Multiscale Turbulence in Fusion and Gyrokinetics.

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What am I going to say?.

- The standard model of tokamak confinement and turbulence.
- Spatial scales -- time scales -- velocity space scales.
- Gyro-kinetic expansion.
- Electron-ion separation.
- Possible problems
  -- nonlocality, time/space
  -- loss of scale separation

BASIC PARAMETERS.
Plasma Major Radius 6.2m
Plasma Minor Radius 2.0m
Plasma Current 15.0MA
Toroidal Field on Axis 5.3T
Fusion Power 500MW
Burn Flat Top >400s
Power Amplification Q>10

Cost is > 12 Billion Euro.
Random walk:
Step = $\rho$, larmor/cyclotron radius.
Decorrelation rate = $\nu$ = collision rate
Radius of plasma = $a$.

$\tau_E \sim \frac{a^2}{\nu \rho^2}$

Collisions are rare and classical confinement can be very good.
Spitzer only needed $a = 20\text{cm}$, ($\tau_E>4\text{s}$) for IGNITION.
Can’t be right. Observed transport is much larger.

ITER neoclassical confinement time $\sim 2000\text{s}$
Density fluctuations

~ 4cm

DIII-D in San-Diego
Plasma is 1m across

Eddies are small compared to the device

Turbulence Imaging, Beam Emission Spectroscopy

George McKee and Ray Fonck
Gyro-kinetic simulation.

DIII-D Shot 121717

GYRO Simulation
Cray X1E, 256 MSPs

GYRO code simulations by Jeff Candy and Ron Waltz GA
Spatial scales

\[ L = \text{Equilibrium scale and parallel scale of turbulence} \]

\[ \rho = \text{Ion larmor radius and perpendicular scale of turbulence} \]

\[ \rho^* = \rho/L \sim 10^{-3} \text{ in ITER} \]
Turbulent Scales

Gyro-kinetic Fluctuations
-- Spatial Scales.

\[ V_{\text{flow}} = \frac{E \times B}{B^2} \]

Parallel scale >> Perpendicular scale, \( L_{\|} \gg L_{\perp} \sim \rho_i \)

Particle displacement, \( \xi_p \sim \) Of Order Field line displacement, \( \xi_B \sim O(L_{\perp}) \)
Typical time for fields to change is $L_{\parallel}/v_{\text{ion}}$ -- i.e. the time for an ion to go one parallel wavelength. Much longer than the gyration period. Ion senses the fields averaged over a **RING** of radius $\rho = v_{\perp}/\Omega_i$. 

\[ V_{\text{flow}} = \frac{E \times B}{B^2} \]

\[ \text{Ring radius} = \rho = v_{\perp}/\Omega_i \]
Guiding Centre Motion

Ring/Gyro-Center Motion

Ion position

"Ring" Center = R

Ring radius = ρ = v⊥/Ω⊥

\[ \frac{d\mathbf{R}}{dt} = v_{\parallel} \mathbf{b}_0 - \frac{\partial}{\partial \mathbf{R}} \left( \mathbf{b}_0 \right) \times \left( \frac{\mathbf{b}_0}{B_0} \right) + v_{\parallel}^2 \left( \frac{\mathbf{b}_0}{\Omega_0} \right) \times \mathbf{b}_0 \cdot \nabla \mathbf{b}_0 + \frac{v_{\parallel}^2}{2B_0} \left( \frac{\mathbf{b}_0}{\Omega_0} \right) \times \nabla B_0. \]

Parallel Motion along Equilibrium Field
Drifts in Perturbed Field
Curvature Drift in Equilibrium Field
VB Drift in Equilibrium Field

\[ \chi = \phi - \mathbf{v} \cdot \mathbf{A} \]
Ambient Gradient Argument

\[ \nabla \delta n \sim \nabla n_0 \]
\[ \frac{\delta n}{\rho} \sim \frac{n_0}{L} \]
\[ \rightarrow \frac{\delta n}{n_0} \sim \frac{\rho}{L} = \rho^* \]

Density \( n(r) \)

Mean Density \( n_0(r) \)

\[ \frac{df}{dt} = 0 \to \text{Collisionless mixing produces finite fluctuations -- collisions are essential.} \]

WARNING without collisions
Energy Confinement -- Random walk of heat/particles.

$L = \text{typical machine size}$

$\Delta = \text{radial eddy size } \propto \text{ion larmor radius } \rho_i = \text{random step.}$

$N = \text{number of steps to random walk out of plasma}$

$$L \sim \sqrt{N} \rho_i$$

$$\rightarrow N = \left( \frac{L}{\rho_i} \right) = \left( \frac{1}{\rho^*} \right)^2$$

For ITER $N \sim 10^6$. 
Energy Confinement -- Random walk of heat/particles.

Eddy turnover time =

$$\tau_{eddy} = \left( \frac{L}{v_{thi}} \right)$$

$$\tau_E \sim N\tau_{eddy} \sim \left( \frac{L^3}{\rho_i^2 v_{thi}} \right) \propto L^3 B^2 T^{-1}$$

Dramatic scaling with size!
Scaling approximately agrees with data BUT geometry dependant.
Timescales -- ITER numbers -- ordering

Cyclotron -- $\sim 4 \times 10^{-9}$ s (ions) -- no activity

Turbulence -- $\sim 10^{-5}$ s -- fluctuating density, temperature, $f$, $E$.

Collisions -- $\sim 10^{-2}$ - $10^{-3}$ -- reestablish Maxwellian

Transport -- $\sim 4$ s -- change mean density, temperature

\[
\frac{1}{\rho^*} \frac{1}{\Omega_i}
\]

\[
\left(\frac{1}{\rho^*}\right) \frac{1}{\Omega_i}
\]
\[ f(r, v, t) = \frac{F_0(r, v, t)}{1} + \frac{q\phi}{T} F_0 + h(R, \mathcal{E}, \mu, t) + \frac{\delta f_2(r, v, t)}{(\rho^*)^2} + \ldots \]

Varies on slow space and timescales
Varies on faster turbulence space and timescales

\[ B(r, t) = B_0(r, t) + \delta B(r, t), \quad E(r, t) = \delta E(r, t) \]

At \( \mathcal{O}(1) \) collisions establish a local Maxwellian.

\[ F_0(r, v, t) = n(t, \psi) \left( \frac{m}{2\pi T(t, \psi)} \right)^{3/2} \exp \left[ -\frac{(1/2)mv^2}{T(t, \psi)} \right] \]

Key issue is to find the slow evolution of \( n(t, \psi) \) and \( T(t, \psi) \)
Solving for Fine Scale Turbulence

Evolution of turbulence comes from solving the gyro-kinetic equation:

\[ \frac{\partial h}{\partial t} + v_{\parallel} \frac{\partial h}{\partial Z} + v_{D} \cdot \nabla \frac{\partial}{\partial R} - \frac{\partial <\phi>}{\partial R} \times \left( \frac{b_0}{B_0} \right) \cdot \nabla \frac{\partial}{\partial R} - \left\langle C(h) \right\rangle = q \frac{F_0}{T_0} \frac{\partial \langle \phi \rangle}{\partial t} - \frac{\partial <\phi>}{\partial R} \times \left( \frac{b_0}{B_0} \right) \cdot \nabla \frac{\partial F_0}{\partial R} \]

Comes from ring averaged kinetic equation \( O(\rho^*) \)

Solved with quasi neutral approximation:

\[ \nabla^2 \phi = -\frac{1}{\epsilon_0} (q n_i - e n_e) \]

In principle this equation should be solved everywhere
With \( F_0 \) held fixed in time letting \( h \) converge to statistical equilibrium.

With weak collisions \( h \) becomes very fine scale in velocity space due to mixing.
Solving for Slow Evolution

\[ \mathcal{O}((\rho^*)^2) \]

\[ \frac{\partial F_0}{\partial t} + \frac{\partial \delta f_2}{\partial t} + \ldots \]

To solve for time evolution of \( F_0 (n \& T) \) we have to annihilate the fast varying parts in space and time. It is easier to first take moments and use exact moment equations.

\[ \frac{\partial n_i}{\partial t} + \nabla \cdot \Gamma_i = 0. \]

\[ \Gamma_i = \int v f_i d^3v. \]

Flux comes From turbulence.

Average flux over annulus (of thickness much greater than turbulence scale much smaller than radius) and time To get mean evolution of \( n \) and \( T \).
Is the turbulence in the flux tube determined by the conditions in the tube?

Does turbulence propagate -- correlated or not?

We should look at turbulent Greens function -- response to stirring.

Beer 1993.
What should we do?

- Transport barriers -- can we use the “standard model”.

- Detailed check of “standard model” needed.

- Full coupling -- (e.g. Trinity) needs finishing.

- The big prize is to find better configurations.