Formation of Giant Planets by Core Accretion

Review

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outline:
- basic concepts & classical models
- improvements & extensions
- population synthesis
1) Basic Concepts and Classical Models
Formation Model: Core Accretion Paradigm

Follow the concurrent growth of an initially small solid core (Ices, Rocks) surrounded by a gaseous envelope (H\textsubscript{2} & HE) in the protoplanetary disk.


Divide problem in three modules
- Solid accretion rate
- Gas accretion (envelope)
- Planetesimal-envelope interaction
Accretion of solids

- **collisional** growth of one big body from small background planetesimals
- **safronov-type rate equation** for \( \frac{dM_Z}{dt} \)

\[
\frac{dM_Z}{dt} = \Omega \Sigma p \pi R_{capt}^2 F_G(e, i)
\]


- **random velocity** \( \sigma(e,i) \) of planetesimals is key parameter (runaway, oligarchic, orderly)

- **accretion from a feeding zone** with spatially constant planetesimal surface density \( \Sigma_P \)

\[
\frac{d\Sigma_P}{dt} = - \frac{(3M_\star)^{1/3}}{6\pi a^2 BLM^{1/3}} \frac{dM_Z}{dt}
\]

e.g. Thommes et al. 2003
Accretion of gas

- one dimensional hydrostatic envelope structure equations (similar to stars)

1) \( \frac{dr^3}{dm} = \frac{3}{4\pi \rho} \) mass conservation

2) \( \frac{dP}{dm} = -\frac{G(m + M_{\text{core}})}{4\pi r^4} \) hydrostatic equilibrium

3) \( \frac{dL}{dm} = -\frac{dU}{dt} + \frac{P}{\rho^2} \frac{d\rho}{dt} + \epsilon_{\text{acc}} \) energy conservation

4) \( \frac{dT}{dP} = \nabla_{\text{ad}} \) or \( \nabla_{\text{rad}} \) energy transfer

Heating by impacting planetesimals
Accretion of gas

• attached case
  • structure goes smoothly to Hill sphere radius
  • low mass, pre-runaway planets
  • boundary conditions: background nebula
  • $dM_{XY}/dt$ given by ability of envelope to radiate away energy ($T_{KH}$)

$R_{out} = \text{Min}(R_H, R_{acc})$

$P(R_{out}) = P_{neb}$

$T(R_{out}) \approx T_{neb}$

• detached case
  • structure has a free surface
  • high mass, post-runaway planets
  • boundary conditions: accretion shock (or circumplanetary disk)
  • disk and gap formation regulate $dM_{XY}/dt$

$v_{ff} = \sqrt{2GM \left( \frac{1}{R_{out}} - \frac{1}{R_H} \right)}$

$P(R_{out}) \approx P_{ram} = \frac{dM_{xy}/dt}{4\pi R_{out}^2} v_{ff}$

$4\pi R_{out}^2 \sigma T(R_{out})^4 \approx \frac{3}{4} L_{\text{shock}} + \frac{1}{2} L_{\text{int}}$

Planetesimals-envelope interaction

• linking the other two modules
  • the effective capture radius
  • determines the location of energy and mass deposition

• complex models on their own
  • gravity and gas drag
  • thermal ablation
  • aerodynamical disruption
    (lateral spreading “pancaking”, hydrodynamic fragmentation
    cf. the impact of SL9 into Jupiter)

Podolak et al. 1988, Zahnle & MacLow 1995, Mordasini et al. 2006
Drag enhanced capture radius

- presence of envelope increases effective capture radius of the planet for planetesimals over the core radius $R_{\text{capt}} (> R_{\text{core}})$.
  - increase by 1 to 2 orders of magnitude
  - important for the growth of the core ($\frac{dM_Z}{dt} \propto R_{\text{capt}}^2$)
  - $R_{\text{capt}}$ dependent on envelope structure, planetesimal size, velocity, composition
  - energy criterion for capture (Podolak et al. 1988)

Inaba & Ikoma 2003, Chambers 2006, Tanigawa & Ohtsuki 2009

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2 Body

- $m_{\text{core}} / m_{\text{earth}} = 13$

3 Body

- Hill Radius

Mordasini & Benz 2004
Planetesimals energy and mass deposition

- The higher the core mass, the more massive the envelope, so the more difficult to penetrate.
- Leads to auto-regulation by self-shielding of max. core mass built by planetesimals in solid form.
- For 100 km icy planetesimals: ca. 6 Earth masses. Additional processes may change this afterwards.

- Huge energies involved .. and frequent ~10 yr⁻¹
- SL9 like plume formation? i.e. can some energy leave the thick protoplanetary atmosphere?
- Reduction of envelope luminosity? => formation time reduced
- Observable from Earth?
Classical formation model: Pollack et al. 1996

Main model assumptions:

• constant ambient T and P (no disk evolution)
• in situ formation (no migration)
• standard opacities

• Phase I: Rapid build up of a core. Planetesimal accretion until solid feeding zone emptied.

• Phase II: Slow accretion of gas and planetesimals

• Phase III: Runaway gas accretion at $M_{\text{core}} > M_{\text{crit}}$

Existance intrinsic to core-envelope setup
(Stevenson 1982)

$M_{\text{crit}}$ ca. 10-15 $M_\oplus$, extreme cases: 1-40 $M_\oplus$
(Papaloizou & Terquem 1999)
Evolution of the internal structure

Pressure

\[ R_{\text{capt}} \]

\[ \text{Min}(R_{\text{acc}}, R_H) \]

Temperature

Density

Mass inside \( R \)
Sensitivity to planetesimal surface density:

Time scale problem

Forms Jupiter in 8 Myrs in a 4 x MMSN, with a core-envelope ratio compatible with internal structure models.

Timescale for formation essentially given by phase II. Comparable to or longer than observed disk lifetimes.

Formation timescale extremely sensitive to disk mass. (Can be shortened by a higher solid surface density, but can lead to too large cores.)

Basic requirement for core accretion to form giant planets:

Assembly of a core having critical mass (≈ 10 M_{Earth}) is fast enough (< gas disk lifetime).
2) Improved and Extended Models
EOS and opacity

• equation of state (reviewed by Saumon & Guillot 2004)
  • EOS Saumon, Chabrier, & van Horn 1995. First principle EOS Militzer et al. 2008

• reduced grain opacity in the envelope (dominant at $T<1700$ K)
  • Podolak 2003: grains coagulate, settle and get destroyed in the envelope.

Hubickyj, Bodenheimer & Lissauer 2005

$\Sigma_P (5.2 \text{ AU}) = 10.0 \text{ g/cm}^2$ (4 MMSN)

Grain opacity 2 % interstellar.
Time to runaway: factor 3-4 reduced

Reduced grain opacity greatly speeds up the gas accretion timescale.
Incorporating constraints from disk-planet interactions

- only particles inside ~ 0.25 \( R_H \) are bound
- accretion rate limited by gap formation at > 30-100 \( M_{\text{Earth}} \)

\[
\frac{dM}{dt}_{\text{max}} = 3\pi \nu \Sigma \left( 1.668 \left( \frac{M_p}{M_J} \right)^{1/3} e^{-\frac{M_p}{1.5M_J}} + 0.04 \right)
\]

rate in disk

Planetesimal accretion rate

- dedicated talk Harold Levison on Thursday

- runaway growth self-limited by increasing heating by the runaway body: oligarchic growth regime (Ida & Makino 1993, Kobubo & Ida 1998)
  - Thommes et al. 2003: Transition at $M<<M_{\text{iso}}$

- Pollack et al. 1996 assumed rather low planetesimal random velocities
  - Planetesimal eccentricities increased by protoplanet (like in oligarchic growth), but inclinations given by planetesimal-planetesimal interactions only (as in runaway).
**Planetary accretion rate cont.**

- Using basic oligarchic $\frac{dM_p}{dt}$ leads to much longer formation timescales ($\gg$ disk lifetimes).

**Oligarchic growth not necessarily slow:**
- Chambers 2006 oligarchic rates almost as fast as Pollack’s growth rate

- no plateau phase (phase II). Isolation mass not reached.
  - can also be absent due to migration (Alibert et al. 2004)
- formation time no more given by contraction rate, but solid accretion rate.
  - therefore, opacity doesn’t matter much (Dodson-Robinson et al. 2008)
Planetesimal accretion rate cont.

- in an anticyclonic vortex (Klahr & Bodenheimer 2006)
- opening of gaps in the planetesimal disk (Zhou & Lin 2007, Shiraishi & Ida 2008)
- speed up by planetesimal fragmentation (Kenyon & Bromley 2009)
- early scattering of cores to large semimajor axes (monarchical growth, Weidenschilling 2008)
- accretion of other oligarchs (Chambers 2006)
- ....
Disk evolution and migration

similar timescales of various processes:

\[ \tau_{\text{migration}} \leq \tau_{\text{formation}} \approx \tau_{\text{disk evolution}} \]

\[ \rightarrow \text{extend model to include in a self consistent way (Alibert, Mordasini, Benz 2004)} \]

1) type I and type II planetary migration

(Lin & Papaloizou 1986; Ward 1997; Tanaka et al. 2002). Type I reduced by constant factor \( f_1 \) (free parameter). New promising results exist (e.g. Kley, Bitsch & Klahr 2009).

2) disk evolution

(1+1 D) \( \alpha \)-disk with photoevaporation (Papaloizou & Terquem 1999)

simplifications (most important)

- One embryo per disk, no systems
- Formation only until the gas disk disappears: No evolution after disk dispersal (Terrestrial planets, Ice giants, evaporating planets)
- No eccentricity, planets on circular orbits
- No particular stopping mechanism, \( a_{\text{min}}=0.1 \) AU
Effects of migration

Migration greatly speeds up the formation timescale: skip phase 2

Migration prevents the depletion of feeding zone
Models meet observations

Jupiter

Alibert et al. 2005b

Saturn

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<th>computed</th>
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<tr>
<td>Kr</td>
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<tr>
<td>Xe</td>
<td>2.1 ± 0.4</td>
<td>2.6</td>
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<tr>
<td>C</td>
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<tr>
<td>N</td>
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<td>S</td>
<td>2.7 ± 0.6</td>
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3) Planetary Population Synthesis
Extrasolar planet population synthesis: Observational Motivation I

extreme diversity
- super Jupiters
- hot Jupiters
- hot Neptunes
- eccentric planets
- planetary systems

no more single objects, but a population
- distributions of masses
- semimajor axes
- host star metallicities
- eccentricities
- correlations

diversity of protoplanetary disks, too. Planetary diversity simple consequence?
Reproducible?
Extrasolar Planet Population synthesis:

**Principle**

Extended core accretion model
Formation model quantitatively tested in the Solar System (Alibert et al. 2004)

Initial Conditions: Probability distributions & parameters
- Disk gas mass
- Disk dust mass
- Disk lifetime

From observations

Draw and compute synthetic planet population

Apply observational detection bias

Observed population

Comparison:
- Observable sub-population
  - Distribution of semi-major axis
  - Distribution of masses
  - Fraction of hot/cold Jupiters
  - Metallicity effect

Match

No match: improve, change parameters

Cross check
Couple to other detection methods

Predictions
(going back to the full synthetic population)

Model solution found!

Mordasini et al. 2009a
Mordasini et al. 2009b
Other recent works: Miguel et al. 2008a,b & Edward Thommes et al. we just heard about

Ida & Lin (2004a,b, 2005,2008a,b)
- Planet desert
- Separation of types in mass-distance
- Metallicity effect (correlation between metallicity and giant planet detection probability)
- Effects of type I migration
- Effects of dead zone

Ida & Lin planet formation model differs significantly from Pollack et al. and from Alibert et al.
→ Effect on population synthesis

Pioneering Monte Carlos
Population Synthesis: Initial conditions

Some can be constrained by observations some from theoretical arguments and some are just “educated” guesses.

Four Monte Carlo variables with probability distributions

- Dust-to-gas ratio (solid surface density). Constrained by observed stellar metallicities. Distribution in the CORALIE planet search sample (Santos et al. 2003).
- Photoevaporation rate. Constrained by observed disk lifetimes. Distribution adjusted to obtain (with fixed alpha) disk lifetime distribution found by Haisch et al. 2001.
- Initial semimajor axis of the small planetary seed put into the disk. Analytical arguments only. Uniform in log a (Ida & Lin 2004)

Parameters (fixed for one synthetic population)

- Type I migration rate reduction factor $f_1$
- Disk viscosity parameter $\alpha$ (0.007)
- Planetesimal size ($R=100$ km)
- Initial solid surface distribution ($\propto r^{3/2}$)
- Stellar mass
Planetary formation tracks

- Type I migration
  (Analytical rate reduced by $f_i$)
- Type II migration
  (Disk dominated: $M_p < M_{\text{disk,loc}}$)
- Type II migration
  (Planet dominated: $M_p > M_{\text{disk,loc}}$ & disk limited gas accretion)

Population Synthesis

$M_{\text{star}} = 1 \, M_\odot$

Nominal model

Mordasini et al. 2009a

Possible evaporation
Synthetic Population

Nominal Model: $\alpha = 7 \times 10^{-3}$, $f_1 = 0.001$, $M = 1 M_\odot$

The variation of the initial conditions within the observed limits (protoplanetary disk properties) produces synthetic planets of a very large diversity.
**RV detection bias**

Instrumental precision = 10 m/s

![Graph showing detection probability vs. semimajor axis](image)

\[ P_{\text{detec}}(a,M) \]

Includes effects of:
- Orbital eccentricity
- Stellar rotation rate
- Stellar jitter
- Actual measurement schedule

Radial velocity method:
Strong bias towards detection of massive, close-in planets.

Naef, Mayor, Beuzit et al. 2004
(Originally for the spectrometer ELODIE, the 51 Peg b detection instrument)
Comparison with observations

**a-M**

Best fitting models are chosen by statistical comparison with observation.

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**Core accretion is able to reproduce well the mass-distance distribution of the actual extrasolar planet population.**

**Non-trivial because 1) many constraints at one time 2) few parameters 3) probability distributions fixed 4) cannot force arbitrarily**

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Tip of the iceberg

**Overall detection probability: 9% (~as obs.)**

- \(0.7 < M_{\text{star}} < 1.3\)
- \(e < 0.3\)
- One planet / star
- Single host stars
- \(K_{RV} > 10 \text{ m/s}\)

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KS a-Msini: 88%

**Observe** 10 yrs at 10 m/s

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Full synth. population

Detectable sub-population
Comparison with observations

Mass distribution

Blue lines: Observational comparison sample
Black lines: Detectable synthetic sub-population

The mass distribution is very well reproduced.
Observations show: Extrasolar planets are preferentially found around stars with a high heavy element abundance: “Metallicity effect”

Well reproduced by the synthetic population.

- Dependence not strong enough: Additional mechanisms? Anders Johansen’s talk.

Large metallicity effect on RV detections

- Metal rich systems tend to produce more massive planets
- Radial velocity method favors massive objects

Does not mean there are no planets around low [Fe/H] stars... we just can’t detect them at the moment...

Comparison with observations

“Metallicity effect”

Blue: Observation (Fischer & Valenti 2005)
Red: Observation (Udry & Santos 2007)
Black: Observable synthetic planets

Mordasini et al. 2009b

cf. also Santos et al. 2004, 2005
Planetary Initial Mass Function (PIMF)

- Model incompleteness
- Minimum: Planetary desert
- Peak at low masses
- Neptunian Bump
- Flat Giant's Plateau
- Superjupiter Tail

<table>
<thead>
<tr>
<th>Type</th>
<th>Mass ($M_{\text{Earth}}$)</th>
<th>%</th>
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<tbody>
<tr>
<td>(Super)-Earth</td>
<td>&lt; 7</td>
<td>58</td>
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<tr>
<td>Neptunian</td>
<td>7-30</td>
<td>17</td>
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<tr>
<td>Intermediate</td>
<td>30-100</td>
<td>6</td>
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<td>Jovian</td>
<td>100-1000</td>
<td>14</td>
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<tr>
<td>Super-Jupiter</td>
<td>&gt; 1000</td>
<td>4</td>
</tr>
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</table>

Model predicts that planets with $M < 30 M_{\text{Earth}}$ account for over 75% of all planets.

- Complex structure, dominated by low mass planets
- Consistent w. non-detection of Jupiters around 90-95% stars.
- Maxima at masses similar to Solar System planets.
PIMF: Dependence on disk properties

Metallicity

Low mass planets
Udry, Mayor, Benz et al. 2006

No metallicity effect for Neptunes

[Fe/H] dist. of Hot Neptunes: flat!
[Fe/H] dist. of all known planets
P<20 d

- Metal poor systems produce more small bodies
- Minimum metallicity effect for Super-Earths & Neptunes

Fe/H mainly just scales

PIMF for giant planets:
Fe/H: threshold, but final mass not given by Fe/H
- higher number of giants
- but not more massive
Towards the **underlying** mass distribution

**Observation**

- Udry & Santos 2007
- All instruments
- HARPS (1 m/s)
- Observational bias

**Synthetic**

- 10 m/s (KS)
- 1 m/s
- 0.1 m/s
- Full Population

Hints of the Neptunian bump and the minimum at 30 $M_\oplus$?

**Dryness** of the planetary desert
Conclusions

★ The principles of core accretion explain in an **unified way** the formation of giant and terrestrial planets.
  ▶ No need for a **special** mechanism for giant planets. “Direct imaging” planets: core accretion + (scattering, tidal outward migration,..) ?

★ Improved/extended core accretion models form giant planets well **within disk lifetime**
  ▶ No need for a **faster** formation mechanism at small distances.

★ Improved/extended core accretion models allow **quantitative tests** with observations
  -for the giant planets of our **own** solar system
  -for **extrasolar** planets

★ **Population synthesis** is a powerful method for such tests
  ▶ The **whole** population of known planets can be used to constrain model parameters and to **improve** the models.
  ▶ Exploit the full observational investment to **feed back** on theory

★ **Populations obtained from extended core accretion models can reproduce** the most important observed properties & correlations in a quantitative significant way.
  ▶ This is not obvious as formation models are still a much simplified description .. much can be improved and extended
Thanks!