# Untangling Tanglegrams

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(Thanks to Dan Gusfield for suggesting & discussing this work)

# How are these species related?

Brighamia insignis

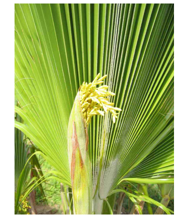
Delissea rhytidosperma

Erythrina sandwicensis

Hibiscus saintjohnianus

Hibiscus waimeae

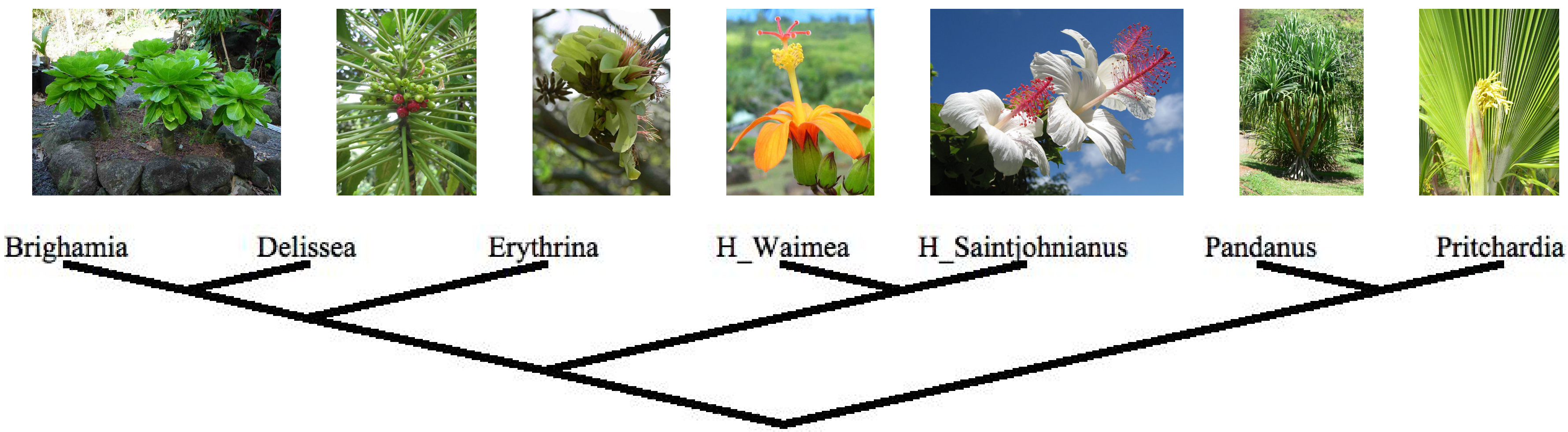
Pandanus tectorius



Pritchardia perlmanii

(Images courtesy of the National Tropical Botanical Gardens.)

# Displaying Trees




(Images courtesy of the National Tropical Botanical Gardens. Trees drawn with Dendroscope.)

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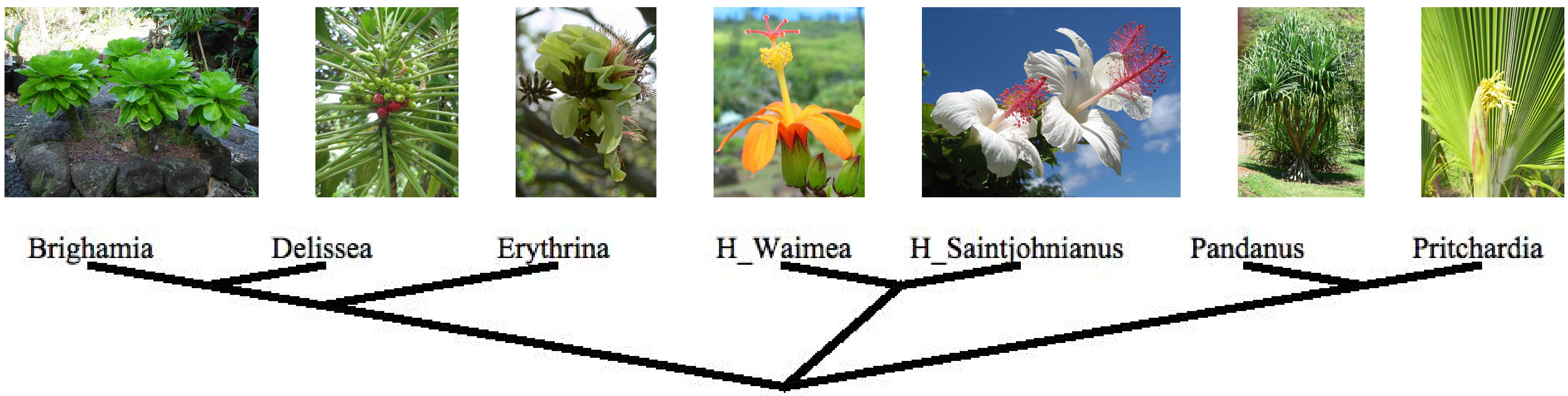


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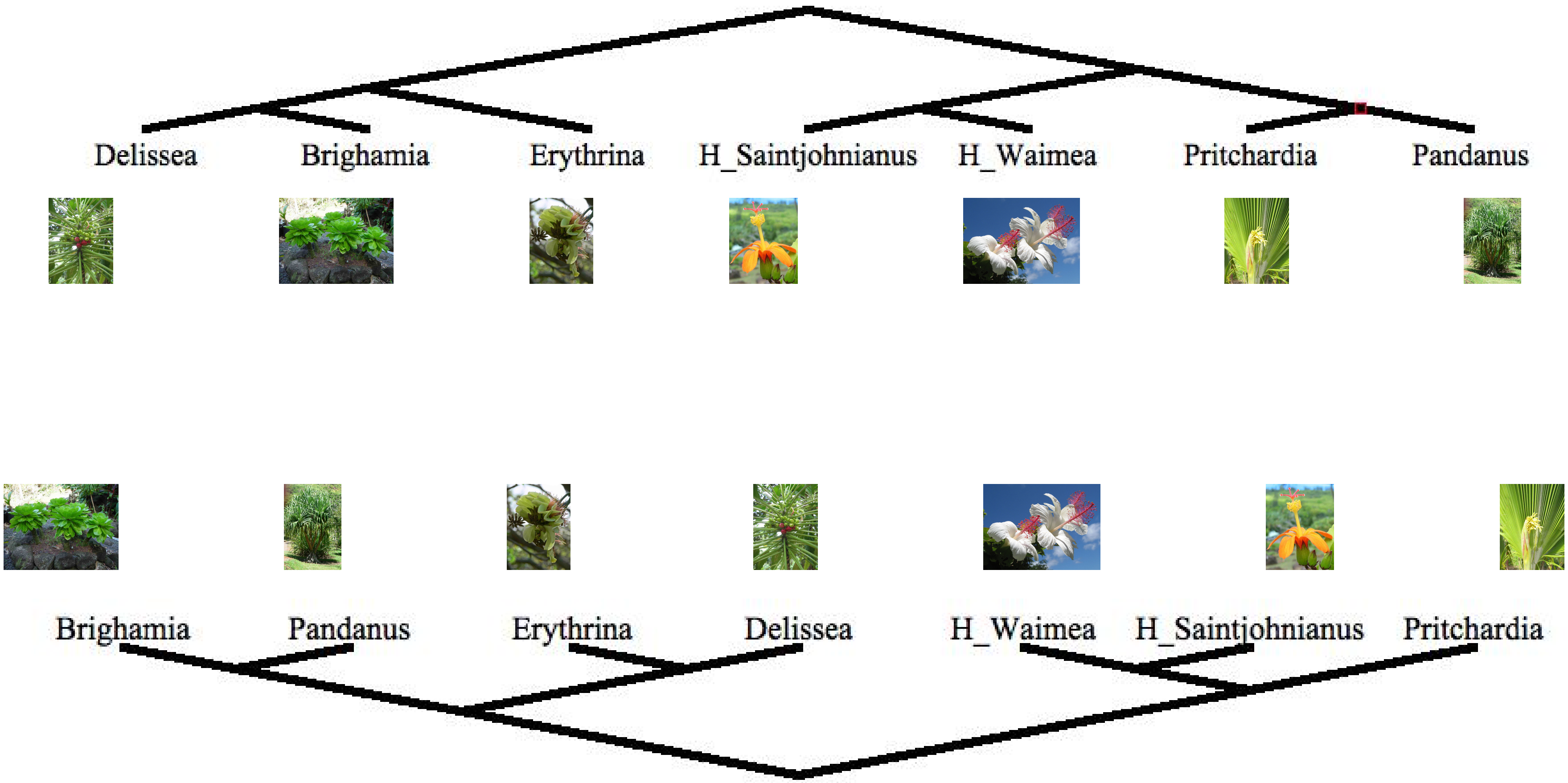
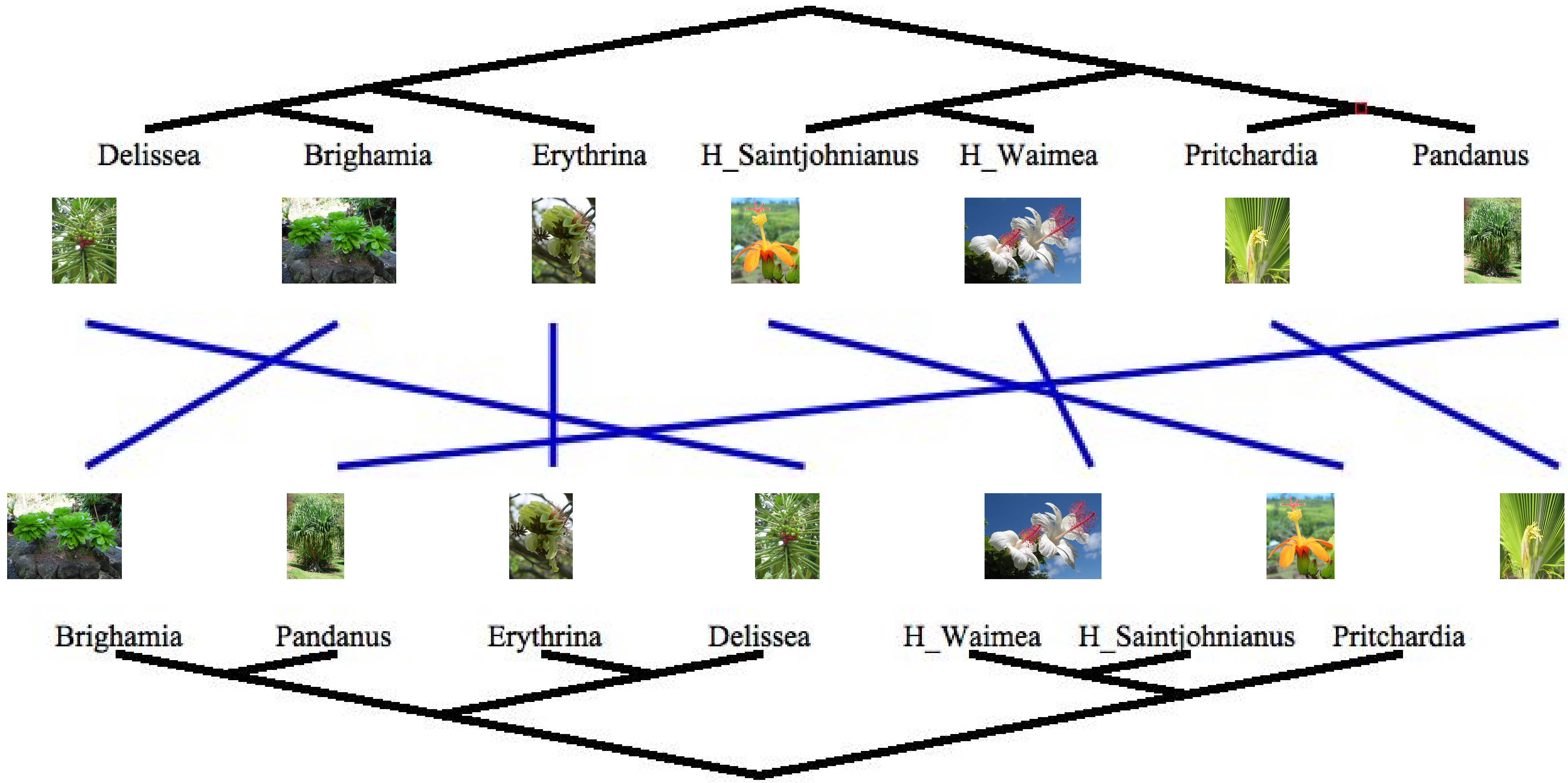
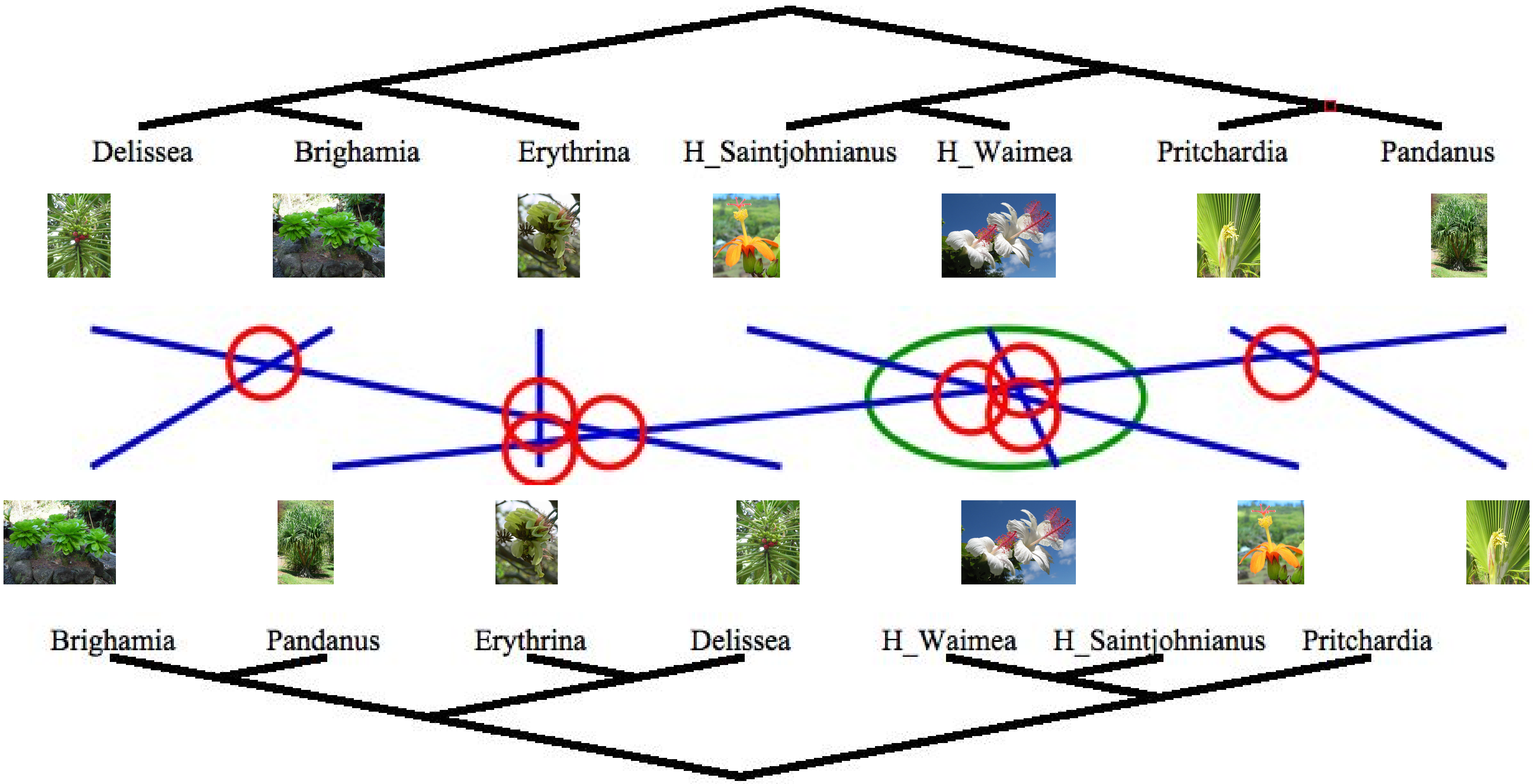


# Tanglegrams



# Crossings in Tanglegrams



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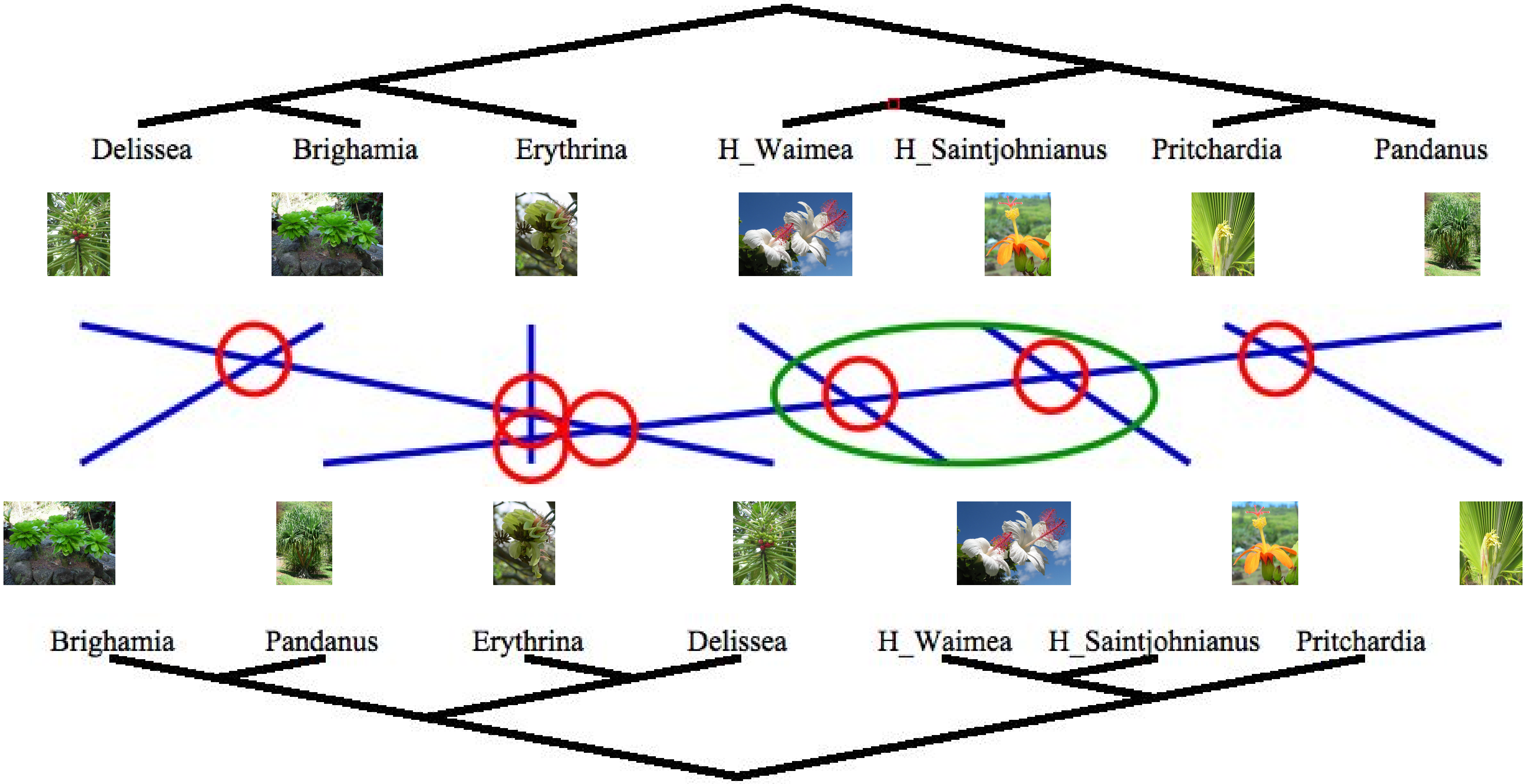

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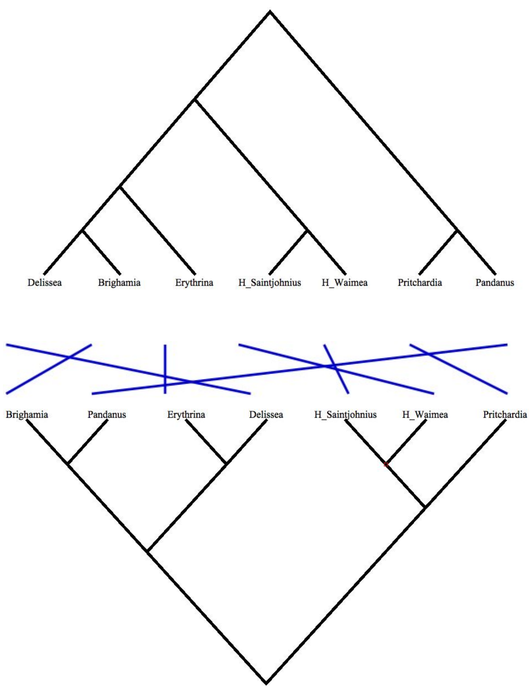


- Goal: Find the layout with the minimal number of crossings.

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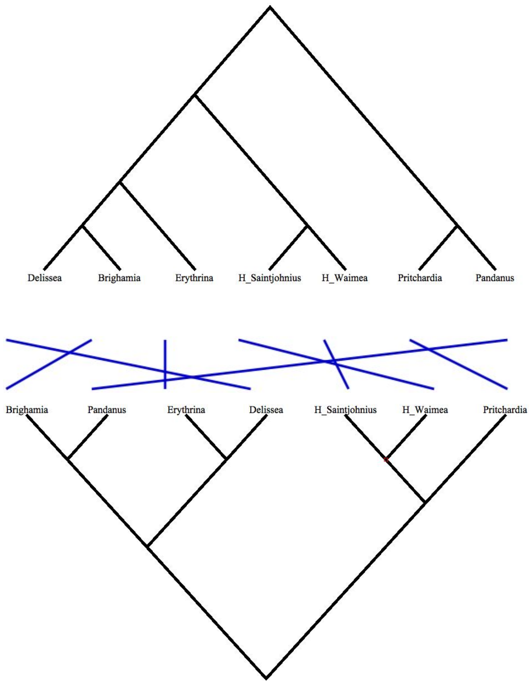


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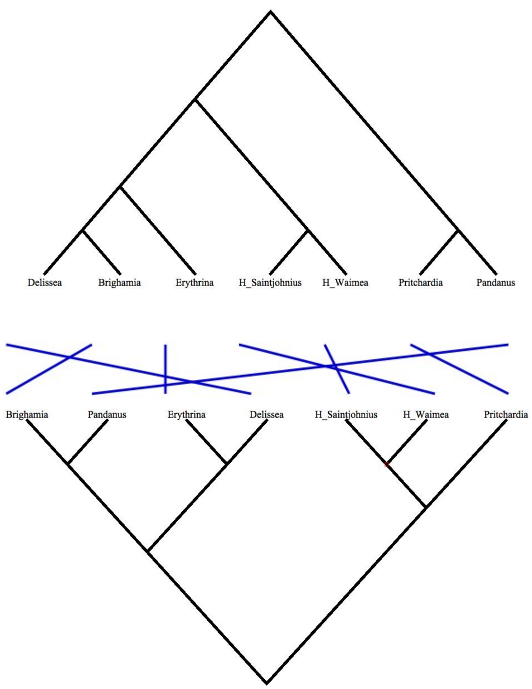


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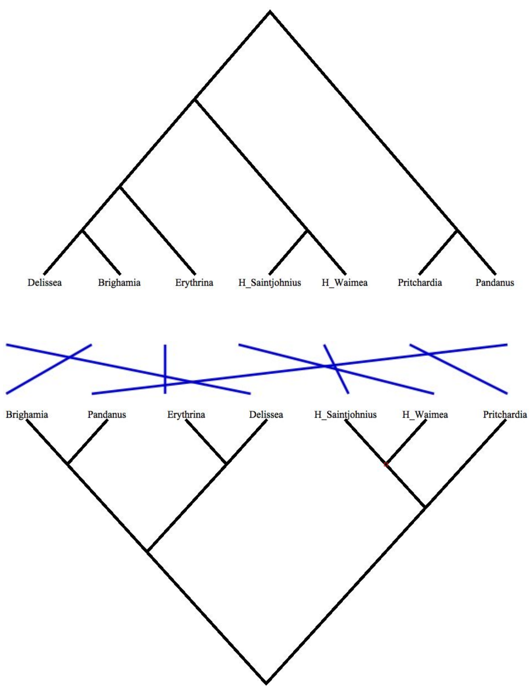


- Goal: Find the layout with the minimal number of crossings.
- Planarity: Is there a planar layout?
- "1-Layer" or Fixed: One tree remains fixed, the other's layout can change.
- "2-Layer" or General: Both trees' layouts can change.

# Planarity

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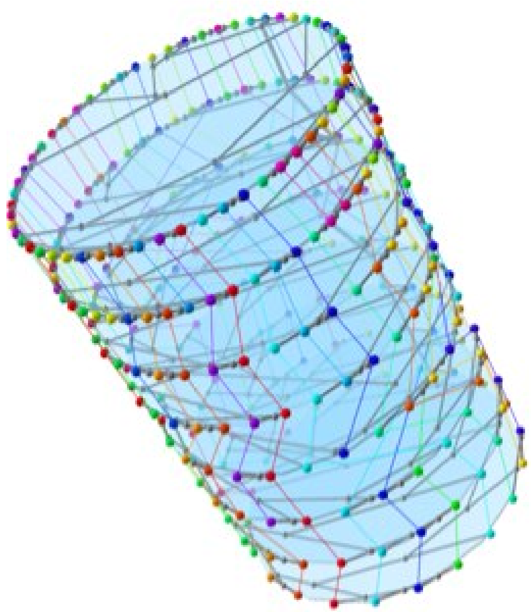
- Easily reduces to planarity question for graphs.
- Simple application of Hopcroft & Tarjan '74 on planarity of graphs, noted in Fernau, Kaufmann & Poths '05.
- (Also, rediscovered and shown $O(n^2)$ in Lozano *et al.* WABI '07.)

# One Tree Fixed: Minimizing Crossings



Dwyer and Schreiber '05

- "On-line" version of the problem: align new tree with previous loaded tree.
- Dwyer and Schreiber '05: $O(n^2)$.
- Fernau, Kaufmann & Poths '05: $O(n \log^2 n)$.

# General Question

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- Via different arguments, we get the result for all trees and improve the running time of the fixed parameter tractability.

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- We have a simpler reduction to Bipartite Graph Crossing Number:

  Every bipartite graph can be encoded as a tanglegram in polynomial time.

# General Question is NP-Hard

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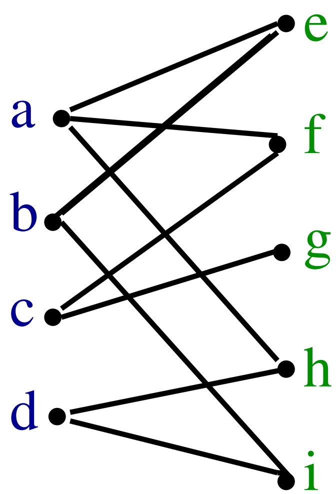

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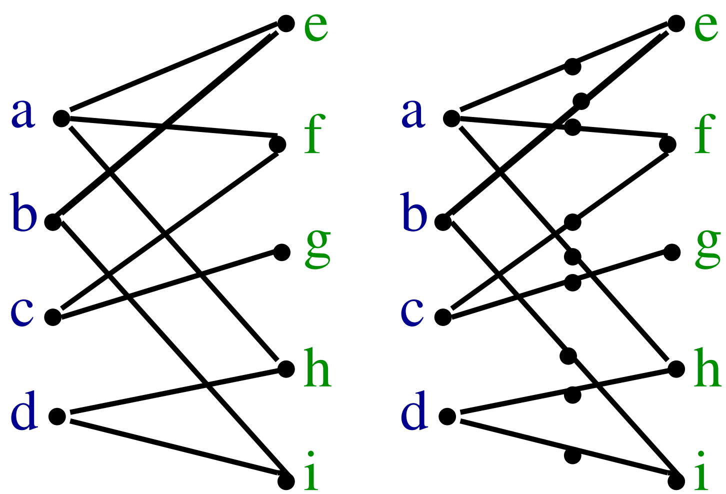

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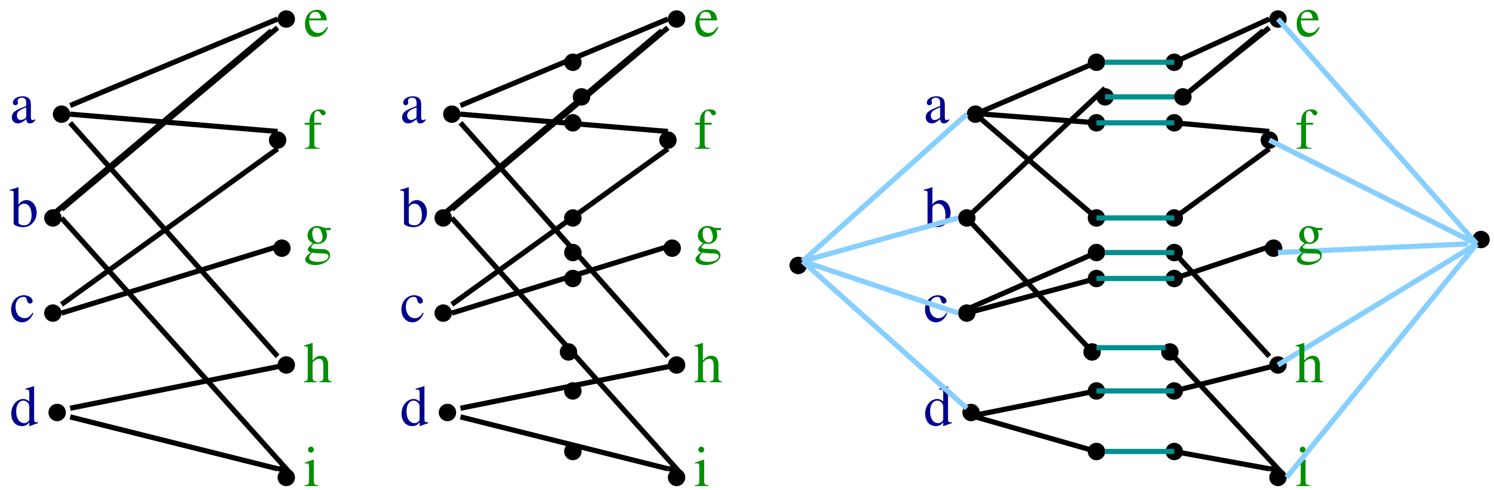

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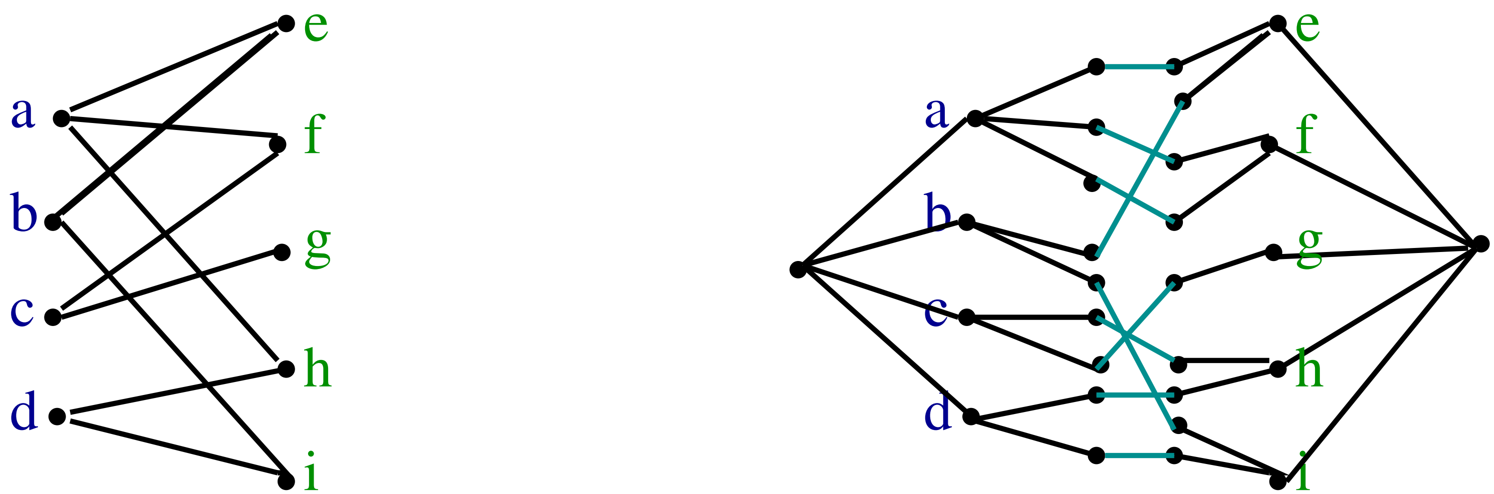

# General Question is Fixed Parameter Tractable

- Ferneau *et al.* '05 give poly-time fixed parameter tractability for binary trees only, and conjecture difficulties for $d$-ary trees, $d > 2$.

- We show a quadratic time fixed parameter tractability for all (including non-binary) trees.

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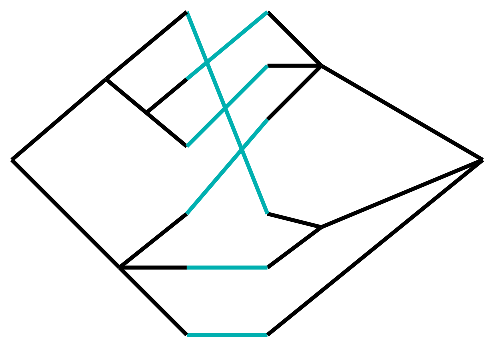
- Though NP-hard, some problems can be solved in time polynomial in the size of the input size but exponential in the size of a fixed parameter.
- In this talk, the parameter, $k$, will be the minimal crossing number of the tanglegram.

# Idea for FPT



- Grohe ‘04 and Kawarabayashi & Reed ‘07 show that computing the graph crossing number is FPT.

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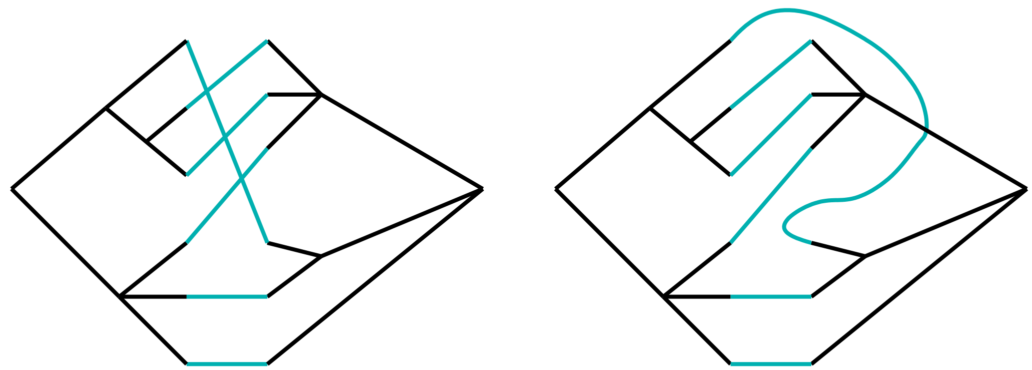

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- This does not work directly, since in graphs, crossings can occur anywhere.

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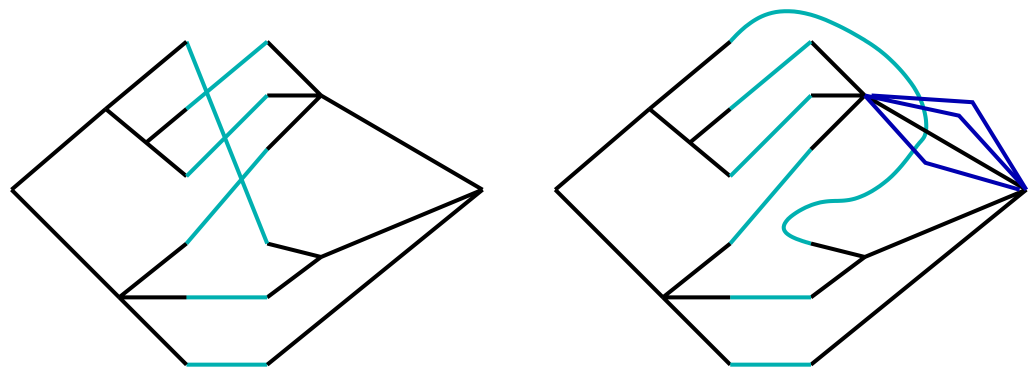

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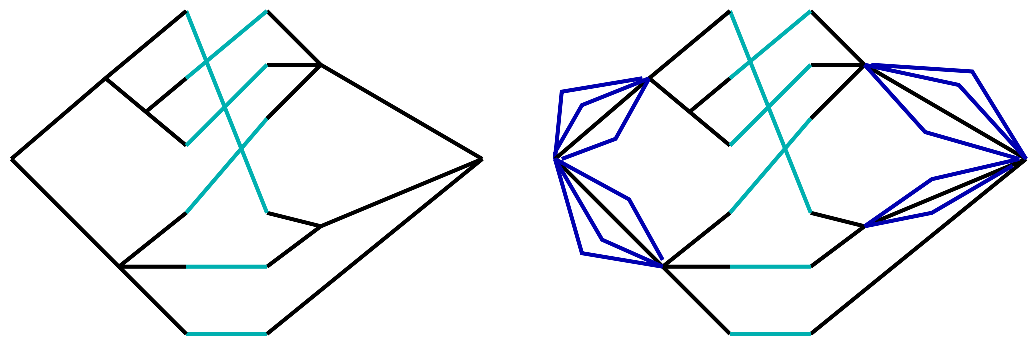

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# Applications

Fernau *et al.* '05 suggested several problems that our encoding should shed light on:

- Weighted version: "crossings have higher weights if they occur between edges of larger different subtrees".
- Determine the complexity of the maximum planar subgraph problem.
- Is there an approximation algorithm?

# Acknowledgements

H. saintjohnianus


- The Isaac Newton Institute and the special year in phylogenetics
- The United States National Science Foundation for their generous support