Transition to turbulence and turbulent bifurcation in a von Karman flow

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VKS team
Turbulent von Karman flow

- Axisymmetry
- $R\pi$ symmetry / radial axis

- $R_c = 100$ mm
- $H = 180$ mm
- $f = 2$-$20$ Hz
- $Re = 2\pi R_c f^2 / \nu = 10^2$ – $10^6$
- fluid: water and glycerol-water

Inertial stirring
TM60 propellers
Velocity regulation
$Re \approx 10^6$
First bifurcations and symmetry breaking

**Re = 90**
Stationary axisymmetric

**Re = 185**
$m = 2$; stationary

**Re = 400**
$m = 2$; periodic

meridian plane: poloidal recirculation

Tangent plane: shear layer

Re = 90
Stationary axisymmetric

Re = 185
$m = 2$; stationary

Re = 400
$m = 2$; periodic
Time spectra as a function of Re

$v_\theta$ en \{r = 0.9; z = 0\}

Re = 330  Periodic

Re = 380  Quasi-Periodic

Re = 440  Chaotic
Time spectra as a function of Re

\( v_\theta \) en \( \{r = 0.9; z = 0\} \)

\( \text{Re} = 1000 \)

Chaotic

\( \text{Re} = 4000 \)

Turbulent

2000 < \text{Re} < 6500

Bimodal distribution: signature of the turbulent shear
Transition to turbulence:
Azimuthal kinetic energy fluctuations

Developed turbulence

Globally supercritical transition via a Kelvin-Helmholtz type instability of the shear layer and secondary bifurcations

$Re_c = 330 \quad Re_t = 3300$

Ravelet et al. JFM 2008

Wall Bounded Shear Flows, Newton Institute, Cambridge, September 2008
Multiplicity of solutions

\[ K_p = \frac{\text{Torque}}{\rho R_c^5 (2\pi f)^2} \]

\[ Re_c = 330 \quad Re_t = 3300 \]

\[ Re^{-1} \]

\[ 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \]

\[ 10^0 \quad 10^{-1} \]

(-) b

(-) s

(+) s
Turbulent Bifurcation of the mean flow

Symmetry broken: 2 different mean flows exchange stability.

Bifurcated flow (b) : no more shear layer broken symmetry

Re = $3 \times 10^5$

(s) two cells one state

(b2) one cell two states
**Turbulent Bifurcation**

- $K_p = \frac{\text{Torque}}{\rho R_c^5 (2\pi f)^2}$
- $\Delta K_p = K_p1 - K_p2$
- $\theta = \frac{f_2-f_1}{f_2+f_1}$
- $Re = (f_1+f_2)^{1/2}$

Re = $3.10^5$

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Wall Bounded Shear Flows, Newton Institute, Cambridge, September 2008
Stability of the symmetric state

Cumulative distribution functions of bifurcation time $t_{\text{bif}}$:

$$P(t_{\text{bif}}>t)=A \exp\left(-\frac{(t-t_0)}{\tau}\right)$$

- $t_0^f \sim 5$
- $\tau$ : characteristic bif. time

Statistics on 500 runs for different $\theta$
Stability of the symmetric state

- symmetric state marginally stable
  \[ \tau \to \infty \text{ when } \theta \to 0 \]

exponent = -6
Mutistability = f(Re)

\[ \Delta K_p = \frac{Re}{Re_c} \]

- $Re = 800$
- $Re = 5000$
- $Re = 10000$

\[ Re = 3 \times 10^5 \]
Forbidden zone with velocity regulation

\[ \gamma = \frac{(Kp_1 - Kp_2)}{(Kp_1 + Kp_2)} \]

\[ \theta = \frac{(F_1 - F_2)}{(F_1 + F_2)} \]

Forbidden zone for stationary regimes
Forbidden zone with torque regulation

\[ \gamma = \frac{(K_p_1 - K_p_2)}{(K_p_1 + K_p_2)} \]

\[ \theta = \frac{(F_1 - F_2)}{(F_1 + F_2)} \]

Intermittent states (i)

- 1 cell (velocity)
- 1 cell (torque)
- 2 cells (torque)
$K_{p_1} \neq K_{p_2} \Rightarrow 1 \text{ cell}$

$K_{p_1} \sim K_{p_2}$, “intermittence” between 2 states

$K_{p_1} \approx K_{p_2} \Rightarrow 2 \text{ cells}$
VKS dynamo experiment

Meridional annulus

Oil cooling circulation

Na at rest

Small curvature, diameter 3/4

Propellers TM73
Position of the shear layer = f(θ)

The annulus stabilizes the separatrix

Position z of the separatrix

SPIV measurements

θ = 0.05, without annulus

θ = 0.09, without annulus

θ = 0.175, with annulus

θ = (F_1 - F_2) / (F_1 + F_2)
Transition 1 cell - 2 cells at $\theta_c = \pm 0.175$

- quasi-continuous transition
- small hysteresis
- small $K_p$ difference

very different from TM60 propellers

$\theta = (F_1 - F_2)/(F_1 + F_2)$
Transition at $\theta_c = \pm 0.175$

Mean $\Delta K_p$ is continuous

stochastic transition

$1 \rightarrow 2$ cells
Transition at $\theta_c = \pm 0.175$

Measurements: 2 – 200 min

$\theta = \frac{(F_1 - F_2)}{(F_1 + F_2)}$
Transition at $\theta_c$

$\theta_c = 0.174$

$\theta = \frac{(F_1 - F_2)}{(F_1 + F_2)}$

Cf. de la Torre & Burguete
PRL 2007 and Friday talk
Origin of erratic field reversals observed in VKS experiment?

\[ \theta = 0.17 \]
1. von Karman: turbulent bifurcation

2. VKS: dynamo action and B reversals

3. Couette flows: turbulent stripes and spirals

Prigent, Dauchot et al. PRL 2002