Issues with convection: What is a useful framework beyond bulk models of large N, non-interacting, scale-separated, equilibrium systems?

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Outline

- Introduction: current treatments of convection
- Concerning bulk models
- Concerning $N \not\to \infty$
- Concerning spatial correlations
- Concerning time-dependence and equilibrium
- Summary

Current treatments of convection

The cumulus ensemble

The Arakawa and Schubert (1974) picture

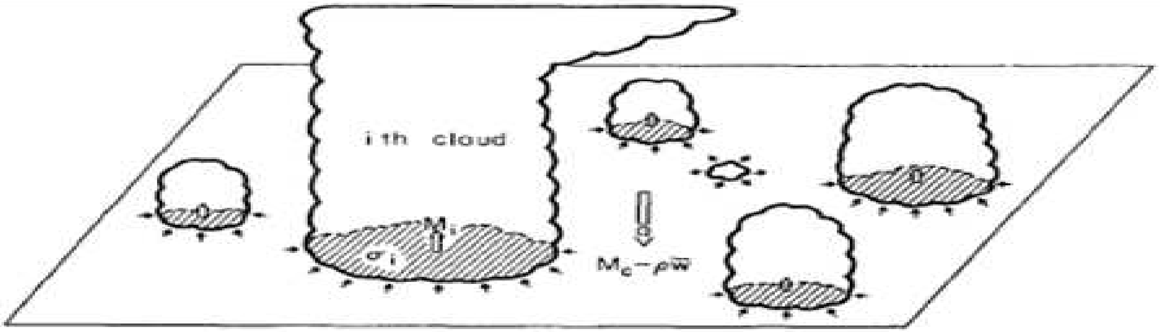
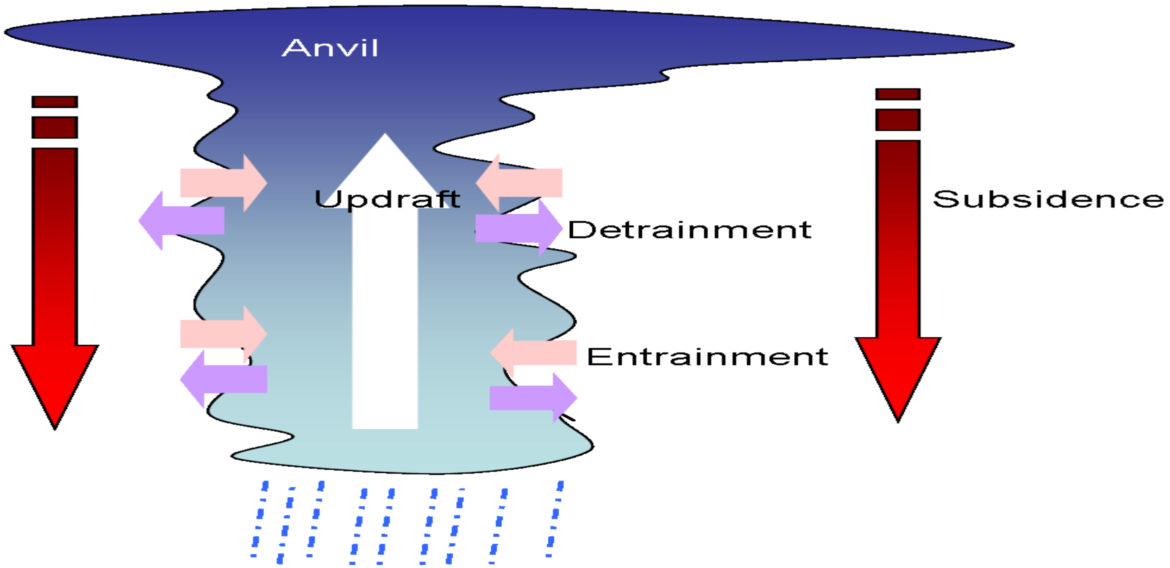


FIG. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

- Convection characterised by ensemble of cumulus clouds
- Scale separation in both space and time between cloud-scale and the large-scale environment

Entraining/detraining “plume”



Key variable is the mass flux,

$$M_i = \rho \sigma_i w_i$$

$$\rho \overline{\chi' w'} \approx \sum_i M_i (\chi_i - \chi_{\mathrm{env}})$$

Equations for a plume

- Various plume models based on this picture
- Differ in formulation of entrainment/detrainment and microphysics
- Integrate from cloud base up to terminating level where the in-cloud buoyancy vanishes
- Hardest part is the lower-boundary condition, or closure: i.e., what is the mass flux at cloud base for each i?

Closure

- The convection is being forced by some large-scale processes that act to destabilize the atmosphere
- If convection occurs, it will act to try to restore stability
- At equilibrium, the large-scale and convective tendencies are in balance

Concerning bulk models

Bulk parameterizations

- A more common approach in practice (MetUM, ECMWF, WRF...)
- Start from the plume equations, and sum over plumes
- Get back essentially the same equations with in-plume values replaced by bulk values,

$$\chi_B = \frac{\sum_i M_i \chi_i}{\sum_i M_i}$$

Just one “bulk plume” now, so all is much simpler...

The price of a bulk scheme

A bulk method works because the plume equations are (almost!) linear

- Hinges on an extra "gross assumption" about the detraining cloud liquid water

  assumed equal to bulk value, which means condensate detrainment is systematically overestimated in a bulk model

- Linearity is needed in the microphysics and radiation terms

  By construction, cumulus microphysics and cumulus-radiation interactions are supposed to be very crude

- No simplification occurs for chemical transports

Concerning $N \not\to \infty$

Typical N values

- Convective instability is released in discrete events
- A typical mass flux for one cloud is $\sim 10^7$kgs^{-1}
- To stabilize a typical convective forcing in the tropics needs a total mass flux of $\sim 10^{-2}$kgm^{-2}s^{-1}
- So a typical number of clouds is $\sim 10^{-9} \times$ area
- $\sim$ 10 for a typical "grid box" of area (100km)^2

$\Rightarrow$ The number of clouds in a GCM grid-box is not large enough to produce a steady response to a steady forcing

e.g. Xu et al 1992; Shutts and Palmer 2004

Variability

- Let's retain the equilibrium assumption, which determines total cloud-base mass flux required on average, $\langle M \rangle$
- Want to describe variability arising from fluctuations about equilibrium
- Must consider the partitioning of $\langle M \rangle$ into individual clouds
- i.e., we will need the pdf for the mass flux m of a single cloud
- and the pdf for the number of clouds present

pdf for m

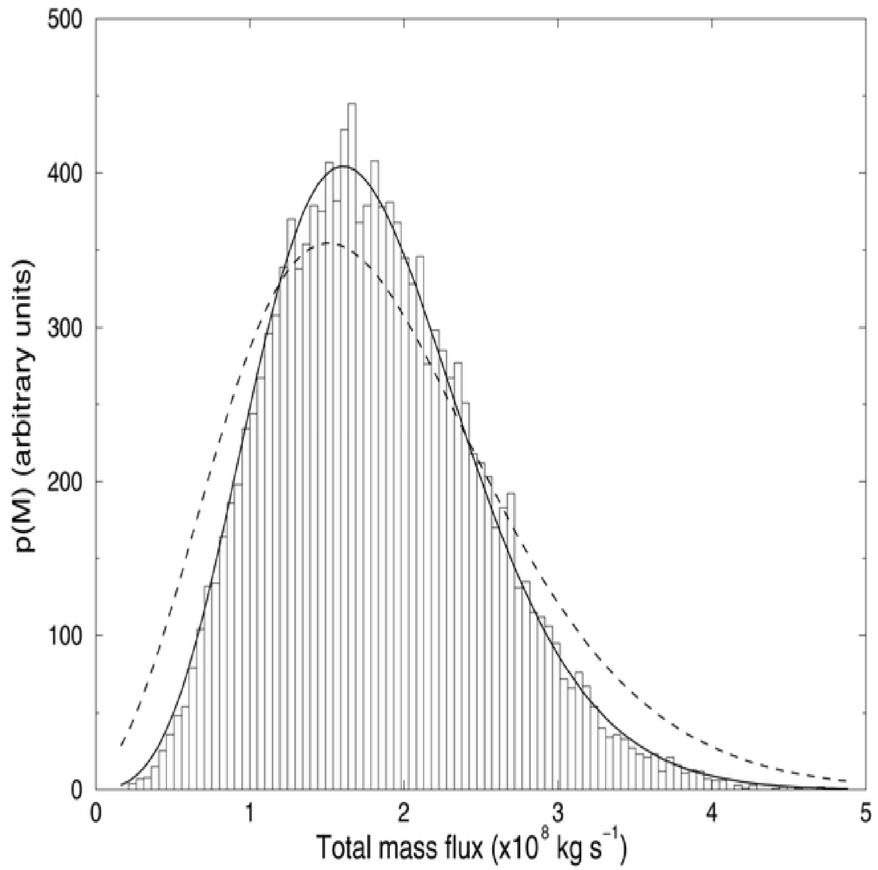
- Our assumptions about clouds as discrete, independent objects in a statistical equilibrium with a large-scale, macroscopic state are directly equivalent to those for an ideal gas
- So the pdf of m is a Boltzmann distribution

$$p(m)dm = \frac{1}{\langle m \rangle} \exp\left(\frac{-m}{\langle m \rangle}\right) dm$$

- Remarkably good and robust in CRM data

  Cohen and Craig 2006; Shutts and Palmer 2007; Plant and Craig 2008; Davies 2008; Davoudi et al 2010

pdf for M



- Number of clouds is not fixed, unlike number of gas particles
- If they are randomly distributed in space, number in a finite region given by Poisson distribution
- pdf of the total mass flux is a convolution of this with the Boltzmann

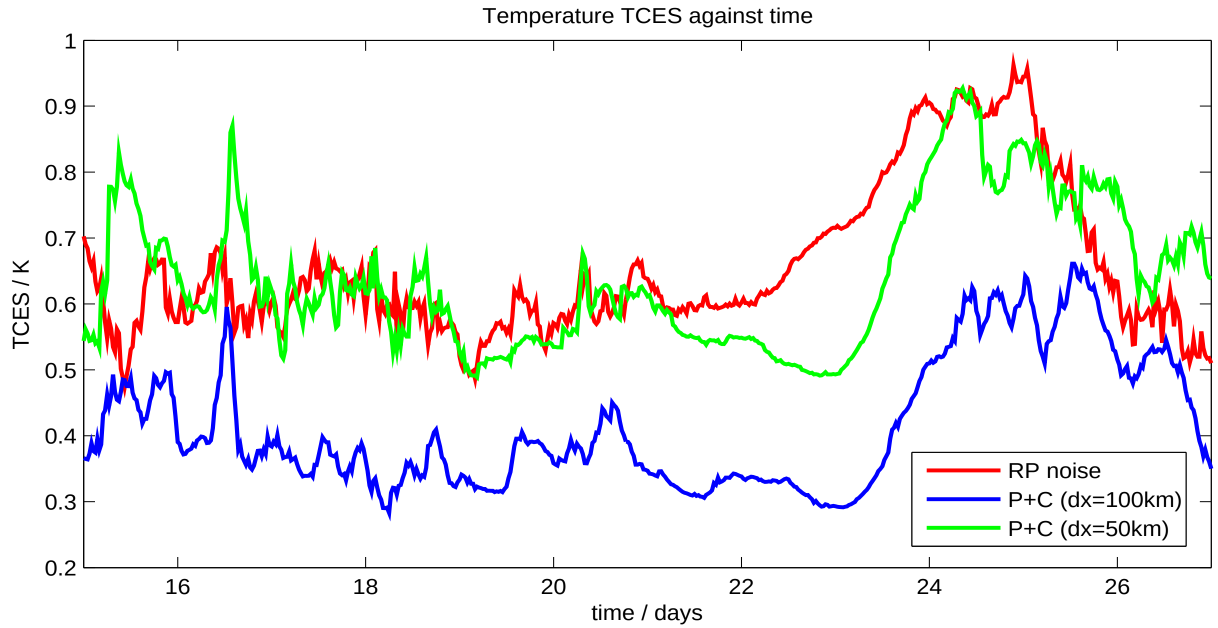
Stochastic parameterization

- Grid-box state $\neq$ large-scale state
  space average over $\Delta x \neq$ ensemble average
- We must parameterize convection on the grid-scale as being unpredictable, but randomly sampled from a known pdf dictated by the large-scale

Important note: **None** of these scales is fixed in a simulation!

Practical implementation

Single-column test with Plant-Craig (2008) parameterization



- Spread similar to random parameters or multiplicative noise for $\Delta x = 50$km
- Stochastic drift similar to changing between deterministic parameterizations

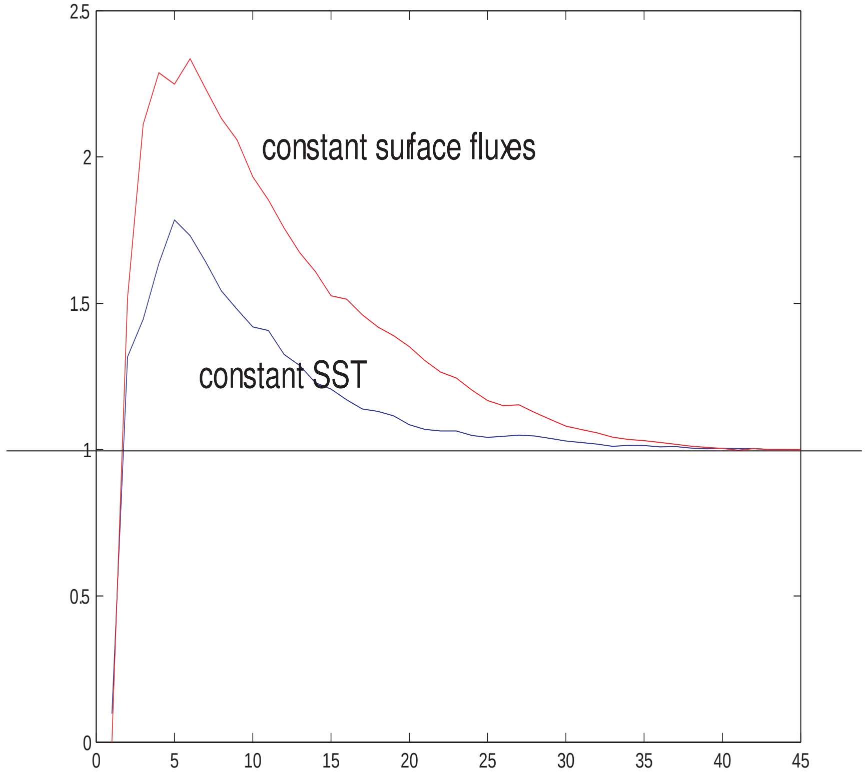
Concerning spatial correlations

Spatial correlation

- We have assumed the clouds to be independent and randomly distributed in space
- In reality, they can readily (self-)organize, even in a uniform environment with uniform forcing
- This will affect both the mean response and the variability, but can we account for it?

Non-random distribution

Consider $w_{12}(r)$, the expectation of finding a 2nd cloud a distance r from the 1st, normalized by that for a random spatial distribution



- Plot is in the equilibrium state of a CRM subject a forcing constant in space and time
- Self-organization with clumping at $\sim$ 10 to 20km

Non-ideal gas analogy

- Statistical mechanics of gas particles easily generalized to include weak interactions between them
- First order correction is to consider only pairwise interactions between particles
- Each cloud is subject to an effective interaction potential

$$V_{12}(r) = -\langle m \rangle \ln w_{12}(r)$$

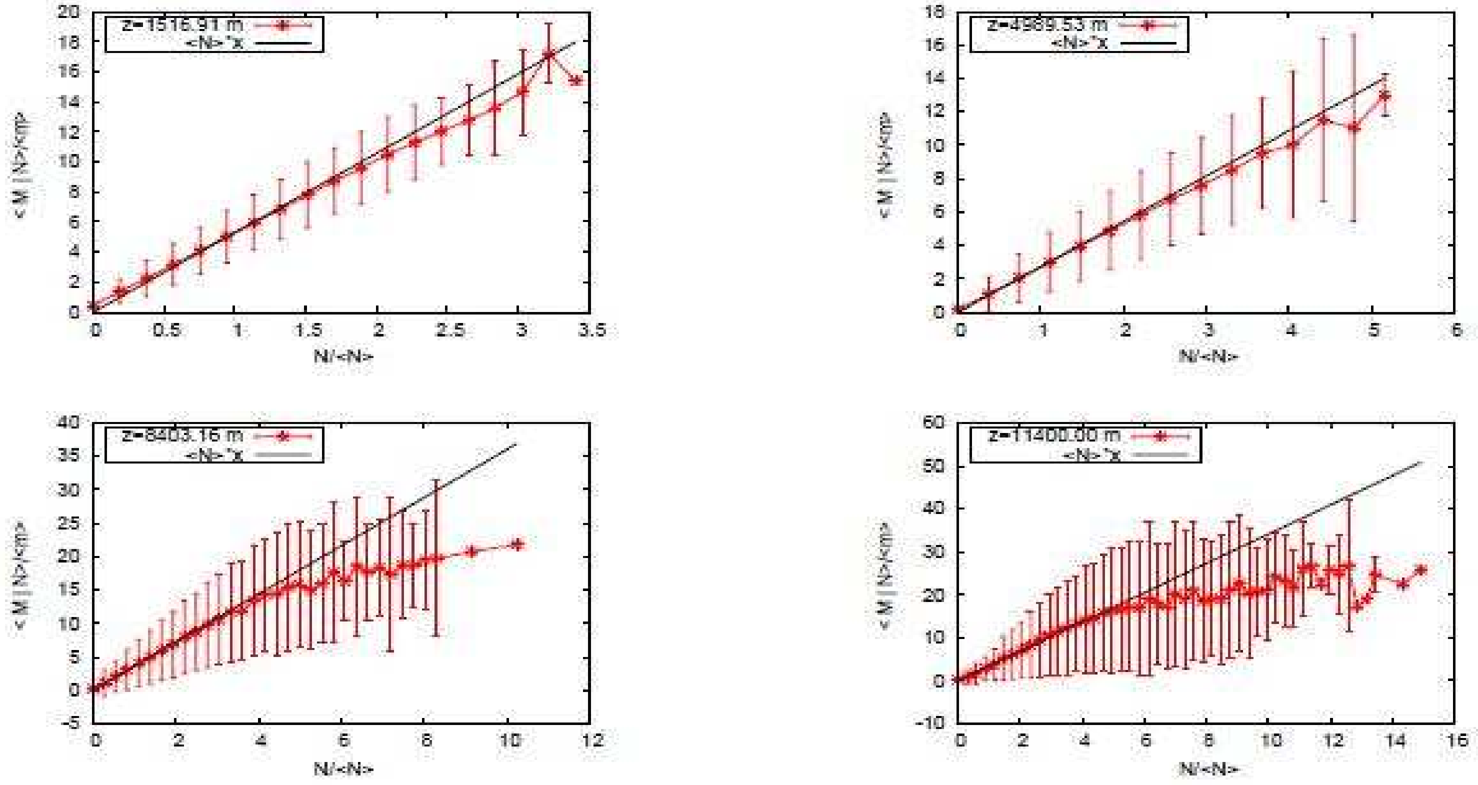
What to expect?

$$\langle M|N\rangle = N\langle m\rangle \left[1 - \frac{N}{2A}\int 2\pi r\,(w_{12}-1)\,dr\cdots\right]$$

- Integral is > 0 for clumping
- Deviations will be largest at large N/A

A hint?

CRM data from Davoudi 2008



$$\langle M|N\rangle = \langle m\rangle N$$

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Variance

$$\frac{\langle (\Delta N)^2 \rangle}{\langle N \rangle^2} = \frac{1}{\langle N \rangle} \left[1 + \frac{\langle N \rangle}{A} \int 2\pi r (w_{12} - 1) dr + \cdots \right]$$

- Enhanced number variance if clouds clump together
- Plant-Craig parameterization makes a moderate underestimate of the variability
- Could parameterize organization straightforwardly from CRM experiments designed to study w_{12}

Concerning time-dependence and equilibrium

Why consider time dependence?

- For relatively rapid forcings, we may wish to consider a prognostic equation for cloud-base mass flux
  Pan and Randall 1998; Piriou et al 2008
- Even for steady forcing, it is not obvious
  - that an equilibrium must be reached
  - which equilibrium might be reached

Time dependence

- Let B_i be a vertical integral of in-cloud buoyancy over the depth of cloud i
- After some algebra

$$\frac{dB_i}{dt} = F_i - \sum_j \gamma_{ij} M_j$$

where B, F and γ are all calculable given a cloud model

- Also, the convective kinetic energy equation is

$$\frac{dK_i}{dt} = B_i M_i - \frac{K_i}{\tau_D}$$

Closing this system

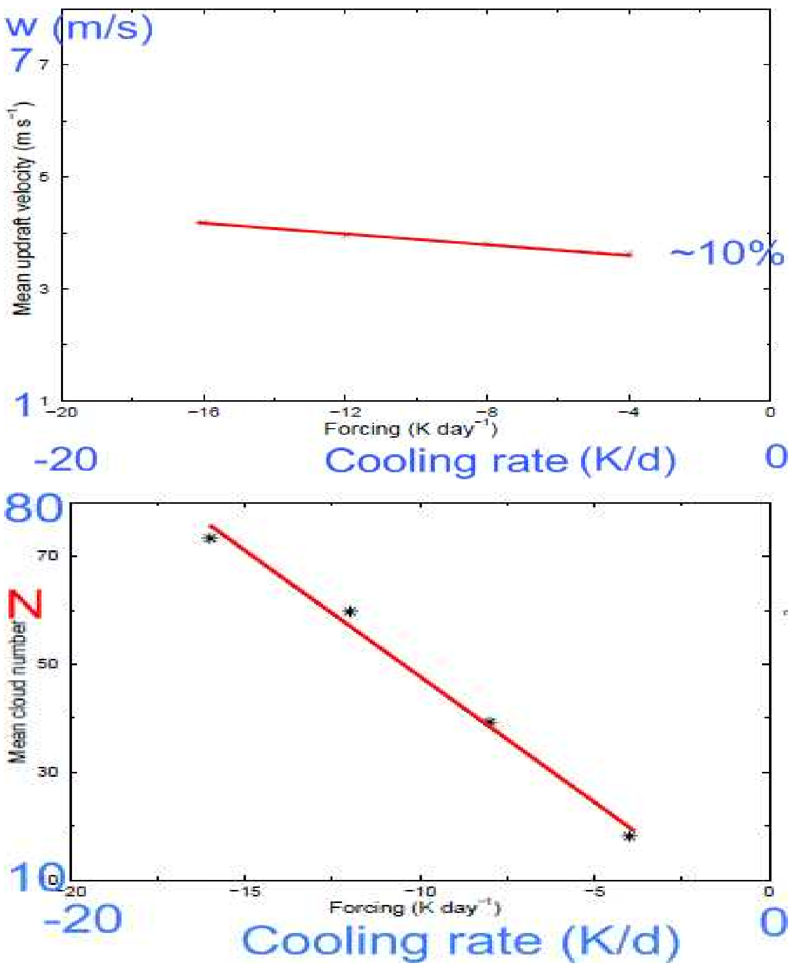
- Pan and Randall (1998) and others postulate

$$K_i \sim M_i^2$$

(Recall $K_i \sim \sigma_i w_i^2$ and $M_i = \rho\sigma_i w_i$)

- For a bulk system, the time dependence is a damped oscillator that approaches equilibrium after a few τ_D

But this is wrong!



Increased forcing linearly increases the mass flux, $\rho\sigma w$

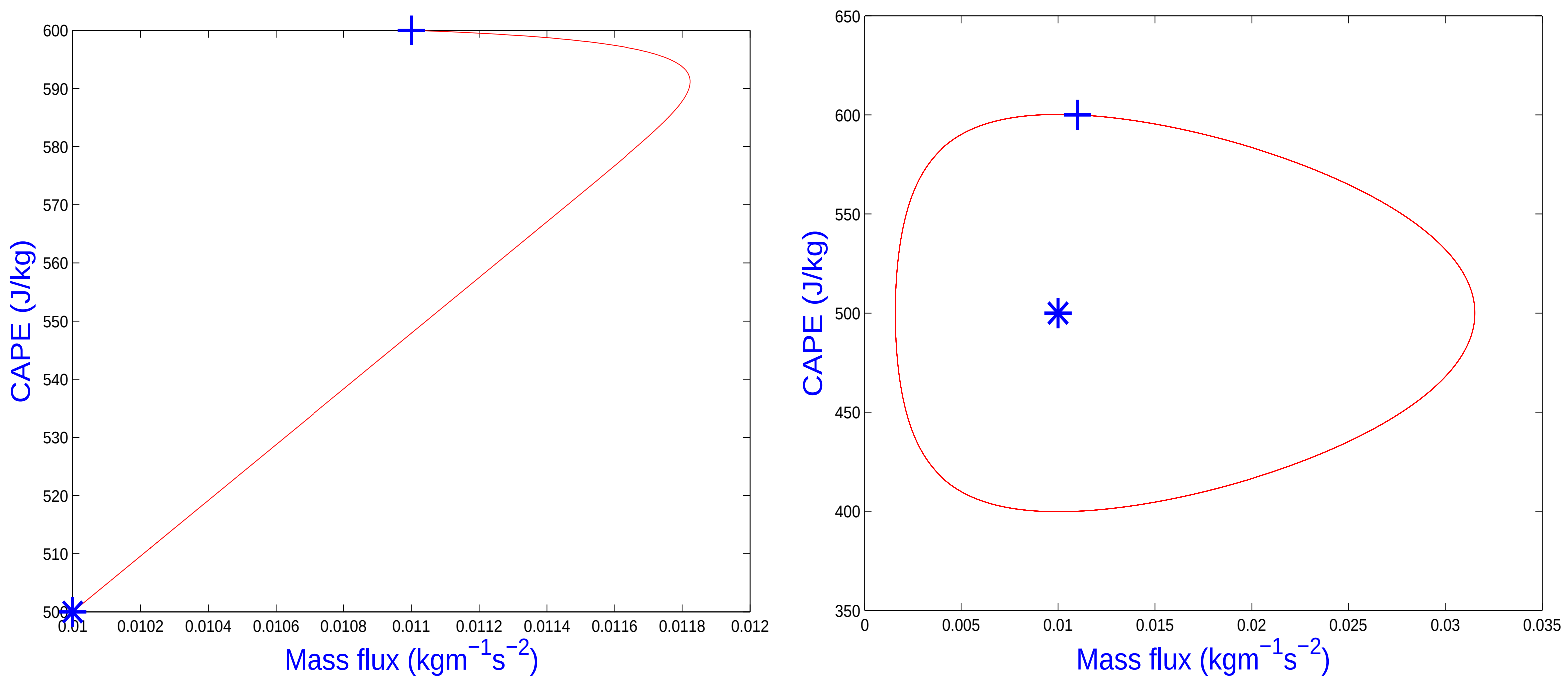
- achieved by increasing cloud number $\langle N \rangle$
- not the in-cloud velocities
- nor the sizes of clouds

Scalings and CRM data of Emanuel and Bister 1996; Robe and Emanuel 1996; Grant and Brown 1999; Cohen 2001; Parodi and Emanuel 2009

Closing the system

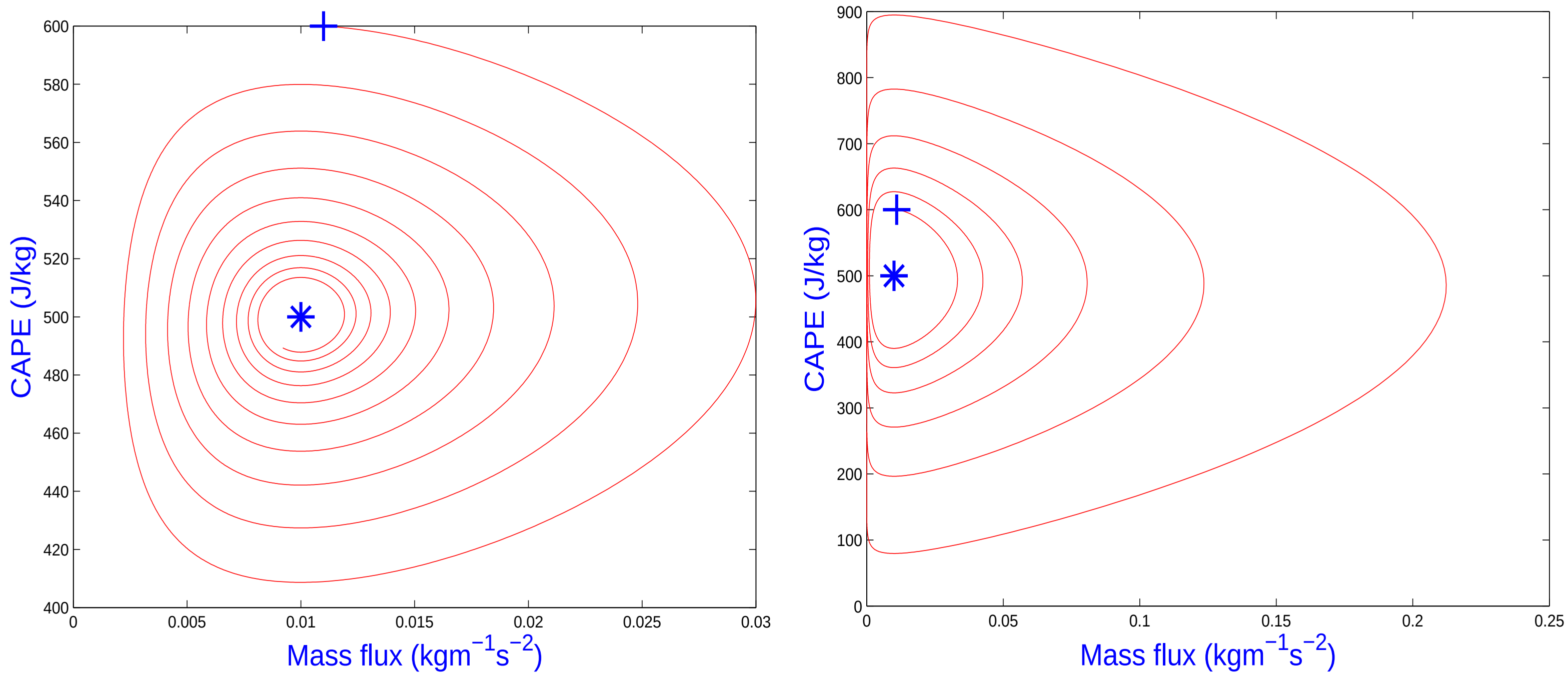
- To respect the above results, should choose $K_i \sim M_i$
- Let's consider $K_i \sim M_i^p$
- Results for a bulk system (one cloud type only)...

Illustrative results



$p = 2$ (left) and $p = 1$ (right) in $M - B$ space

Illustrative results



$p = 1.01$ (left) and $p = 0.99$ (right)

- The CRM data supports $p \approx 1$ but > 1
- Equilibrium is reached but more slowly as $p \longrightarrow 1$ from above

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Population Dynamics

- Truncate the prognostic system at dM_i/dt (neglect $(dM_i/dt)^2$ and d^2M_i/dt^2) and write $M_i = N_i m_i$ to get

$$(p-1)B_i\frac{dN_i}{dt} = F_i N_i - \sum_j \gamma_{ij} m_j N_i N_j$$

- For $p > 1$, a Lotka-Volterra (LV) system of biological populations competing for resource
- i.e., of cloud types competing to remove the instability
- extensively studied by mathematical ecologists

Simple Example

- Consider two cloud types, shallow cumulus and deep cumulus
- Described by different parameterization schemes in GCM
- Which one (or both?) to call typically based on ad hoc criteria
- Transitions between them are not well described or understood

Simple Example

- The LV system has a globally-stable equilibrium state with co-existing shallow and deep clouds if

$$\frac{\gamma_{11}}{\gamma_{12}} < \frac{F_1}{B_2} \quad \text{and} \quad \frac{\gamma_{12}}{\gamma_{22}} < \frac{F_2}{B_1}$$

- Otherwise one type will be driven to extinction
- So in equilibrium-based parameterization we should be using these criteria...

What is a useful framework beyond bulk models of large N, non-interacting, scale-separated, equilibrium systems?

Prognostic system for finite N?

- The prognostic systems above assumed infinite N_i
- Necessary for M_i to be continuous and dM_i/dt well defined
- How to generalize to finite N?

Methodology

- Construct an individual-based model with a difference equation for $P(\{N_i\},t)$ that evolves according to transition probabilities for births, deaths, competitive exclusion etc
- Choose the probabilities such that in the limit of large system size, we recover the deterministic ode's from before
- Leading correction for a non-infinite system is stochastic and accounts for fluctuations in N
- Explicit demonstration for the biological case has been done (McKane and Newman 2004)

  Straightforward to generalize to a lattice and modulate transition probabilities with w_{12}

Summary

- The archetypal convective parameterization is based on a bulk model of entraining/detraining plumes
- If grid boxes are not large, fluctuations about statistical equilibrium become important
- If cloud-cloud interactions are important, can account for them if we can say something about w_{12}
- Worthwile to ask which (or if) equilibrium is reached as this leads to useful constraints
- Proposed framework for a non-equilibrium, spatially-correlated, finite N model of cumulus
- Could be a useful intermediate system to study, sitting between CRM/observations and parameterization?