Predicting extremes in the midlatitudinal atmospheric circulation using regime-dependent statistical modelling

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Outline

1. Extreme events
2. Model system
3. Methodology
4. Results
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Extreme events in complex systems

- deterministic or stochastic dynamics
- irregular
- endogeneous
- stationary dynamics
- no bifurcations or tipping points
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Atmospheric low-order model

Barotropic flow over topography in $\beta$-plane channel:

\[
\begin{align*}
\dot{x}_1 &= \gamma_1^* x_3 - C(x_1 - x_1^*) \\
\dot{x}_2 &= -\alpha_1 x_1 x_3 + \beta_1 x_3 - Cx_2 - \delta_1 x_4 x_6 \\
\dot{x}_3 &= \alpha_1 x_1 x_2 - \beta_1 x_2 - \gamma_1 x_1 - Cx_3 + \delta_1 x_4 x_5 \\
\dot{x}_4 &= \gamma_2^* x_6 - C(x_4 - x_4^*) + \varepsilon (x_2 x_6 - x_3 x_5) \\
\dot{x}_5 &= -\alpha_2 x_1 x_6 + \beta_2 x_6 - Cx_5 - \delta_2 x_3 x_4 \\
\dot{x}_6 &= \alpha_2 x_1 x_5 - \beta_2 x_5 - \gamma_2 x_4 - Cx_6 + \delta_2 x_2 x_4
\end{align*}
\]

*Charney and DeVore 1979; DeSwart 1989; Crommelin et al. 2004*
Time series of first PC
Extreme values of $||x||$
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1. **Extreme events**
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Regime-dependent prediction model

Cluster-weighted modelling (Gershenfeld et al. 1999)

$$p(c^0, e^\tau_\alpha) = \sum_{k=1}^{K} w_k p(c^0|k) p(e^\tau_\alpha|y^0, c^0, k)$$

Predictive probability density:

$$p(e^\tau_\alpha|c^0) = \sum_{k=1}^{K} g_k(c^0) p(e^\tau_\alpha|c^0, k) \quad \text{with} \quad g_k(c^0) = p(k|c^0)$$

non-linear, non-Gaussian probabilistic modelling

parameter estimation with expectation-maximisation (EM) algorithm

relating to precursor patterns
Information content of forecasts for $b = 0.1$
Probabilistic skill scores

Brier score:

\[ Br = \sum_{\alpha} (f_{\alpha} - e_{\alpha})^2 \]

Ignorance score:

\[ \text{ign} = - \log f_{\alpha} \]
Forecast skill for $b = 0.1$
ROC curves for $b = 0.1$ and $\tau = 10, 30$
Model parameters

\[ b = 0.05, \quad \tau = 25, \quad K = 10: \]

\[ w_1 = 0.124, \quad \rho_1 = 0.340 \]
\[ w_2 = 0.073, \quad \rho_2 = 0.038 \]
\[ w_3 = 0.082, \quad \rho_3 = 0.021 \]
\[ w_4 = 0.104, \quad \rho_4 = 0.018 \]
\[ w_5 = 0.078, \quad \rho_5 = 0.010 \]
\[ w_6 = 0.126, \quad \rho_6 = 0.003 \]
\[ w_7 = 0.158, \quad \rho_7 = 0.000 \]
\[ w_8 = 0.103, \quad \rho_8 = 0.000 \]
\[ w_9 = 0.081, \quad \rho_9 = 0.000 \]
\[ w_{10} = 0.072, \quad \rho_{10} = 0.000 \]
ROC curves for different event rarity; $\tau = 30$, $K = 15$