Bayesian estimation of the climate sensitivity based on a simple climate model fitted to global temperature observations

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Work in progress, preliminary results
Climate sensitivity $S$

Definition:

Climate sensitivity $= S$
$= \text{The temperature increase due to a doubling of } CO_2 \text{ concentrations compared to pre-industrial time (1750)}$
Radiative forcing

- $CO_2$ is only one of several factors that affect the global temperature
- Radiative forcing = The change in net irradiance into the earth relative to 1750
- Measured in Watts per square meter
- The global temperature depends on the radiative forcing
- The climate sensitivity measures the strength of this dependency
Aim of study

To estimate the climate sensitivity

• by modelling the relationship between
  ★ estimates of radiative forcing since 1750 and
  ★ estimates of global temperature
    based on measurements since 1850
• using a climate model based on physical laws
Climate model

Could use

• an Atmospheric Ocean General Circulation Model, but complex and very computer intensive
• an approximation to an AOGCM, an emulator based on Gaussian processes
• a simple climate model, our approach
The “true” global state of the earth in year $t$

- $TNH_t$ - Temperature at the northern hemisphere
- $TSH_t$ - Temperature at the southern hemisphere
- $OHC_t$ - Ocean heat content
Simple climate model

- Deterministic computer model (Schlesinger et al., 1992)
- based on
  - energy balance
  - upwelling diffusion ocean
- where the earth is divided into
  - atmosphere and ocean
  - northern and southern hemisphere
- with
  - radiative forcing into the system
  - energy mixing
    - between the atmosphere and the ocean
    - within the ocean
Simple climate model cont.

\[ m_t(x_{1750:t}, S, \theta) \]

- Yearly time resolution
- Output
  - temperature northern hemisphere
  - temperature southern hemisphere
  - ocean heat content
- Input
  - \( x_{1750:t} \) - yearly radiative forcing from 1750 until year \( t \), separate for northern and southern hemisphere
  - \( S \) - the climate sensitivity, the parameter of interest
  - \( \theta \) - 6-dimensional vector of other model parameters
Response data

- $y_t$ - 7-dimensional vector with yearly observed temperatures and ocean heat content
- Three pairs of series with temperature measurements for northern and southern hemisphere
  - 1850-2007 (HadCRUT3, Brohan et al., 2006)
  - 1880-2007 (GISS, Hansen et al. 2006)
  - 1880-2007 (NCDC, Smith and Reynolds 2005)
- One series with ocean heat content measurements
- These observations are not the truth, but are estimates of the underlying “true” global state of the earth
- $s_t$ - 7-dimensional vector of corresponding standard errors
Observations

Observed temperatures, northern hemisphere
- NH1, HadCRUT3
- NH2, GISS
- NH3, NCDC

Observed temperatures, southern hemisphere
- SH1, HadCRUT3
- SH2, GISS
- SH3, NCDC

Observed global ocean heat content

1850 1900 1950 2000
−0.5 0.0 0.5 1.0
Temp [°C]

1850 1900 1950 2000
−5 0 5 10
Ocean heat content [10^22 J]

1850 1900 1950 2000
−5 0 5 10
Ocean heat content [10^22 J]
Standard errors

Standard error of observed temperatures, northern hemisphere

Standard error of observed temperatures, southern hemisphere

Standard error of observed global ocean heat content
Radiative forcing

- We will specify our best knowledge about historical radiative forcing as prior distributions of 9 independent components, based on temperature-independent estimates of each component, including uncertainties
  - long-lived greenhouse gases
  - direct aerosols
  - indirect aerosols
  - solar radiation
  - volcanoes
  - land use
  - tropospheric ozone
  - stratospheric ozone
  - stratospheric $H_2O$
Priors of components of radiative forcing

LLGHG NH
Radiative forcing [W/m²]

LLGHG SH
Radiative forcing [W/m²]

Sun NH
Radiative forcing [W/m²]

Sun SH
Radiative forcing [W/m²]

Volcanos NH
Radiative forcing [W/m²]

Volcanos SH
Radiative forcing [W/m²]
Prior of total radiative forcing

Radiative forcing [W/m²]

- Mean
- 95% credibility interval

radiative forcing [W/m²]

-4 -2 0 2

1750 1800 1850 1900 1950 2000
Model for “true” global state of the earth

\[ g_t = (TNH_t, TSH_t, OHC_t)^T \]

Combined deterministic + stochastic model

\[ g_t = m_t(x_{t:1750}, S, \theta) + n_t^m \]

- \( n_t^m \): model error, dimension 3
Model for observations

\[ y_t = A g_t + \beta_0 + n_t^o \]

- **A**: 7x3 matrix copying the northern and southern temperatures 3 times, to compare model with observations
- **\( \beta_0 \)**: intercept, accounts for different reference periods, dimension 7
- **\( n_t^o \)**: observational (measurement) error, dimension 7
- Can be difficult to separate model error and observational error, i.e. careful with too strict interpretation
We assume $n_t^m$ is VAR(1) (vector autoregressive process of order 1)

$$n_t^m = \Phi^m n_{t-1}^m + \varepsilon_t^m$$

$\Phi^m$ is diagonal

$$\varepsilon_t^m \sim N(0, \Sigma^m)$$

elements of $\varepsilon_t^m$ are correlated
Observational error

- We assume $n_t^o$ is a scaled VAR(1)

$$n_t^o = diag(s_t)n_t^{o*}$$

where $s_t$ is the vector of known observational standard errors and $n_t^{o*}$ is VAR(1)
Estimation

- Bayesian approach (Kennedy and O’Hagan 2001), using MCMC
- Vague prior for $S$
- Informative priors for $x_{t:1750}$ and $\theta$
- Vague priors for other parameters
Observed and fitted response values
Posterior vs. prior of radiative forcing

![Plot showing the comparison of prior and posterior means and 95% credibility intervals for radiative forcing over time from 1750 to 2000. The graph displays the variation in radiative forcing with time, indicating how prior estimates have been updated to posterior estimates.](image-url)
Model and observational errors

- Positive autocorrelations - $\phi$’s between 0.5 and 0.7
- Positive correlations between temperature errors
- Temperature errors and ocean heat content errors are uncorrelated
Posterior of the climate sensitivity $S$

![Graph showing posterior distribution of climate sensitivity with main analysis and other analysis options.]

Temperatures, northern hemisphere, NH1, HadCRUT3

- Predicted
- Observed
- Fitted

Temperatures, southern hemisphere, SH1, HadCRUT3

- Predicted
- Observed
- Fitted

Global ocean heat content

- Predicted
- Observed
- Fitted

Ocean heat content [10^22 J]
Posteriors for reduced data

a) Main analysis

b) Data up to 1990

c) Data up to 2000

d) Gauss posterior

e) Wid posterior

f) θPan and θUV constraint

g) Common st.err. profiles

h) Including cloud lifetime = −0.25

i) Including cloud lifetime = −0.5

j) S−prior based on Hegerl's PDF

k) S−prior based on combined PDF

l) Only OHC data

m) Only temperature data

Probability density

Climate sensitivity [ °C ]
Posteriors for only temperature or OHC

![Graphs showing posterior distributions for different scenarios.]

- **a) Main analysis**
- **l) Only OHC data**
- **m) Only temperature data**
- **j) S-prior based on Hegerl’s PDF**
- **k) S-prior based on combined PDF**
- **i) Including cloud lifetime = −0.5**
- **g) Common st.err. profiles**
- **c) Data up to 2000**
- **b) Data up to 1990**
• Aerosols change the lifetime of clouds
• Cloud lifetime is *not* included a radiative forcing by IPCC (2007)
• But plays a similar role in our approach
Further work

- Include the cloud lifetime effect with uncertainty
- Update priors of forcing,
  more precise estimates of historical forcing will be available soon
- Include data from 2008 and 2009
Thank you for your attention!
Other approaches i)

Tomassini, Reichert, Kunsch, Buser, Knutti and Borsuk (2009) in Applied Statistics

\[ y_t = m_t(x_t;1750, S, \theta) + n_t^o \]

- Similar model \( m_t \)
- Only one temperature series
- No autocorrelation in observational error \( n_t^o \) for temperature
- No intercept
- No model error term, all model error due to error in forcing
- No informative prior for forcing

\[ x_t = \mu_t + \phi_t = \text{mean} + \text{random} \]

- Medium complex climate model - computer intensive
- Emulator
- Response is aggregated temperature data
  - mean(1946-1955) - mean(1905-1995),
  mean(1956-1965) - mean(1905-1995),
  mean(1966-1975) - mean(1905-1995),
  mean(1976-1985) - mean(1905-1995),
- Earth divided into four zonal bands
- 20 dimensional vector

\[ S = 3.71 \cdot \Delta T / (\Delta RF - \Delta OHC) \]

- \( \Delta T \) = mean temp 1957-1994 - mean temp 1861-1900
- \( \Delta RF \) = mean RF 1957-1994 - mean RF 1861-1900
- \( \Delta OHC \) = mean OHC 1957-1994 - mean OHC 1861-1900