Palaeoclimate advances @ INI: the dynamical system perspective

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with thanks to:
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Outline

Posing the problem

A case study: the Van der Pol oscillator

Conclusion
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Conclusion
The Pliocene - Pleistocene Earth History

-5 -4 -3 -2 -1 Present
closure of
tropical sea-ways
(Panama and Indonesia)
first major
glaciation of the NH
ardapithecus
Homo erectus
Homo sapiens
Changes in regime: the mid-Pleistocene

1. Orbital-scale African climate variability persisted throughout the entire interval, extending in some cases into the Miocene and Oligocene [52,53].

2. The onset of large-amplitude African aridity cycles was closely linked to the onset and amplification of high-latitude glacial cycles [15,30,31,54-57].

3. Eolian concentration and supply (flux) increased gradually after 2.8 Ma [30,31,54,55].

4. Step-like shifts in the amplitude and period of eolian variability occurred at 2.8 (±0.2) Ma, 1.7 (±0.1) Ma, and 1.0 (±0.2) Ma [15,30,31].

5. Evidence for 10⁴⁻¹⁰⁵ year ‘packets’ of high- and low-amplitude paleoclimatic variability which were paced by orbital eccentricity [15,30]. The marine record of African climate variability is perhaps best described as a succession of wet-dry cycles with a long-term shift toward drier conditions, punctuated by step-like shifts in characteristic periodicity and amplitude. Prior to 2.8 Ma subtropical African climate varied at the 23⁻¹⁹ kyr period (Figs. 2 and 3) which has been interpreted to reflect African monsoonal variability resulting from low-latitude (precessional) insolation forcing of monsoonal climate [15]. After 2.8 Ma, African climate varied primarily at

Fig. 3. Detail of terrigenous input (flux) variability at West African Site 659 and East African Site 721/722 spanning three intervals with differing patterns of eolian variability. The terrigenous (eolian) flux records (in units of g/cm²/kyr) were calculated at each site as the product of terrigenous percentage data (Fig. 2), and interval sedimentation rate and dry bulk density data [15,30,31,54,56]. Dashed curves in each panel show the combined 100 kyr and 41 kyr period bandpass filters of the terrigenous flux series to illustrate changes in the amplitude and dominant period of eolian supply during the Pliocene-Pleistocene. Note the predominance of precessional (23⁻¹⁹ kyr) variability for the 3.2⁻².8 Ma interval, whereas larger 41 kyr cycles dominate the 1.6⁻¹.2 Ma interval, and still larger 100 kyr and 41 kyr cycles dominate the 0.4⁻⁰ Ma interval (particularly at Site 721/722). Long-term trends towards increased eolian flux variability are most evident at Site 721/722, although they are also apparent at West African Sites 659, 662, and 664 [15,30,31,54,56].
Changes in regime: the mid-Brünhes

Luethi et al., Nature, 2008 (EPICA and Vostok projects)
The Last Glacial: Dansgaard-Oeschger Events

- First migration of fully modern humans out of Africa
- Aborigines arrive in Australia
- Migrations of fully modern humans from South Asia to Europe
- Beginning of agriculture
- Great European civilisations: Greek, Roman
Climate records as climate sensors

Lang and Wolff,
www.clim-past-discuss.net/6/2223/2010/
Model = Forcing – Dynamical System – Sensors + Priors

The problem
Identify – Calibrate – Select – Analyse

SUPRANET paper, C. Buck, T. Edwards et al., in prep
Stochastic dynamical systems to generate palaeoclimate histories

\[ x(t) := (x_1(t), x_2(t), \ldots) \]

\[ \tau_1 \frac{dx_1}{dt} = f_1(x_1, x_2, \ldots, t; \theta_1) \]

\[ \tau_2 \frac{dx_2}{dt} = f_2(x_1, x_2, \ldots, t; \theta_2) \]

\[ \vdots \]

\[ \tau_n \frac{dx_n}{dt} = f_n(x_1, x_2, \ldots, t; \theta_n) + \dot{\omega} \]

with \( \tau_n \ll \tau_1 \).

Goal: Keep slow modes only.

See Majda, Franzke and Crommelin, PNAS, 2009 for a modern view of this theory, and Saltzman and Maasch, 1991 for application in the palaeoclimate context.
Stochastic dynamical systems to generate palaeoclimate histories $x(t)$

Generate a **very complex** joint structure between the $x(t_i)$

**example**

3 climate histories generated with the Saltzman and Maasch 1991 model

Further assumptions needed to generate potential palaeoclimate records

\[ \text{climate} \leftrightarrow \text{record} \quad \text{i.e.} \quad \{ x(\cdot), \phi \} \} \leftrightarrow y[\cdot] \]

**Example**

\[
\begin{align*}
d[i] &= -\phi_1 t[i] \quad \text{(depth)} \\
y[i] &= \phi_2 x(d[i]) + \phi_3 \epsilon_i, \quad \epsilon_i \sim \text{Normal i.i.d.}
\end{align*}
\]

3 climate records generated with the Saltzman and Maash 1991 model

... and some further assumptions
You will never exactly match all observations.

Another layer is needed between the model and reality. Assumptions about discrepancies may be introduced explicitly at different levels of the model:

1. in the stochastic parametrisations (but, pay attention)
2. in the likelihood function

or, implicitly, as in Approximate Bayesian Computations.

**Example**

“model discrepancies at different time steps are Normal i.i.d with covariance $\Sigma$”
We need to think hard about what we want to learn

- dynamical structure
- changes in regimes
- the effect of the astronomical forcing
- ‘noise’ structure
- …

**Warning:**
We have to pay attention about how we formulate discrepancy, because once adopted they influence subsequent inferences, including model selection.
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A modified, stochastic van der Pol oscillator

‘normal form’ for many conceptual palaeoclimate models

\[ \begin{align*}
x & = (x, y) \\
\tau \dot{x} & = -(y + \beta + \gamma F(t)) \\
\frac{\tau}{\alpha} \dot{y} & = -([y^3 - y/3] - x) + \sigma \omega
\end{align*} \]
Effect of stochastic perturbations on astro-forced oscillator

relaxation oscillator as palaeoclimate model

\[ \alpha = 30, \quad \beta = 0.75, \quad \gamma = 0.4 \quad \text{and} \quad \tau = 36 \text{ ka} \]
Hand-Fit on observations (deterministic)

relaxation oscillator as palaeoclimate model

\[
\begin{align*}
\alpha &= 30, \quad \beta = 0.75, \quad \gamma = 0.4 \text{ and } \\
\tau &= 36 \text{ ka}
\end{align*}
\]
Further analysis: synchronisation on astronomical forcing

Multi-frequency astronomical forcing

Insolation on summer solstice at 65° N

\[ \sum a_i \sin(\omega_i t + \phi_i) \]

Obliquity terms
Precession terms

Applies to the deterministic system driven by astronomical forcing. More in NI10044-CLP INI preprint series.
Further analysis: synchronisation on astronomical forcing

Multi-frequency astronomical forcing

Complex synchronisation structure

Applies to the deterministic system driven by astronomical forcing. More in NI10044-CLP INI preprint series.
Calibrate: update priors on parameters by observations

$$\Pr(\Theta = \theta | Z) = \frac{\Pr(Z = z | \Theta = \theta) \Pr(\Theta = \theta)}{\Pr(Z = z)}$$
Van der Pol likelihood surface

The difficulty of parameter estimation can be explained by looking at the likelihood surface for \((\tau_1, \alpha)\) given the true initial conditions.
Van der Pol likelihood surface

The difficulty of parameter estimation can be explained by looking at the likelihood surface for \((\tau_1, \alpha)\) given the true initial conditions.

"I simply have no idea why one would want to fit a simple deterministic dynamical model to real data.” (J. Rougier)
Particle Filter: the general idea

Particle filter, one timestep

Swarm at time t-1  Propagate to time t  Reweight by likelihood  Resample to cull low-weights

\[ x_{t-1}^{(i)} \sim x_t^{(i)} \sim \pi_t(x_{t-1}^{(i)}) \]

\[ w_i \propto L_t(x_t^{(i)}) \]

\[ w_i \text{ rebalanced} \]

\( w = 0.2 \)
\( w = 0.2 \)
\( w = 0.2 \)
\( w = 0.2 \)
\( w = 0.2 \)

\( w = 0.05 \)
\( w = 0.50 \)
\( w = 0.40 \)
\( w = 0.03 \)
\( w = 0.02 \)

\( w = 0.2 \)
\( w = 0.2, 0.2 \)
\( w = 0.2, 0.2 \)
\( w = 0.2, 0.2 \)
\( \dagger \)
\( \dagger \)
Method #1: Particle Filter for joined state and parameter estimation

(Liu and West, 2001 for theory; Crucifix and Rougier 2009 for application)
Failure to capture ‘slow dynamics’ in future climate simulations

Further notes: tutorial by Jonty Rougier during INI
Method #2: Approximate Bayesian Computation methods

ABC - Van der Pol Oscillator

Rather than use the entire time-series of data $d_0:T$ we use a couple of summaries:

- $C = (C_x, C_y)$ number of crossings of zero by the $x$ and $y$ trajectories
- $x_0$ the initial condition.

For the observed values of these summaries we can count 17 crossings for both $x$ and $y$, and we take the first observation as the true value of $x_0$. For the metric I tried a simple weighted absolute difference

$$\rho(S, S') = \sum |C_\alpha - C'_\alpha| + 10|x_0 - x'_0|$$

where the primes indicate simulated values. The factor of 10 is there to give both components roughly equal importance.

- With exploration and better understanding, better choices could be made - for example by considering the error on $x_0$
After $10^5$ model evaluations we find we accept 191 runs when $\delta = 1$, and 1151 runs for $\delta = 2$. Plotting the posteriors for $\delta = 2$ we find:

The posterior mode is close to every true parameter (red = true, green = prior).

further notes: talk by Wilkinson during INI and Wilkinson (2008)
Method #3: Particle Marginal Metropolis Hastings

PMMH = Particle Marginal Metropolis-Hastings.

1. Random walk in the parameter space, $q(\theta \to \theta')$;
2. Use the particle filter to propose $x' \sim \Pr(x | z, \theta')$, and to approximate the marginal likelihood $\hat{p}(\theta') \approx \Pr(z | \theta')$;
3. Accept or reject $\{\theta', x', \hat{p}(\theta')\}$ according to

$$\frac{\hat{p}(\theta') \Pr(\theta') q(\theta' \to \theta)}{\hat{p}(\theta) \Pr(\theta) q(\theta \to \theta')}$$

The stationary distribution of this chain is $\Pr(\theta, x | z)$, even though $N$, the number of particles, may be small.

Caveat

In practice one has to use every trick in the book to make this work efficiently on a single machine (e.g. my four-year-old laptop).

(thanks to Jonty)
PMMH seems to be working in a simple problem

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Summary

- We use a dynamical skeleton to model palæoclimatic dynamics.
- This dynamical skeleton needs to be linked to palæoclimatic data within a Bayesian framework.
- Calibration is not simple. Methods may vary depending on goals.
- Newton Institute was very useful:
  - 'think-tank' on model discrepancy, likelihood, statistical summaries
  - intercomparison exercise
  - towards concrete applications
  - datation models
More steps

- Model design
  - link dynamical system structure with simulators
  - consider additional time-scale interactions (e.g.: Dansgaard-Oeshger events)
  - consider dating uncertainties (e.g.: sedimentation model)
  - specific applications (carbonate cycle: R. Rickabbby)

- Model calibration and identification
  - Polish intercomparison
  - Consider further statistical summaries (e.g.: wavelet-based)
The bibliography


