Rotation and zonal flows in stellarators

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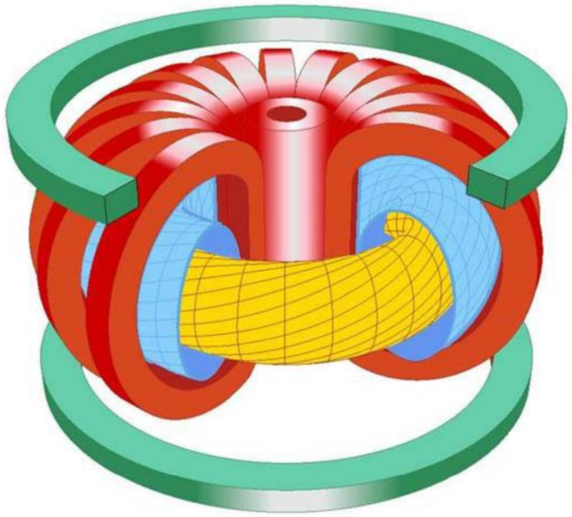
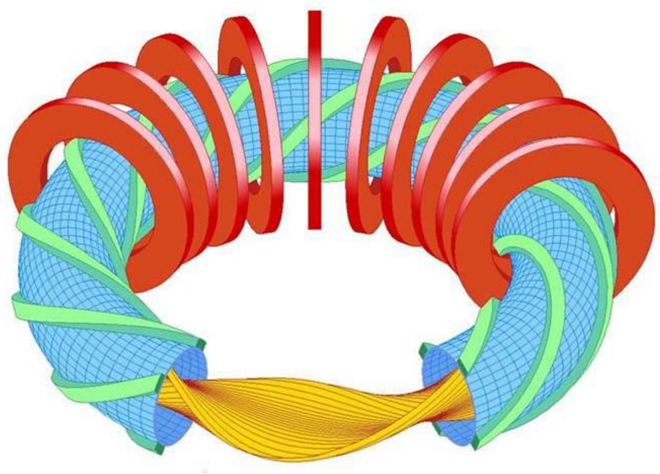
What is a stellarator?

- In a tokamak (Tamm and Sakharov, 1951), the magnetic field lines twist around the torus because of the toroidal plasma current.
- In the stellarator (Spitzer, 1951), this twist is imposed by external coils.
  - Magnetic field is necessarily 3D.
  - No toroidal current is necessary.
  - Less „free energy“ in the plasma.
  - Greater degree of control

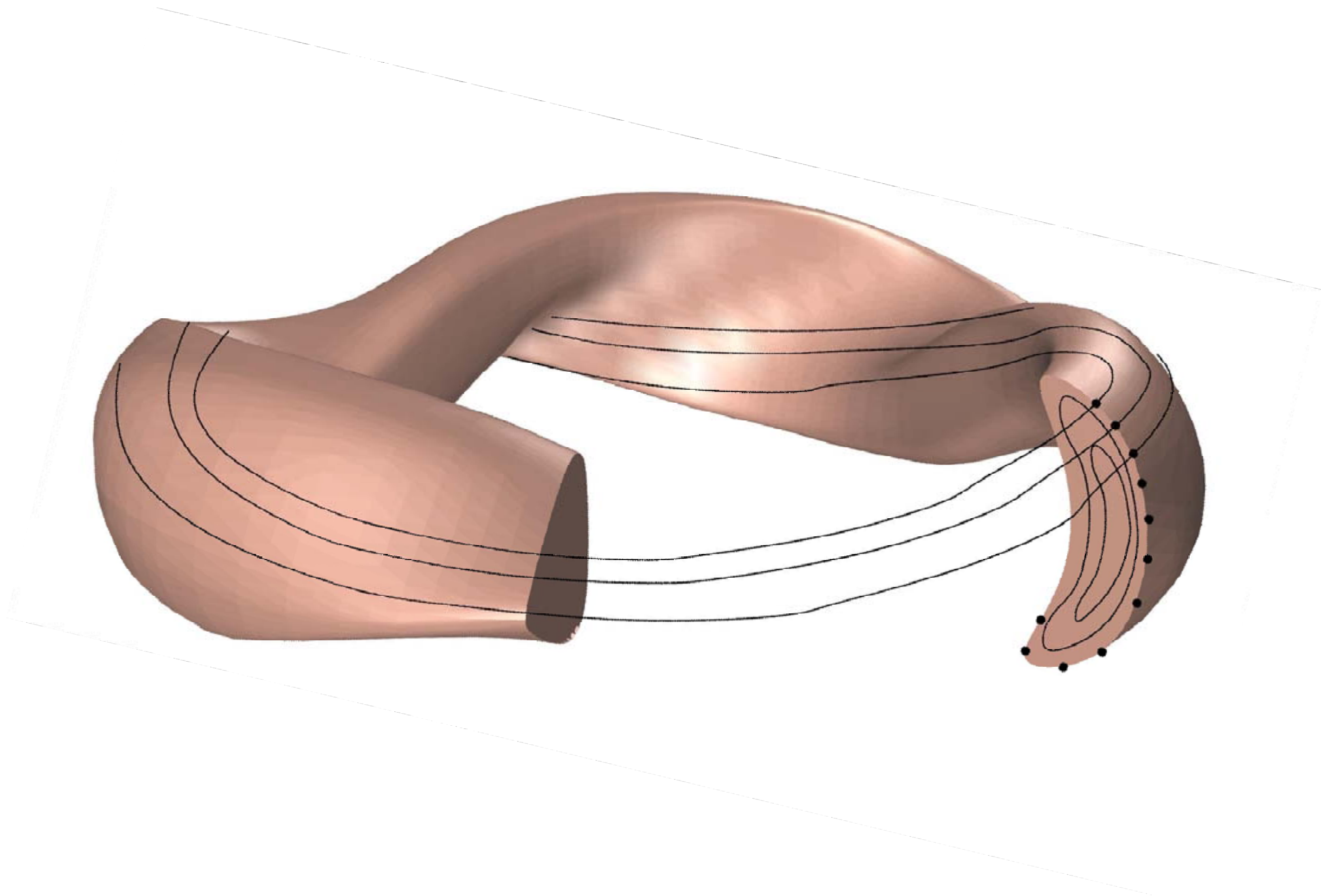


Tokamak



Stellarator

Theoretician's stellarator



Questions in this talk

- Can a stellarator plasma rotate?
  - Effect of turbulence?

- If so,how quickly?

  - Fast rotation

$$V \sim v_{Ti} = \text{ion thermal speed}$$

  - Slow rotation

$$V \sim \delta v_{Ti}, \qquad \delta = \frac{\rho_i}{L}$$

- Empirically
  - in tokamas, toroidal rotation tends to be fast and poloidal rotation slow
    - expected theoretically
  - in stellarators, the rotation tends to be slow

Fast rotation

Theorem I:

Fast equilibrium rotation is only possible in certain (so-called quasisymmetric) magnetic fields if $\delta \ll 1$.

This conclusion holds independently of any turbulence that may be present, as long as the fluctuations are small.

Proof: The Vlasov equation

$$\frac{\partial f}{\partial t} + (\mathbf{V}+\mathbf{v})\cdot\nabla f + \frac{e}{m}\left(\mathbf{E} + (\mathbf{V}+\mathbf{v})\times\mathbf{B} - \frac{\partial \mathbf{V}}{\partial t} - (\mathbf{V}+\mathbf{v})\cdot\nabla\mathbf{V}\right)\cdot\frac{\partial f}{\partial \mathbf{v}} = C(f)$$

and the ordering

$$\Omega^{-1}\frac{\partial}{\partial t} \ll \rho_i \nabla \ll 1$$

lead to the drift kinetic equation (Hazeltine and Ware, 1978)

$$\frac{\partial f_0}{\partial t} + (v_\parallel \mathbf{b} + \mathbf{V})\cdot\nabla f_0 + \dot{w}\frac{\partial f_0}{\partial w} = C(f_0)$$

$$\mathbf{V}(\mathbf{r},t) = V_\parallel \mathbf{b} + \frac{\mathbf{B}\times\nabla\Phi_0}{B^2}$$

$$\dot{w} = eE_\parallel v_\parallel - m v_\parallel \mathbf{V}\cdot\nabla\mathbf{V}\cdot\mathbf{b} - m v_\parallel^2 \mathbf{b}\cdot\nabla\mathbf{V}\cdot\mathbf{b} + \mu B \mathbf{V}\cdot\nabla \ln B$$

- An equilibrium with

$$\frac{\partial}{\partial t} \ll \frac{\delta v_T}{L}$$

must have the the following properties:
  - isothermal flux surfaces
  - either |**V**| << ion thermal speed
  - or

$$|\mathbf{B}| = f(\psi, l) + O(\delta) \text{ corrections}, \qquad l = \text{ arc length along } \mathbf{B} \qquad (1)$$

- Since this follows in 0th order in δ, it is independent of any turbulence!

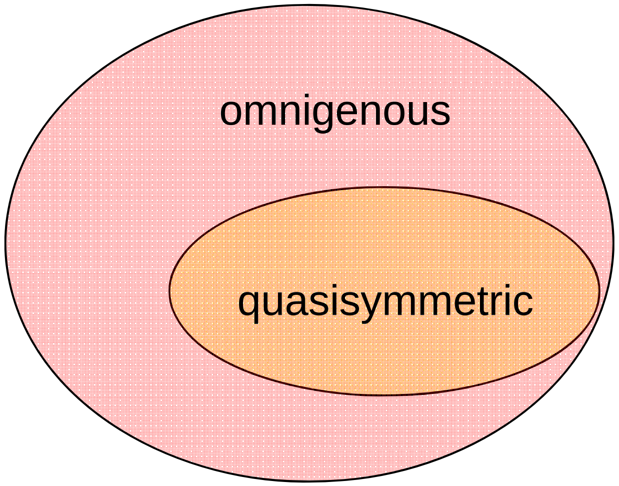
- In a scalar-pressure equilibrium, (1) means that <u>B is quasisymmetric</u>.

Special magnetic fields:

- Quasisymmetric
  - |B| symmetic in Boozer coordinates:

$$B = B(\psi, m\theta - n\varphi)$$

  - Neoclassical properties identical to those in a tokamak
  - Isomorphism between stellarator and tokamak drift kinetic equations (Boozer 1984)

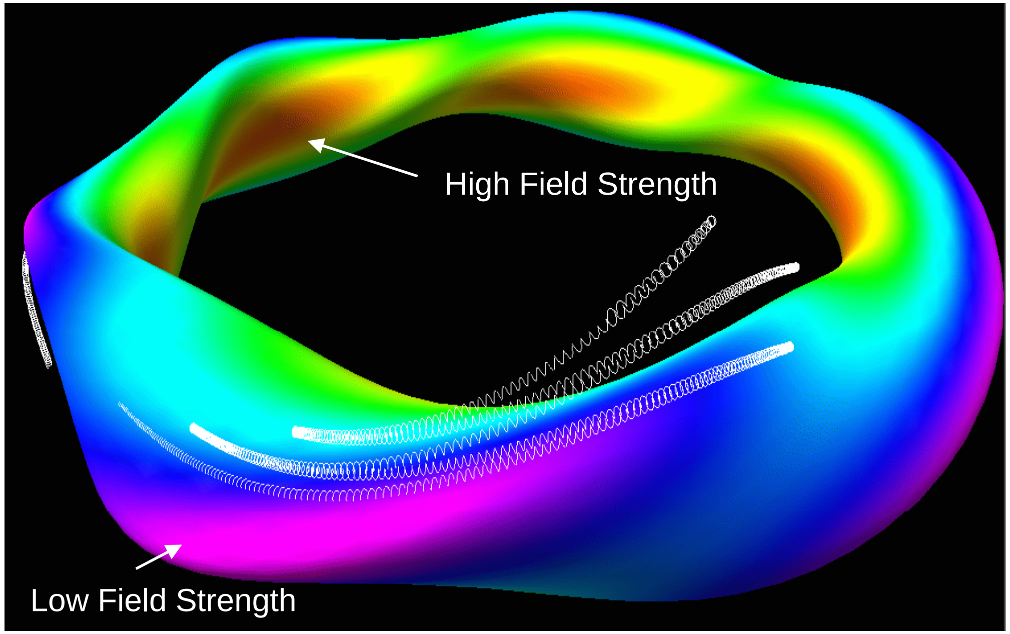


- Omnigenous (Hall and McNamara 1972)
  - No radial magnetic drift on a bounce average

$$\int_0^{\tau_b} \mathbf{v}_d \cdot \nabla\psi \; dt = 0$$

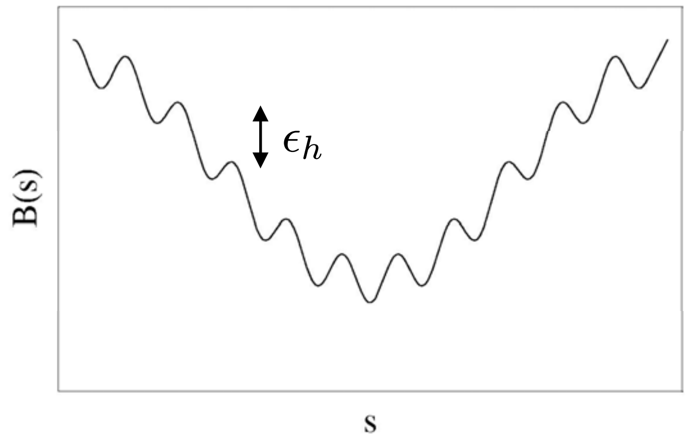
Particle orbits

- Particle motion in WEGA (an unoptimised stellarator)



Transport due to helically trapped particles

- Complicated parallel variation of the magnetic field strength
  - Local trapping
  - Unconfined orbits



- Random walk:
  - Step length $\Delta r \sim v_d \Delta t$
  - Time between steps $\Delta t \sim \epsilon_h / \nu$
  - Diffusion coefficient

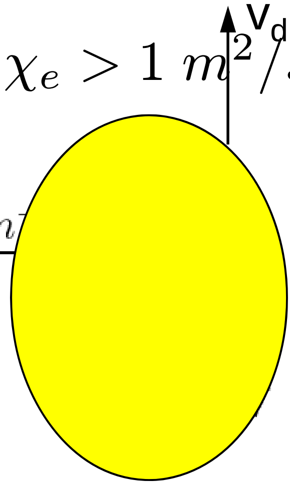
$$D_{1/\nu} \sim \epsilon_h^{1/2} \frac{\Delta r^2}{\Delta t} \sim \frac{\epsilon_h^{3/2} v_d^2}{\nu}$$

  - Always dominates over turbulence at high enough temperature, since

$$D_{1/\nu} \propto \frac{m^{1/2} T^{7/2}}{n B^2 R^2}$$

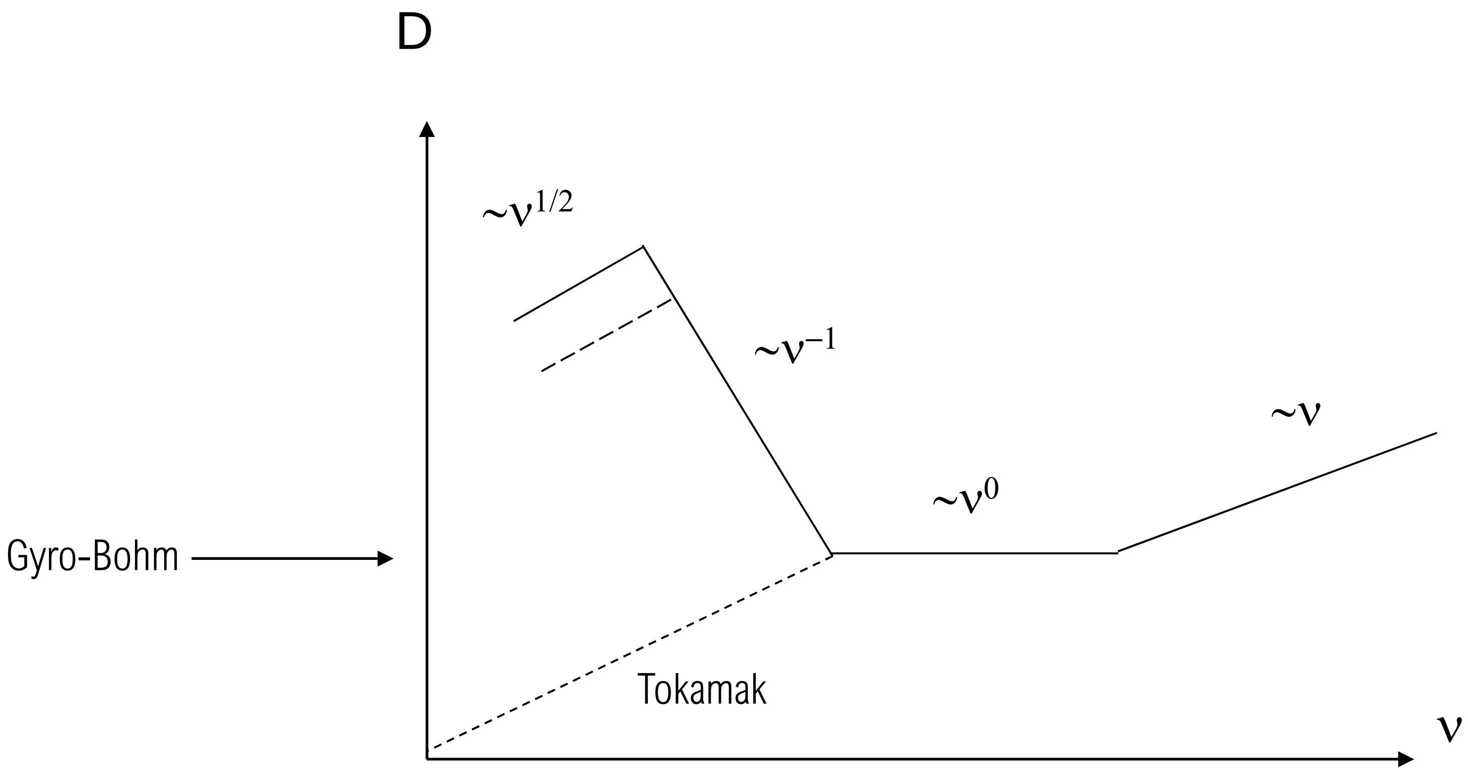
Temperatures beyond which

$$\chi_e > 1\ m^2/s$$



n (10^{20} m	W7-X
0.1	3.50 keV
0.4	5.20 keV
1.0	6.70 keV

Transport vs collisionality



Slow rotation

Theorem II:

Gyrokinetic turbulence cannot affect the macroscopic equilibrium rotation in a stellarator, except if B is quasisymmetric. This rotation is instead determined by neoclassical theory.

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Projections of the momentum equation

$$\frac{\partial\langle\rho\mathbf{V}\cdot\mathbf{J}\rangle}{\partial t} = -\left\langle\nabla\cdot(\rho\mathbf{V}\mathbf{V}+\pi+\mathsf{M})\cdot\mathbf{J}\right\rangle$$

$$\frac{\partial\langle\rho\mathbf{V}\cdot\mathbf{B}\rangle}{\partial t} = -\left\langle\nabla\cdot(\rho\mathbf{V}\mathbf{V}+\pi+\mathsf{M})\cdot\mathbf{B}\right\rangle$$

where

$$\rho\mathbf{V}\mathbf{V} = \text{Reynolds stress}$$

$$\pi = \text{viscous stress}$$

$$\mathsf{M} = \frac{1}{\mu_0}\left(\frac{\tilde{\mathsf{B}}^2}{2}\mathsf{I}-\tilde{\mathbf{B}}\tilde{\mathbf{B}}\right) = \text{Maxwell stress}$$

The gyrokinetic ordering

$$\frac{V}{v_{Ta}} \sim \frac{\tilde{f}_a}{f_a} \sim \frac{\tilde{B}}{\beta B} \sim \frac{e_a \tilde{\phi}}{T_a} \sim \delta \ll 1, \qquad k_\perp \rho_i = O(1)$$

gives

$$\nabla \cdot \pi_{\rm neocl} \sim \nabla \cdot (\rho \mathbf{V} \mathbf{V})_{\rm turb} \sim \nabla \cdot \mathsf{M}_{\rm turb} \ll \nabla \cdot \pi_{\rm turb}$$

$$\pi_{\rm neocl} = (p_\| - p_\perp)(\mathbf{bb} - \mathsf{I}/3) \sim O(\delta p)$$

The turbulent gyroviscous force dominates locally.

- However, on a volume average over a volume ΔV
  - between two flux surface several gyroradii apart

$$\rho \ll \Delta r \ll r$$

  - neoclassical viscosity dominates

$$\frac{\partial}{\partial t} \int_{\Delta V} \rho \mathbf{V} \cdot \begin{pmatrix} \mathbf{J} \\ \mathbf{B} \end{pmatrix} dV \simeq \int_{\Delta V} \pi_{\mathrm{neocl}} : \nabla \begin{pmatrix} \mathbf{J} \\ \mathbf{B} \end{pmatrix} dV$$

- Hence, gyrokinetic turbulence cannot affect macroscopic rotation!
  - one exception…

Non-ambipolar neoclassical transport

- Toroidal plasma rotation corresponds to a radial electric field

$$V_\varphi \sim E_r / B_p$$

- If the neoclassical transport is not intrinsically ambipolar, there is a radial current

$$J_r \sim -neD \left(\frac{d \ln n}{dr} + \frac{e}{T} \frac{d\phi}{dr} \right), \qquad D \sim \nu_i \rho_i^2$$

producing a toroidal torque

$$J_r \times B_p \sim \nu_i \rho_i^2 \frac{ne^2 E_r B_p}{T}$$

that slows down the rotation on the time scale

$$\frac{m_i n_i V_\varphi}{J_r \times B_p} \sim \nu_i^{-1}$$

Intrinsic ambipolarity

- The radial neoclassical current

$$\langle \mathbf{J} \cdot \nabla \psi \rangle = \langle \pi_{||} : \nabla \mathbf{J} \rangle / p'(\psi)$$

vanishes in case of intrinsic ambipolarity (for any E_r).

- In what configurations does intrinsic ambipolarity hold?

- **Theorem III:** B is intrinsically ambipolar if, and only if, it is quasisymmetric.

Boozer, PoP 1983 (→ "if") our present concern (→ "only if")

- **Proof:** From the drift kinetic equation

$$v_{||}\nabla_{||}\bar{f}_{a1} + \mathbf{v}_d \cdot \nabla f_{a0} = C_a(\bar{f}_{a1}) \qquad (1)$$

In an isothermal plasma follows an entropy production law

$$\phi_0'(\psi)\left\langle \mathbf{J}\cdot\nabla\psi \right\rangle = \sum_a T_{a0}\left\langle \int d^3v \bar{f}_{a1} C_a(\bar{f}_{a1})/f_{a0} \right\rangle \le 0.$$

But then the H theorem implies

$$\left\langle \mathbf{J}\cdot\nabla\psi \right\rangle = 0 \quad \Rightarrow \quad \bar{f}_{a1} = (\alpha_a + \beta_a v_{||} + \gamma_a v^2) f_{a0}$$

Re-insert into (1):

$$\nabla_{||}\left(\frac{(\mathbf{B}\times\nabla\psi)\cdot\nabla \ln B}{\nabla_{||} B} \right) = 0 \qquad \Rightarrow \qquad \text{B is quasisymmetric}$$

Proximity to quasisymmetry

- How close to perfect quasisymmetry must a stellarator come, in order to have tokamak-like rotation?
  - depends on collisionality and on the radial length scale considered

- In the $1/\nu$ regime, the diffusion coefficient is

$$D \sim \epsilon_h^{3/2} \delta^2 \frac{T_i}{m_i \nu_i}$$

- The current becomes

$$\langle \mathbf{j} \cdot \nabla \psi \rangle \sim \epsilon_h^{3/2} \delta_i^2 \frac{p_i \Omega_i}{\nu_i}$$

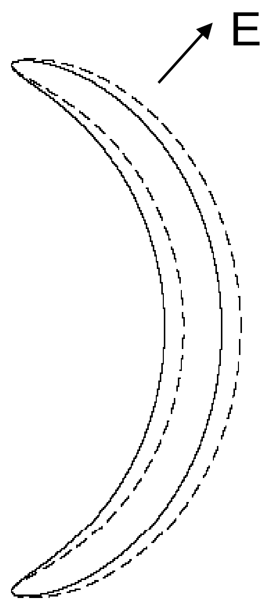
and its torque exceeds the Reynolds stress, averaged over the volume between two flux surfaces N ion gyroradii apart if

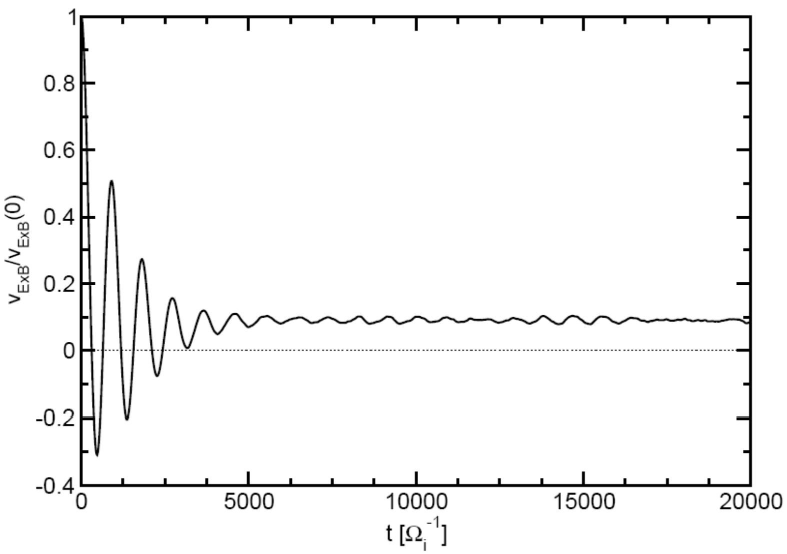
$$\epsilon_h > \left(\frac{\nu_*}{N} \right)^{2/3}$$

Zonal flows

Zonal flow oscillations

- Rosenbluth-Hinton problem:
  - perturb the plasma density and radial electric field at t=0
  - watch the linear evolution
- Analytical preditiction:
  - geodesic acoustic modes (GAMs)
  - damping of initial perturbation because of banana-orbit polarisation
  - finite residual perturbation level as $t \to \infty$
- Qualitatively different in stellarators
  - different types of orbits
  - end state oscillatory





Mathematical problem

- The linearised drift kinetic equation

$$\frac{\partial f_{a1}}{\partial t} + (v_\parallel \mathbf{b} + \mathbf{v}_d) \cdot \nabla f_{a1} = -\frac{e_a \phi'}{T_a} (\mathbf{v_d} \cdot \nabla r) f_{a0}$$

- is to be solved on time scales exceeding the bounce time, coupled to gyrokinetic quasineutrality

$$\sum_a \left\langle n_a e_a + \nabla \cdot \left(\frac{m_a n_a \nabla_\perp \phi}{B^2} \right) \right\rangle = 0$$

$$\frac{\partial n_a}{\partial t} = -\frac{1}{V'} \frac{\partial}{\partial r} \left\langle V' \int f_a (\mathbf{v}_d \cdot \nabla r) d^3 v \right\rangle$$

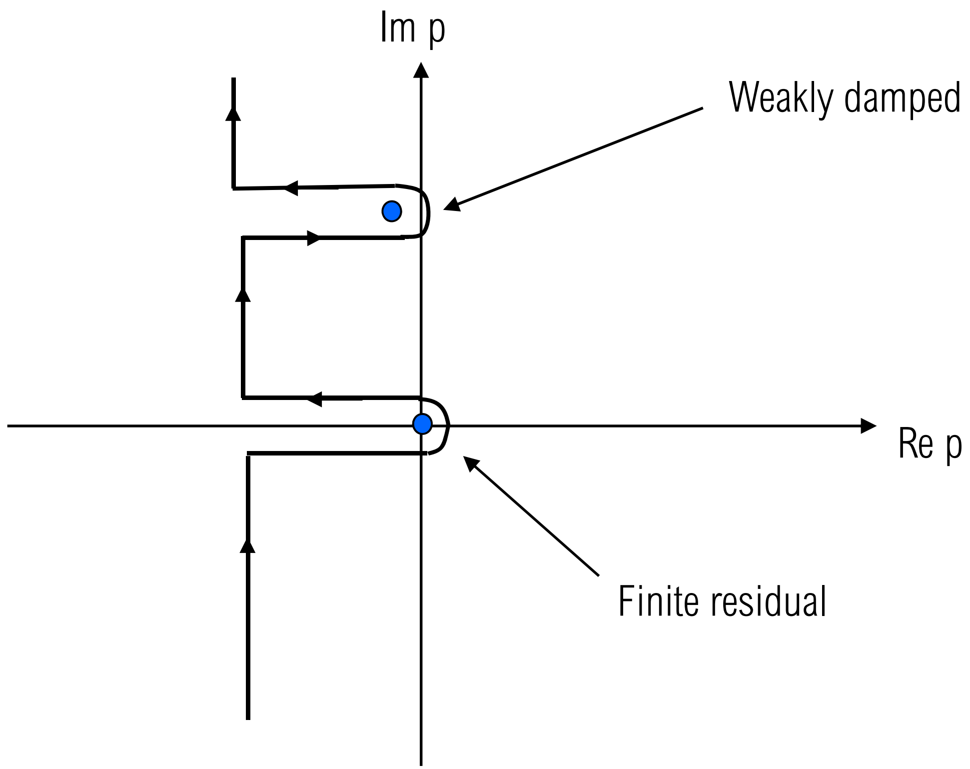
- Laplace transformation gives

$$L(p) \hat{\phi}'(p) = \phi_0'$$

$$L(p) = p + \sum_a \frac{e_a^2}{T_a} \left\langle \int f_{a0} \frac{\bar{v}_r^2 + p^2 \delta_r^2}{p + ik\bar{v}_r} d^3 v \right\rangle / \left\langle \frac{|\nabla r|^2}{B^2} \right\rangle \sum_a m_a n_a$$

$$\mathbf{v}_d \cdot \nabla r = \bar{v}_r + v_\parallel \nabla_\parallel \delta_r$$

Inversion of the Laplace transform



$$\phi(t) = \frac{\phi_0'}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{e^{pt}dp}{L(p)} = \phi_0'\sum_j a_j e^{p_j t}$$

$$a_j = \mathrm{Res}\left[1/L(p_j)\right]$$

Zonal flow oscillations

- Analytical predictions:
  - Oscillation frequency < GAM frequency and

    $$\Omega \to 0$$

    in the limit of perfect confinement.
  - Landau damping of zonal flow oscillations. Damping rate

    $$\gamma \sim \exp(-\Omega / k\bar{v}_r)$$

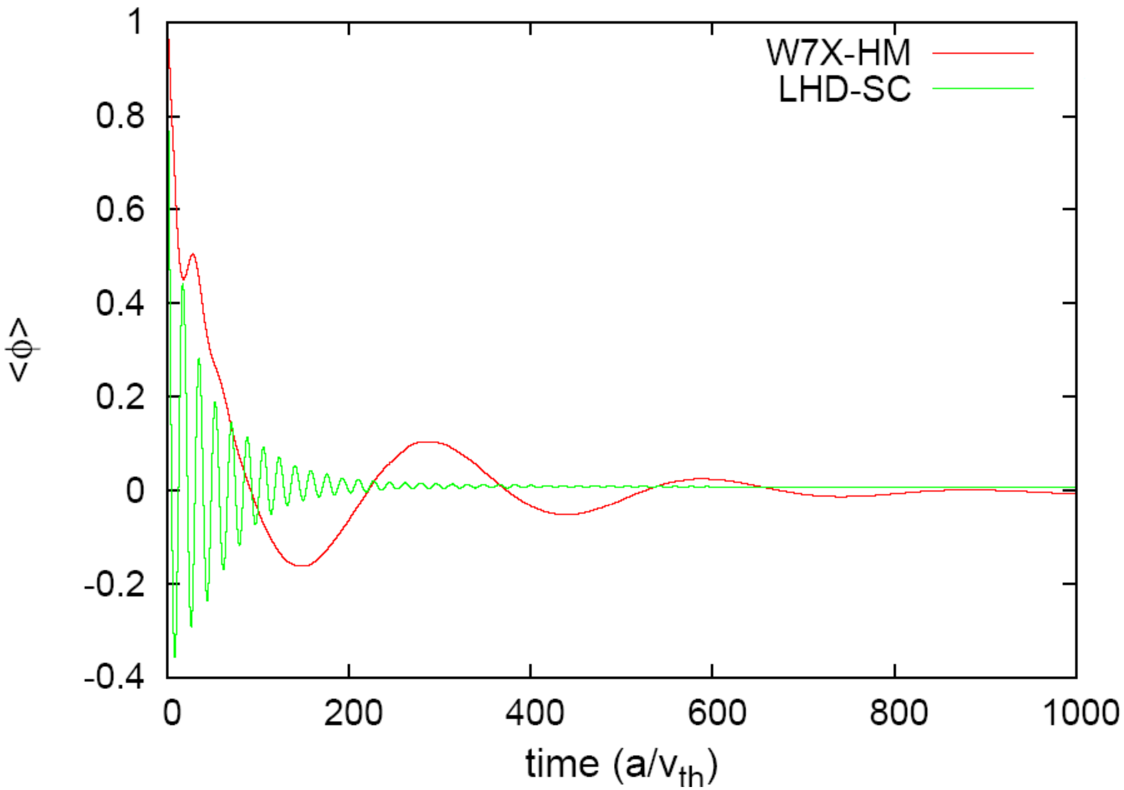
    is sensitive to magnetic geometry: higher in LHD than in W7-X
  - Residual level dependent on radial wavelength

    $$\lim_{t\to\infty} \frac{\phi(t)}{\phi_0} = \left(1 + \frac{\alpha q^2}{\epsilon^{1/2}} + \frac{\beta \epsilon^{1/2}}{k^2 \rho_i^2}\right)^{-1}$$

- Confirmation by EUTERPE and GENE:
  - differences between LHD and W7-X qualitatively in line with expectations

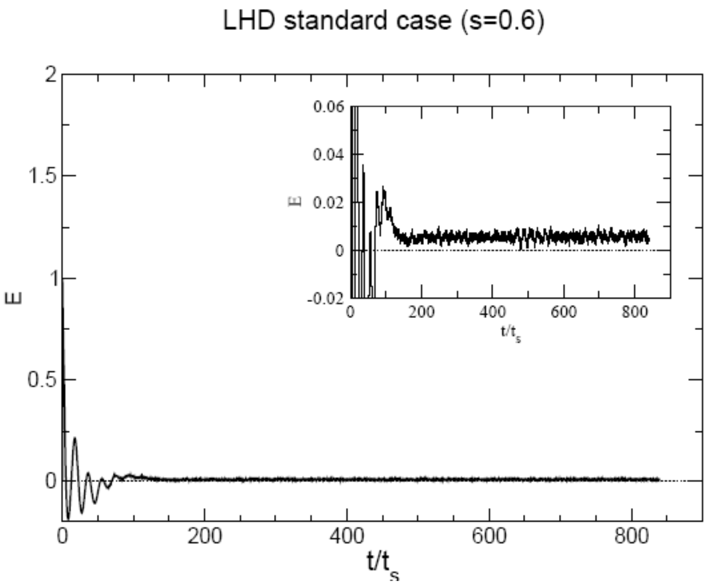
Zonal flows and GAMs in W7-X and LHD

- LHD
  - GAMs weakly Landau damped
  - Zonal-flow oscillations strongly damped
- W7-X:
  - GAMs strongly Landau damped
  - Clear zonal-flow oscillations
    - Landau damping of these

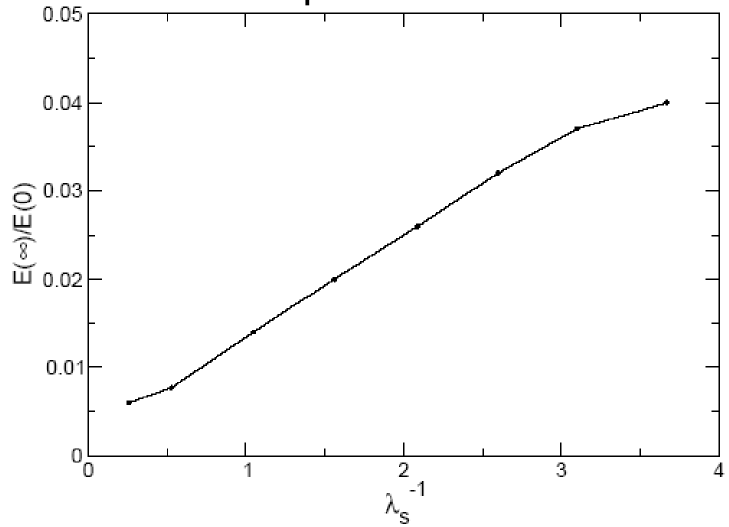


GENE Simulations

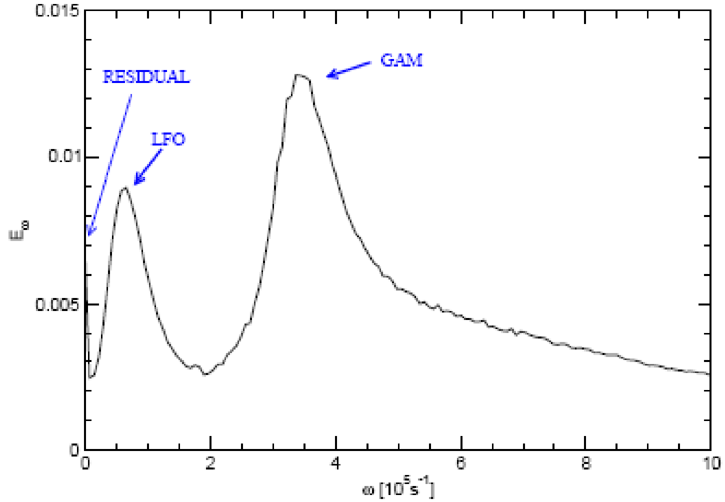
LHD (global simulations)



Dependence on k

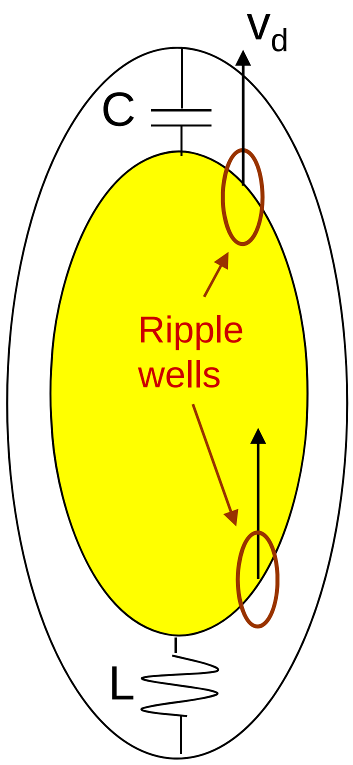


Fourier transform



$$\lim_{t\to\infty}\frac{\phi(t)}{\phi_0}=\left(1+\frac{\alpha q^2}{\epsilon^{1/2}}+\frac{\beta\epsilon^{1/2}}{k^2\rho_i^2}\right)^{-1},$$

- Consider the plasma between two flux surfaces



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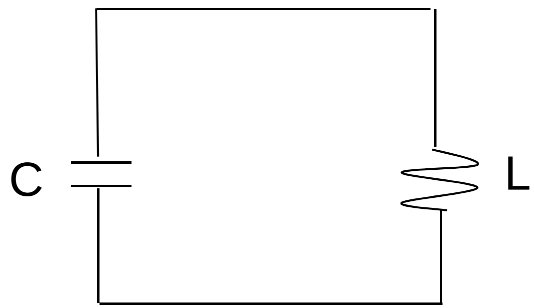
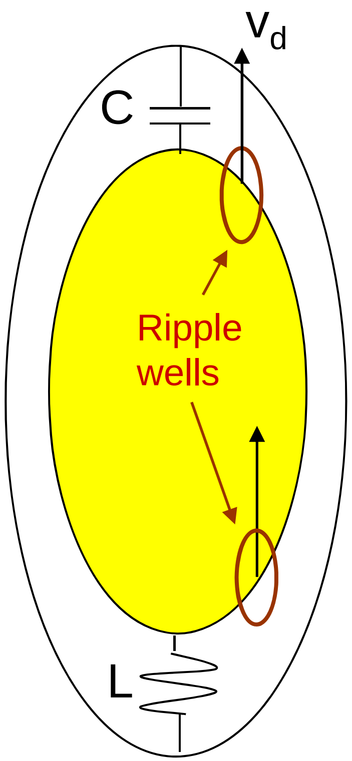


$$i(t) = L \int_0^t u(t')dt'$$

Physical picture

- Consider the plasma between two flux surfaces



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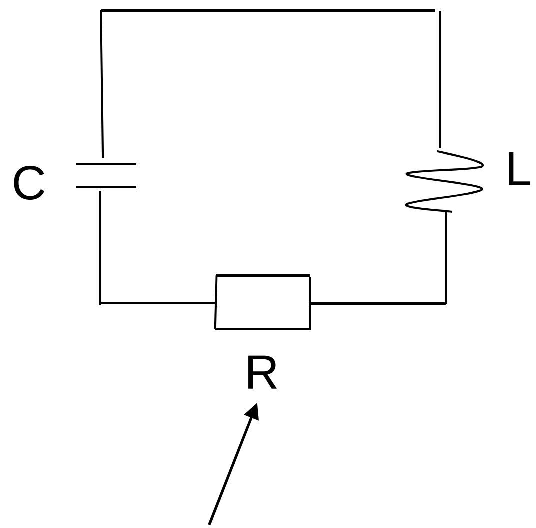


$$i(t) = L \int_0^t u(t')dt'$$

Summary

- Only in quasi-symmetric configurations can the plasma rotate rapidly or freely
  - very robust result: insensitive to turbulence

- In all other stellarators
  - E_r and rotation are clamped at the value required for neoclassical ambipolarity
  - gyrokinetic turbulence only matters on small scales
  - momentum transport unimportant on large scales
  - It is much easier to calculate E_r in a stellarator than in a tokamak!

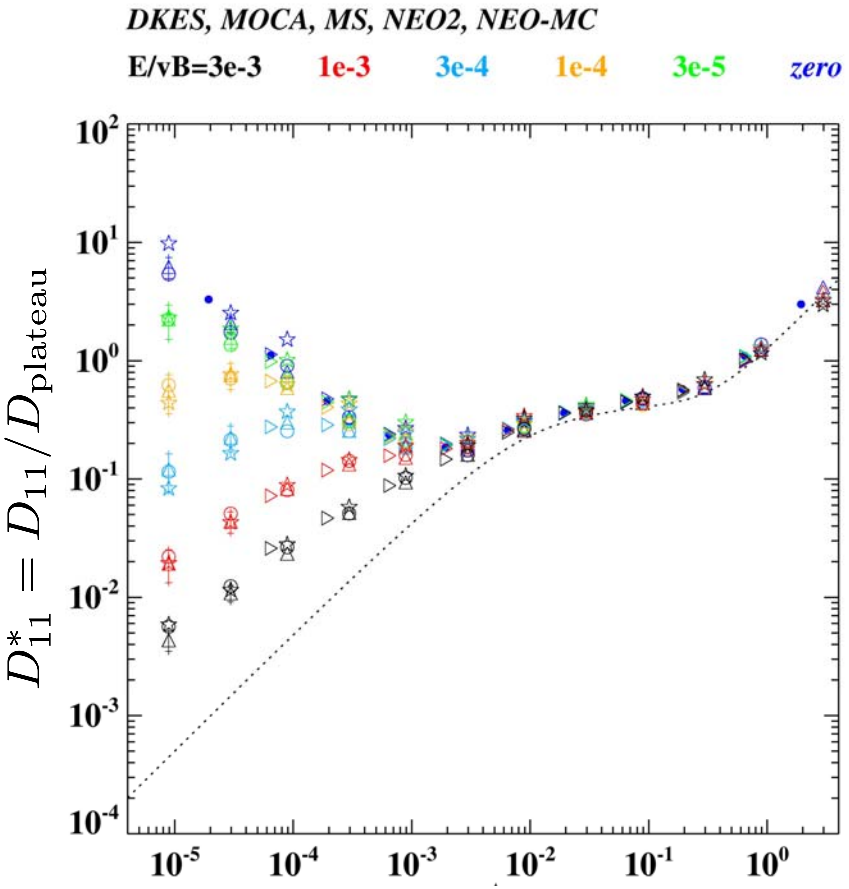
- Linear zonal-flow physics qualitatively different in stellarators
  - oscillatory response to an applied electric field

References:

- Theorem I: Helander PoP 2007
- Theorems II and III: Helander and Simakov PRL 2008
- Zonal flows: Sugama, Watanabe et al, PoP and PRL 2005-2010,
  Mischenko, Helander and Könies, PoP 2008
  Helander, Mischenko, Kleiber and Xanthopouls, unpublished (2010)

Extra material

- Monoenergetic particle diffusivity



$$D_{11}^* = D_{11}/D_{\mathrm{plateau}}$$

$$\nu^* = \frac{\nu R_0}{\iota v}$$

Tokamak rotation

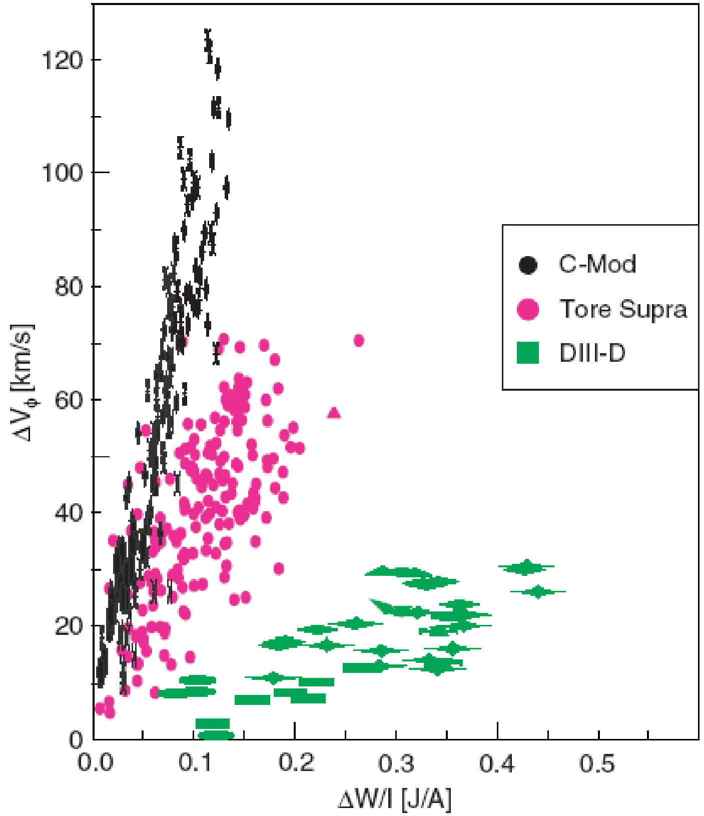
- Tokamak plasmas rotate freely, even spontaneously, in the toroidal direction.
  - Mach numbers ~1/3, up to ~1 in spherical tokamaks with unbalanced NBI

- Poloidal rotation much slower
  - Damped by collisions
  - Friction between trapped and passing ions

- Toroidal rotation determined by E_r

$$\mathbf{V} = u(\psi)\mathbf{B} - \left(\frac{d\Phi}{d\psi} + \frac{1}{ne}\frac{dp_i}{d\psi} \right) R\hat{\varphi}$$

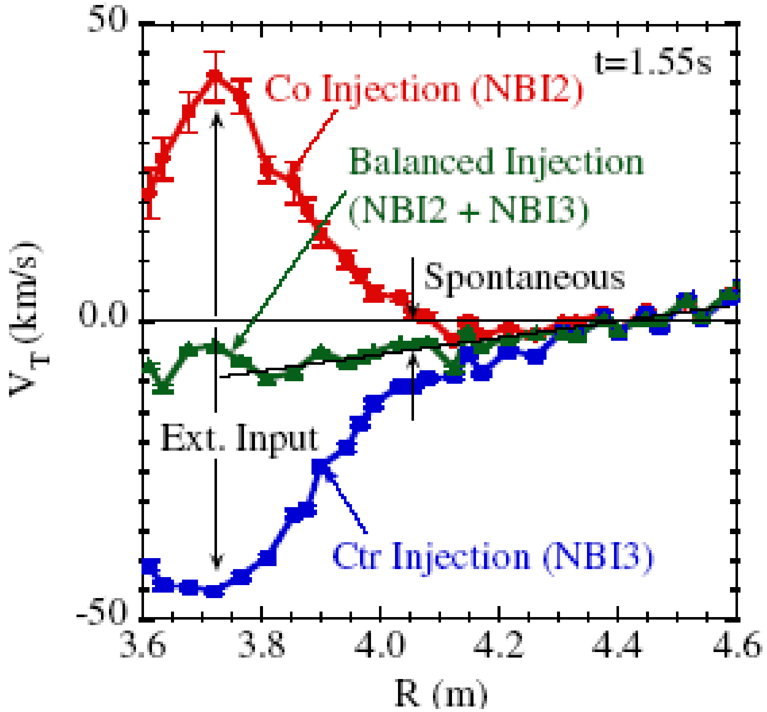
small



Rice et al, Nucl. Fusion 2007

Rotation in LHD

- Much smaller rotation in LHD



Yoshinuma et al, Nucl. Fusion 2009