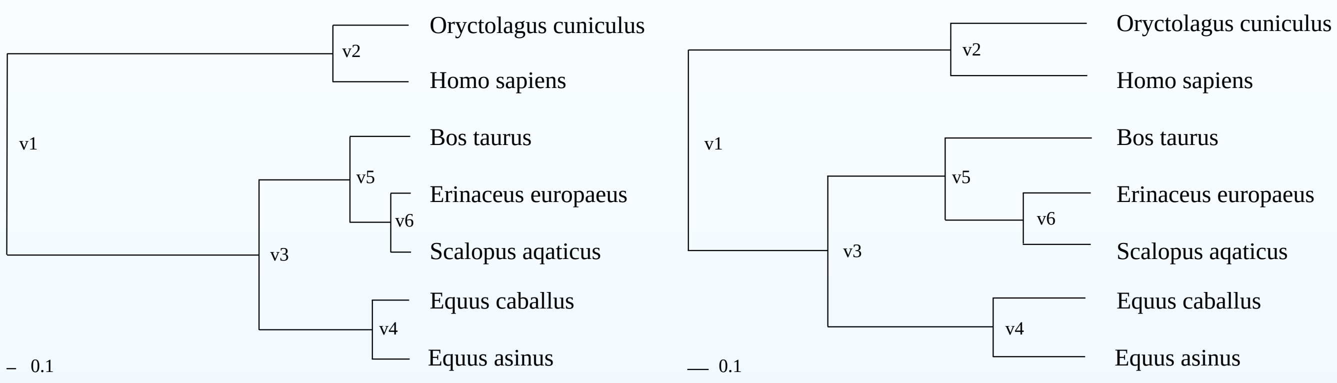
in particular:

$$\mathbb{E}_{Yule}[\mathcal{A}_n^k] = \sum_{i=k+1}^{n} \frac{1}{\lambda i}$$

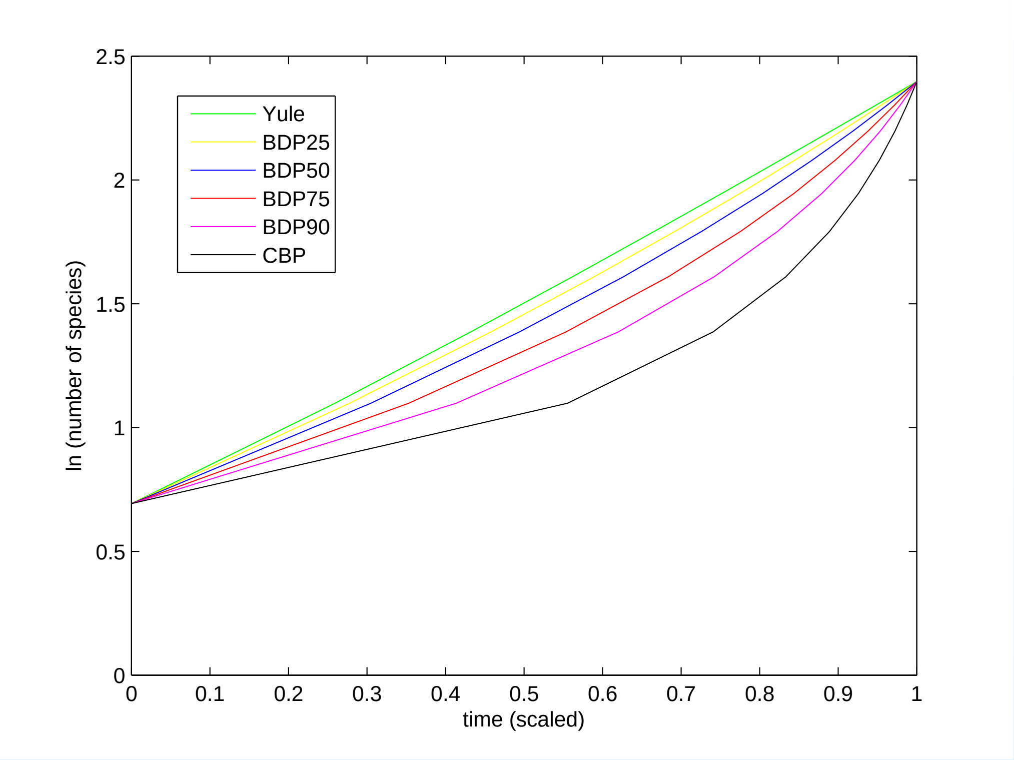
$$\mathbb{E}_{CBP}[\mathcal{A}_n^k] = \frac{n-k}{\lambda k}$$

Application - Dating trees



Given the tree shape, we obtained the displayed speciation times under the CBP (left) and Yule model (right).

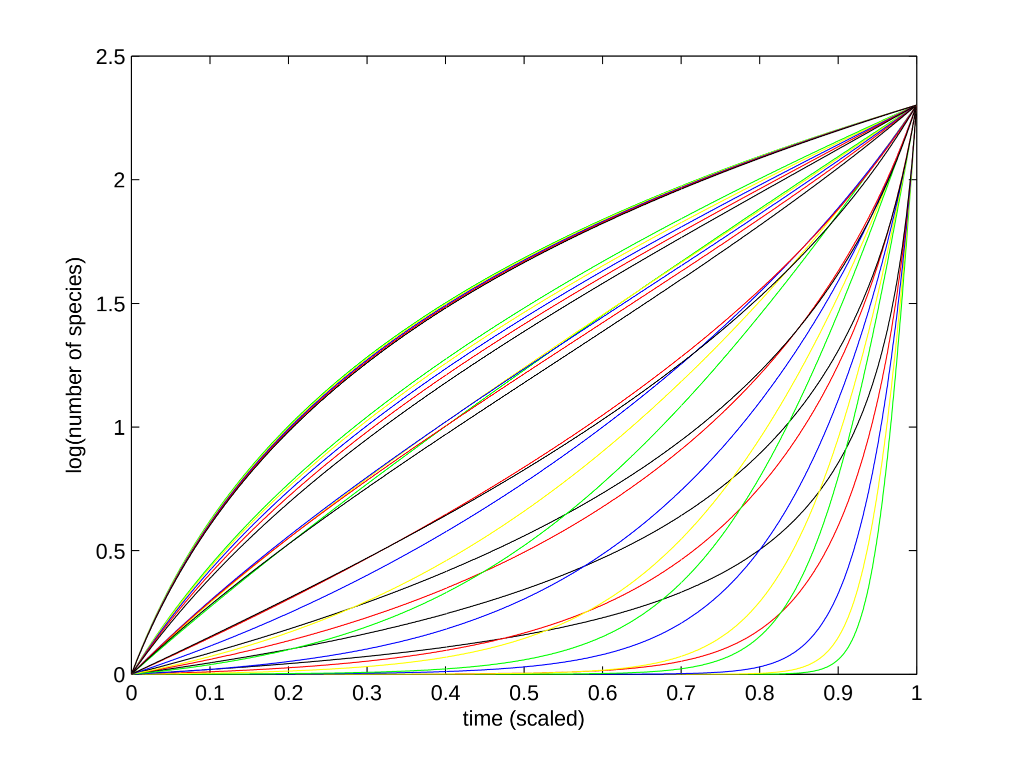
Expected LTT plots (“horizontal”)



$n = 10$.

Most recent common ancestor is at 0 and today is 1.

Expected LTT plots ("vertical")

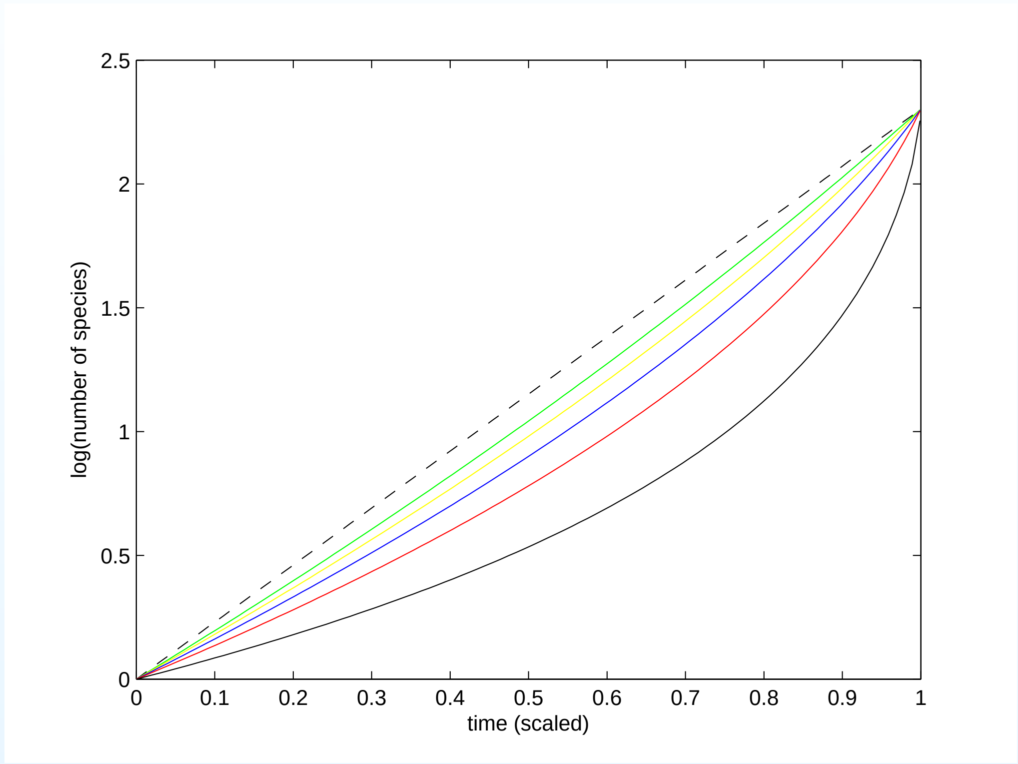


$n = 10$, $t = 10$.

$\lambda = 5, 2, 1, 0.5, 0.2, 0.1, 0.01$, from bottom to top.

green: $\mu = 0$, yellow: $\mu = \lambda/4$, blue: $\mu = \lambda/2$, red: $\mu = 3/4\lambda$, black: $\mu = \lambda$.

Expected LTT plots (“vertical”)



$n = 10$.

Expectation only depends on $\rho = \mu/\lambda$.

$\rho = 0, 1/4, 1/2, 3/4, 1$ (from top to bottom).

Dankeschön

Mike Steel

Dennis Wong

Anusch Taraz