Inertial Range Dynamics in Density-Stratified Turbulent Flows

James J. Riley

University of Washington

Collaborators: Steve deBruynKops (UMass)
Kraig Winters (Scripps IO)
Erik Lindborg (KTH)

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Examples of Instabilities – Atmosphere

Examples of Instabilities – Atmosphere (cont’d)

Denver, Colorado, 1953 (photo by Paul E. Branstine)

Examples of Instabilities – Ocean

Examples of Instabilities – Laboratory

Research Focus

- dynamics of larger-scale motions that lead to instabilities and turbulence

- horizontal scales of interest
  - typically, in the ocean
    * on horizontal scales from a few meters to several hundred meters (potentially up to 10 km)
  - in strongly stable atmosphere boundary layers
    * at horizontal scales from a few meters to several hundred meters (or more)
  - in the upper troposphere, stratosphere
    * at horizontal scales from tens of meters to kilometers (potentially up to 100 km)
Description of Motions

• Some general characteristics
  – strongly affected by stable density stratification
  – high Reynolds number (compared to laboratory experiments)
  – usually there is a significant internal wave component
  – ‘classical’, smaller-scale 3D turbulence is very intermittent, sporadic
  – often horizontally meandering motions are observed
    * e.g., plumes in stable atmospheric boundary layers
  – larger-scale component velocities are highly non-isotropic
    * larger-scale motions do not overturn

• Termed ‘Stratified Turbulence’ (Lilly, 1983)
  – as distinguished from ‘classical’ 3D turbulence
• Controlling parameters
  
  – Reynolds number: \( R_\ell = u'\ell_H/\nu \)
    * \( u' \) – characteristic rms velocity
    * \( \ell_H \) – horizontal scale of energy-containing motions
  
  – Froude number: \( F_\ell = u'/N\ell_H \sim T_B/T_{FM} \)
    * \( N \) – buoyancy frequency, \( N = \sqrt{-\frac{g}{\rho_o} \frac{d\bar{\rho}}{dz}} \)
  
  – Gradient Richardson Number: \( Ri = N^2 \left/ \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right] \right. \)

• Stratified turbulence: \( F_\ell \leq \mathcal{O}(1), R_\ell \gg 1 \)
  
  – \( Ro = u'/\Omega\ell_H \gg 1 \) no effect of rotation (in this study)


**Scaling Arguments**

- Turbulent flows are expected to exist, locally, when

$$Ri = N^2 \sqrt{\left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right]} < \mathcal{O}(1)$$

- This can be shown to imply that

$$R_b = F_\ell^2 R_\ell \sim \frac{\epsilon}{\nu N^2} > \mathcal{O}(1) \quad \text{‘buoyancy Reynolds number’}$$

- from oceanographic measurements, laboratory data, numerical simulations

  - turbulence is found to exist when $R_b > \mathcal{O}(10)$
Stratified Turbulence (cont’d)

$F^2 R_f = \epsilon/\nu N^2 = \text{constant}$

- Stratified Turbulence
- Kolmogorov turbulence
- Laboratory turbulence
- Laminar?
- Numerical simulations
- Turbulent
- Laminar


Theoretical Arguments – Stratified Turbulence

- Lilly (1983) used scaling arguments to suggest, for $F_\ell < O(1)$:
  - flows in ‘adjacent’ horizontal layers are somewhat decoupled
  - leads to increasing vertical shearing of horizontal flow
  - and to decreasing Richardson numbers

- Billant and Chomaz (1999)
  - induced velocities lead to strong vertical inhomogeneities and layering

- Even though strong, stable stratification, i.e., $F_\ell < O(1)$, at high Reynolds numbers, both mechanisms lead to
  - smaller vertical scales continually developing
  - local instabilities and ‘classical’ 3D turbulence intermittently occurring
Three-dimensional contour plots of the stream function for the case with $F_\ell = 0.6$, $R_\ell = 3200$ at $t = 0$ (left) and $t = 15$ (right).
Volume-Averaged Gradient Richardson Number versus $t$

$$Ri_V = \frac{N^2}{\left\langle \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right\rangle}$$
Horizontal Kinetic Energy Spectra
Ozmidov Scale

- In a horizontal inertial range, with (constant) dissipation rate $\epsilon$
  - defining KE and PE for each horizontal length scale
  - the horizontal scale $l_o$ at which KE and PE balance is

\[ l_o = \left( \frac{\epsilon}{N^3} \right)^{1/2} \quad \text{Ozmidov scale} \]

- Interpretation of $l_o$:
  - horizontal scales smaller than $l_o$ can overturn
  - horizontal scales larger than $l_o$ cannot
Scaled horizontal energy spectra, Lindborg (2005).
Horizontal kinetic energy spectra at $t = 18.5$ for $R_{\ell} = 9600$ case.
Scaling Arguments

- Assume
  - $F_{\ell} \ll 1$ (strong stratification) --- implies $\ell_O/\ell_H \sim F_{\ell}^{3/2} \ll 1$
  - $F_{\ell}^2 R_{\ell} > R_{B\text{crit}} \sim 40$ --- implies $\eta/\ell_O < R_{b\text{crit}}^{-3/4} < 0.25$
  - $\epsilon \sim u^3/\ell_H$, independent of $N, \nu$ (or $F_{\ell}, R_{\ell}$)

- Implications for horizontal spectra
  - $E = E(k_H, \epsilon)$, so, analogous to Kolmogorov, $E = C \epsilon^{2/3} k_H^{-5/3}$
Evidence for Stratified Turbulence Inertial Range

- Numerical simulations (all with constant $N$)
  - Riley & deBruynkops, DNS, no shear decaying flow
  - Lindborg, LES, no shear, forced flow
  - Diamessis, DNS, wake of sphere

- Field measurements
  - Ocean
    * Klymak & Moum, off Hawaiian ridge, shear, temperature, $\epsilon$
    * Ewart, various locations, temperature
    * Hollbrook & Fer, displacement spectra
    * Bouruet-Aubertot, van Haren & Lelong, temperature
  - Atmosphere
    * Frehlich, very stable atmospheric boundary layer
      - velocity, temperature
Shear Spectra – Ocean (Klymak, 2005)
Shear Spectra – Ocean (Klymak, 2005)

$z < 1000 \text{ dbar}$ Binned by $K_{\rho}^\rho$

\[ \frac{(N/N_0) \Phi_{x_x}}{[\text{cpm}^{-1}]} \]

\[ k_x [\text{cpm}] \]

\[ 10^{-1} \quad 10^{0} \quad 10^{1} \]

- $1.4 \times 10^3$
- $3.4 \times 10^4$
- $4.6 \times 10^5$
- $4.9 \times 10^6$

McKean Ewart '74
Hawaii: 495 m
Temperature spectra – Ocean (Ewart, 1976)

Power spectra of temperature off the coast of San Diego (30°N, 124°W).
Displacement spectra – Ocean (Hollbrook & Fer, 2005)

Vertical displacement spectra from open ocean (squares) and near slope (dots)
Temperature spectra – Ocean

Bouruet-Aubertot, van Haren & Lelong, 2008
Temperature frequency (horizontal wave number) spectrum in very stable atmospheric boundary layer
Velocity Spectrum – Atmosphere (Frehlich et al., 2007)

Velocity frequency (horizontal wave number) spectrum in very stable atmospheric boundary layer
**Conclusions**

- Stratified turbulence $F_{e} \leq \mathcal{O}(1), \ R_{e} \gg 1, \ R_{b} \geq \mathcal{O}(10)$
  - e.g., at oceanic horizontal scales larger than a few meters
  - strong tendency for vertical shearing of horizontal motion
  - leads to intermittent, 3D turbulence
  - intermittent occurrence in space leads to a strong downscale transfer of energy
  - potential for stratified turbulence ‘inertial cascade’
    * highly nonisotropic inertial subrange
    * possible explanation of field data
    * replaces notion of a ‘buoyancy subrange’
Some Open Issues

- Dynamics in ‘inertial range’?
  - E.g., except in 3-D turbulent regions, vortex lines are mainly horizontal

- Role of internal waves, especially in the ocean
  - Are the density perturbations ‘slaved’ to velocity?

- How does energy cascade down from the scales dominated by rotation?
  - Is there both upscale and downscale propagation of energy?

- What are the vertical spectra?

- Does horizontal particle separation go as: $\langle D^2 \rangle = C \epsilon t^3$?
  - Does shear dispersion dominate?
Inertial Range Dynamics and Mixing
colors: w in xy plane Lz/2 at time t = 182.8125 s
Inertial Range Dynamics and Mixing

\[ \rho(x, y, z) \quad t = 182.6125 \text{ s} \quad x_0 = 1.8691 \text{ m} \]

\[ w \text{ velocity, positive up} \]
Mean Square Patch Size – Ocean

Okubo, Deep-Sea Research, 1971
Spectra of available potential energy in horizontal (Dugan et al., 1986)
Temperature Structure Function – Ocean

Voorhis and Perkins, Deep-Sea Research, 1966

Fig. 10. Temperature structure function along the eastward tow track.
Temperature Spectrum – Ocean

Lafond and Lafond, 1967, Marine Technical Society

Figure 18. Average power spectrum of 25 sections of isotherm depths in the main thermocline and in isotherm depths just below the main thermocline. The slope of -5/3 in the log-log relationship is shown for comparison.