Carter-Payne maps — example 1

Suppose that $\lambda = (4,4,3,2)$ and $\mu = (6,4,3)$ and $\gamma = 2$. Then

$$L_{\lambda,\mu} = (L_{15} - [5])(L_{15} - [2])L_{15}(L_{14} - [5])(L_{14} - [2])L_{14}.$$

Direct computation shows that

$$\begin{matrix} 1&1&1&1&X&X\\ 2&2&2&2\\ 3&3&3\\ 4&4 \end{matrix} \; L_{\lambda,\mu} = q^{-5}(q^3 - q - 1)[2][2][4] \begin{matrix} 1&1&1&1&2&2\\ 2&2&3&3\\ 3&4&4\\ X&X \end{matrix}$$

$$-q^{-5}[2][2][2][4] \begin{matrix} 1&1&1&1&2&2\\ 2&2&3&4\\ 3&3&4\\ X&X \end{matrix} - q^{-4}[2][2][4] \begin{matrix} 1&1&1&1&2&3\\ 2&2&2&3\\ 3&4&4\\ X&X \end{matrix}$$

$$+q^{-5}[2][2][4] \begin{matrix} 1&1&1&1&2&3\\ 2&2&2&4\\ 3&3&4\\ X&X \end{matrix} + q^{-5}[2][2][4] \begin{matrix} 1&1&1&1&2&4\\ 2&2&2&3\\ 3&3&4\\ X&X \end{matrix} + \dots$$

Now, $\lambda_a - \lambda_b + b - a + \gamma = 4 - 2 + 4 - 1 + 2 = 7$

$\Longrightarrow$ $\rho_{\lambda,\mu}$ is a non-map map in $\mathrm{Hom}_{\mathcal{H}_n^\Lambda}(S^\lambda, S^\mu)$ when $e = 7$ (p arbitrary).

Carter-Payne maps — example 2

Suppose that $\lambda = (4,3,3)$ and $\mu = (7,3)$ and $\gamma = 3$.
Then $L_{\lambda,\mu} = (L_{10} - [6])(L_9 - [6])(L_8 - [6])$ and

$$\begin{matrix} 1&1&1&1&X&X&X\\ 2&2&2\\ 3&3&3 \end{matrix} = -q^6[2][3] \begin{matrix} 1&1&1&1&2&2&2\\ 3&3&3\\ X&X&X \end{matrix}$$

$$+q^5[2] \begin{matrix} 1&1&1&1&2&2&3\\ 2&3&3\\ X&X&X \end{matrix} - q^3[2] \begin{matrix} 1&1&1&1&2&3&3\\ 2&2&3\\ X&X&X \end{matrix}$$

$$+[2][2] \begin{matrix} 1&1&1&1&3&3&3\\ 2&2&2\\ X&X&X \end{matrix}.$$

This time $\lambda_a - \lambda_b + b - a + \gamma = 4 - 3 + 3 - 1 + 3 = 6$.

Taking $e = 2$ and $p = 3$ we have $\rho_{\lambda,\mu} \neq 0$ (after dividing by [2]).

However,if $e = 3$ and p is arbitrary then we still get a non-zero map!
These maps are predicted by Carter-Payne only when $p = 2$!?